OPTIMAL DESIGN FOR EXPERIMENTS WITH POSSIBLY INCOMPLETE OBSERVATIONS

Kim May Lee, Stefanie Biedermann and Robin Mitra

University of Southampton, UK

Supplementary Material

This supplement discusses Theorem 3 in the main paper.

S1 Discussion for Theorem 3.

We consider the restricted range $2/n \leq w_1 \leq 1 - 2/n$. For fixed but arbitrary n, denote the second order approximation to $E[1/Z_i]$ from (3.6) by $f(w_i, P)$, $i = 1, 2$, and $w_2 = 1 - w_1$.

For the D-objective function, consider $f(w_1, P)f(1 - w_1, P)$. Taking derivatives with respect to w_1 and setting equal to zero yields

$$f(1 - w_1^*, P)\frac{\partial}{\partial w_1}f(w_1^*, P) = f(w_1^*, P)\frac{\partial}{\partial w_1}f(1 - w_1^*, P)$$

which is solved by $w_1^* = 1/2$.

Similarly, for the c-objective function, consider $f(w_1, P) + f(1 - w_1, P)$. After taking derivatives with respect to w_1 and setting equal to zero, we find that again the value $w_1^* = 1/2$ satisfies the resulting equation

$$\frac{\partial}{\partial w_1} f(w_1^*, P) = \frac{\partial}{\partial w_1} f(1 - w_1^*, P).$$

So for both optimality criteria, $w_1 = 1/2$ is a critical point. However, the objective functions are not generally convex in w_1. Figure 1 shows the typical behaviour of the c-criterion as a function of w_1, for fixed $n = 20$ and various choices of P, where $0.1 \leq w_1 \leq 0.9$ to ensure at least two runs in each support point. As n increases, the values of P where the shape of the

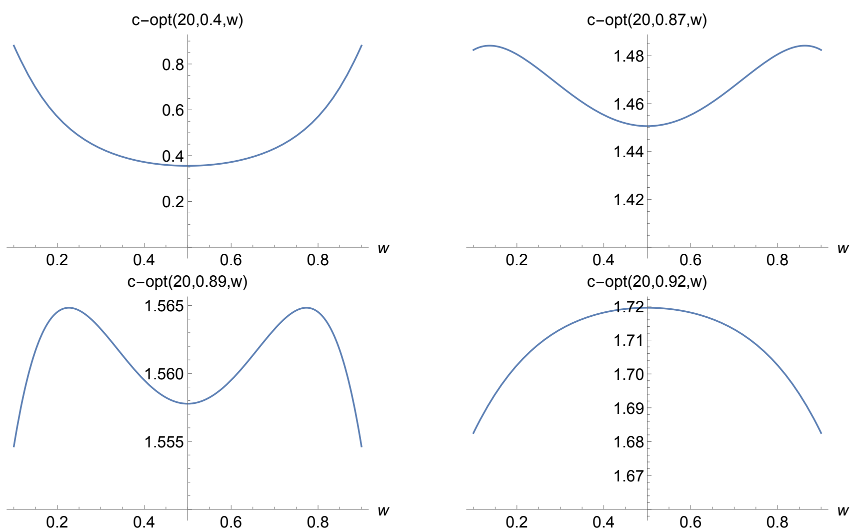

Figure 1: Top left: For small to moderate P, $w_1 = 1/2$ is the only turning point, and is the local and global minimum point. Top right: As P increases, two local maxima emerge while $w_1 = 1/2$ still is a local and the global minimum point. Bottom left: As P increases further, $w_1 = 1/2$ still is a local but no longer the global minimum point. Bottom right: For very large P, $w_1 = 1/2$ is the local and global maximum point.

objective function changes, will also be larger. For various values of n, we have found the respective largest possible values of P such that $w_1 = 1/2$ is still the global minimum point (where $2/n \leq w_1 \leq 1 - 2/n$). These points, together with the function $P(n) = 1 - 2/n$ for comparison, are depicted in Figure 2. While this function is not a perfect fit through the points, it can be used as a guideline.

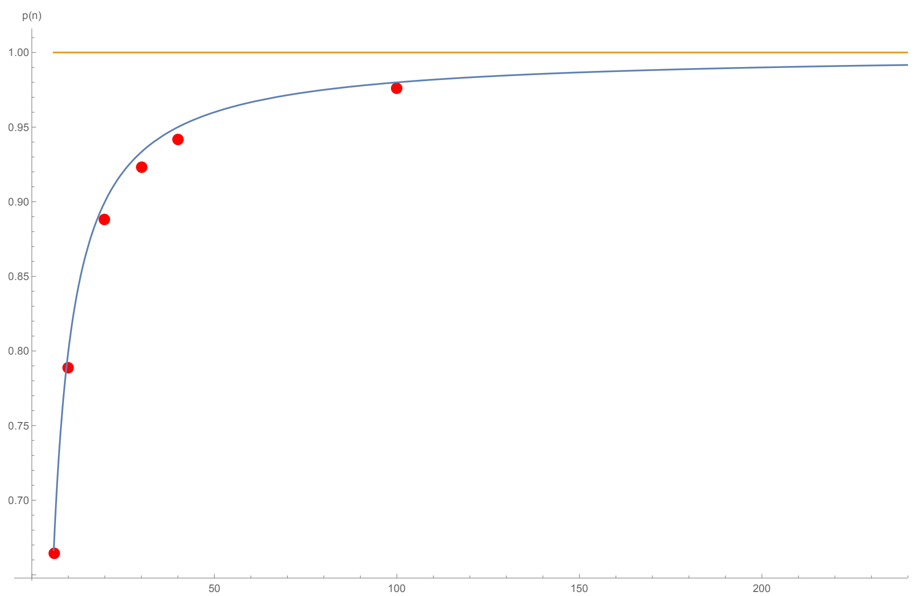

Figure 2: Dots: Largest values of P (given n) such that $w = 1/2$ is the global minimum point. Continuous line: $P(n) = 1 - 2/n$.

For D-optimality, the objective function has the same shape and behaviour as the c-objective function, but the largest values of P that guarantee the global minimum to occur at $w_1 = 1/2$ are slightly smaller. For this criterion, the function $P(n) = 1 - 2/n^{0.8}$ turns out to be a good approximation to the upper bound.