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POISSON APPROXIMATION FOR UNBOUNDED FUNCTIONS, I: INDEPENDENT SUMMANDS

A. D. Barbour*, Louis H. Y. Chen and K. P. Choi

Universität Zürich and National University of Singapore*

Abstract. Let $X_{n1},\dots,X_{nn}, n \geq 1$, be independent random variables with $P(X_{ni} = 1) = 1 - P(X_{ni} = 0) = p_{ni}$ such that $\max\{p_{ni} : 1 \leq i \leq n\} \to 0$ as $n \to \infty$. Let $W_n = \sum_{1\leq k\leq n} X_{nk}$ and Z be a Poisson random variable with mean $\lambda = EW_n$. We obtain an absolute constant bound on $P(W_n = r)/P(Z = r), r = 0,1,\dots$, and using this, prove two Poisson approximation theorems for $Eh(W_n)$ with h unbounded and λ unrestricted. One of the theorems is then applied to obtain a large deviation result concerning $Eh(W_n)I(W_n \geq z)$ for a general class of functions h and again with λ unrestricted. The theorem is also applied to obtain an asymptotic result concerning $\sum_{r=0}^{\infty} h((r-\lambda)/\sqrt{\lambda})|P(W_n = r) - P(Z = r)|$ for large λ.

Key words and phrases: Poisson approximation, unbounded functions, large deviations, asymptotics, Stein's method.

1. Introduction

Let $X_{n1},\dots,X_{nn}, n \geq 1$, be a triangular array of independent Bernoulli random variables with $P(X_{ni} = 1) = p_{ni}$ such that $\tilde{p}_n = \max\{p_{ni} : 1 \leq i \leq n\} \to 0$ as $n \to \infty$. Let $W_n = \sum_{1\leq k\leq n} X_{nk}$ and Z be a Poisson random variable with mean $\lambda = EW_n$. Approximating $Eh(W_n)$ by $Eh(Z)$ for unbounded functions h dates back to Simons and Johnson (1971) who proved that in the case all the p_{ni}'s are equal,

$$d(h, W_n, Z) = \sum_{r=0}^{\infty} h(r)|P(W_n = r) - P(Z = r)| \to 0$$

as $n \to \infty$, provided that $h \geq 0$ with $Eh(Z) < \infty$ and λ remains fixed. The result of Simons and Johnson (1971) was generalized by Chen (1974) to the case where the p_{ni}'s are not necessarily equal, and further generalized by Chen (1975b) to convolutions of probability measures on a measurable Abelian group. Chen (1975b) also obtained a bound on the rate of convergence. The result of Chen (1974) was also generalized by Wang (1991), though Wang's result is in fact a special case of Theorem 3.1 in Chen (1975b).

A crucial step in Chen (1975b) is finding an explicit bound on the Radon-Nikodym derivative which, in the context of W_n, is $P(W_n = r)/P(Z = r), r = 0, 1, 2, \ldots$. The method of using such a bound in the context of Poisson approximation was developed in Chen and Choi (1992) to obtain asymptotic results concerning $d(h, W_n, Z)$ for small and moderate λ, and also a large deviation result concerning $Eh(W_n)I(W_n \geq z)$ for h, a polynomial. Although Barbour (1987) also considered Poisson approximation for unbounded functions, the approach of Chen and Choi (1992) is different and holds promise for successful application in the case of dependent indicators: these possibilities are to be explored in Part II. However, in Chen and Choi (1992), the bound on $P(W_n = r)/P(Z = r)$ depends on λ, and as a result λ had to be assumed to be bounded.

The main objective of this paper is to obtain an absolute constant bound on $P(W_n = r)/P(Z = r)$, and using this, prove two Poisson approximation theorems (Theorems 3.1 and 3.2) for $Eh(W_n)$ with h unbounded and λ unrestricted. Theorem 3.2 is then applied to obtain a large deviation result concerning $Eh(W_n)I(W_n \geq z)$ for a general class of functions h, again with λ unrestricted, thus generalizing the large deviation result of Chen and Choi (1992). Finally, Theorem 3.2 is also applied to obtain an asymptotic result concerning $\sum_{r=0}^{\infty} h((r-\lambda)/\sqrt{\lambda})|P(W_n = r) - P(Z = r)|$ for large λ. This result complements the asymptotic results of Chen and Choi (1992), and generalizes results of Barbour and Hall (1984) and of Deheuvels and Pfeifer (1986). See also Barbour and Jensen (1989) and Deheuvels (1992) for other approaches to the approximation of tail probabilities in this setting.

Although the two approximation theorems are proved for unbounded functions h, the error bounds are more refined than the usual total variation bounds even in the case when h is bounded. This is because the function h is reflected in the error bounds, and as a result these bounds are always relatively small compared to $Eh(W_n)$ or the approximating $Eh(Z)$. This is not the case for total variation bounds.

From now on, we abbreviate $X_{ni}, p_{ni}, \tilde{p}_n$ and W_n to $X_i, p_i, \tilde{p}$ and W respectively. We also let $W^{(i)} = W - X_i$. For any real-valued function h defined on the set of nonnegative integers $\boldsymbol{Z}^+$ such that $E|h(Z)| < \infty$, let $U_\lambda h$ denote a solution to the difference equation $\lambda f(w+1) - wf(w) = h(w) - Eh(Z), w \in \boldsymbol{Z}^+$. We note that $U_\lambda h$ is uniquely defined except at $w = 0$ and that the value of $U_\lambda h$ at $w = 0$ does not enter into our calculations at all. For $w \geq 1$, $U_\lambda h(w) = -E[h(Z) - Eh(Z)]I(Z \geq w)/\lambda P(Z = w-1)$, as is given by Equation (18) in Stein (1986, page 84). Define $V_\lambda h(w) = U_\lambda h(w+2) - U_\lambda h(w+1), w \geq 0$, that is, $V_\lambda h(w) = \triangle U_\lambda h(w+1)$. Let I_A denote the indicator function of A, a subset of $\boldsymbol{Z}^+$. In the case $A = \{r\}, r \in \boldsymbol{Z}^+$, we use I_r instead of $I_{\{r\}}$.

2. Bounds on the Radon-Nikodym Derivative

Let

$$\tilde{C}(p_1,\ldots,p_n)=\sup\{\frac{P(W=r)}{P(Z=r)}:r\geq 0\}$$

and

$$C^*(p_1,\ldots,p_n)=\sup\{\frac{P(W^{(i)}=r)}{P(Z=r)}:r\geq 0, 1\leq i\leq n\}.$$

When ambiguity does not arise, we abbreviate them to $\tilde{C}$ and C^* respectively.

Proposition 2.1. *We have*

$$\tilde{C}(p_1,\ldots,p_n)\leq\begin{cases} e^{\lambda}, & \text{if } 0<\lambda<1,\\ e^{13/12}\sqrt{2\pi}[1-\frac{1}{\lambda}\sum_{i=1}^n p_i^2]^{-1/2}, & \text{if } 1\leq\lambda.\end{cases}$$

Furthermore, $(1-\tilde{p})C^*\leq\tilde{C}$.

Remarks. 1. Note that for $\lambda<1$, $\tilde{C}\leq e$. Therefore, if $\tilde{p}\leq 1/2$, we can see that $\tilde{C}$ is bounded above by an absolute constant independent of the p_i's, and so is C^*.

2. The bounds on $\tilde{C}$ in Proposition 2.1 are far from best possible, but they suffice for our purpose. This is the advantage of the present approach. However, they can be improved by applying Theorem 3.1 or Theorem 3.2.

Proof of Proposition 2.1. Let $A(r)=P(W=r)/P(Z=r)$. From Inequality (5) of Samuels (1965), $A(r)/A(r-1)$ is decreasing in $r\geq 1$. Applying Corollary 2.1 of Hoeffding (1956), one can verify that $A([\lambda])/A([\lambda]-1)\geq 1$ and $A([\lambda]+2)/A([\lambda]+1)\leq 1$. Hence the maximum of $A(r)$ is attained at either $r=[\lambda]$ or $[\lambda]+1$.

For $\lambda\geq 1$,

$$\begin{aligned} A([\lambda]) &= \frac{e^{\lambda}[\lambda]!}{\lambda^{[\lambda]}}P(W=[\lambda])\\ &\leq \frac{\sqrt{2\pi[\lambda]}[\lambda]^{[\lambda]}\exp(\lambda-[\lambda]+1/(12[\lambda]))}{\lambda^{[\lambda]}}P(W=[\lambda])\\ &\leq e^{13/12}\sqrt{\frac{\pi}{2}}[1-\frac{1}{\lambda}\sum_{i=1}^n p_i^2]^{-1/2}.\end{aligned}$$

The last inequality follows from Lemma 1 of Barbour and Jensen (1989, page 78). Similarly,

$$A([\lambda]+1)\leq e^{13/12}\sqrt{2\pi}[1-\frac{1}{\lambda}\sum_{i=1}^n p_i^2]^{-1/2}.$$

For $0 < \lambda < 1$,

$$A([\lambda]) = A(0) = \prod_{i=1}^{n}(1-p_i)e^{p_i} \le 1$$

and

$$A([\lambda]+1) = A(1) = \frac{P(W=1)}{P(Z=1)} \le \frac{\sum_{k=1}^{n} P(X_k=1)}{\lambda e^{-\lambda}} = e^{\lambda}.$$

This completes the proof of Proposition 2.1.

3. Main Theorems

In this section we prove the following two main theorems.

Theorem 3.1. *Let h be a real-valued function defined on $\boldsymbol{Z}^+$ such that $EZ^2\ |h(Z)| < \infty$. We have*

$$|Eh(W) - Eh(Z)|$$
$$\le C^*(\sum_{i=1}^{n} p_i^2)\left[4(1\wedge\lambda^{-1})E|h(Z+1)| + E|h(Z+2)| - 2E|h(Z+1)| + E|h(Z)|\right]/2,$$

where C^* is given in Proposition 2.1.

Theorem 3.2. *Let h be a real-valued function defined on $\boldsymbol{Z}^+$ such that $EZ^4\ |h(Z)| < \infty$. We have*

$$|Eh(W) - Eh(Z) + \frac{1}{2}\sum_{i=1}^{n} p_i^2 E\triangle^2 h(Z)| \le C^*\{(\sum_{i=1}^{n} p_i^2)^2 R_1 + (\sum_{i=1}^{n} p_i^3)R_2\},$$

where C^* is given in Proposition 2.1,

$$R_1 = 12(1\wedge\lambda^{-2})E|h(Z+2)|$$
$$+ \frac{1}{3}(1\wedge\lambda^{-1})\{5E|h(Z+3)| - 9E|h(Z+2)| + 3E|h(Z+1)| + E|h(Z)|\}$$
$$+ \frac{1}{8}\{E|h(Z+4)| - 4E|h(Z+3)| + 6E|h(Z+2)| - 4E|h(Z+1)| + E|h(Z)|\},$$

and

$$R_2 = 2(1\wedge\lambda^{-1})\{E|h(Z+2)| + E|h(Z+1)|\}$$
$$+ \frac{1}{3}\{E|h(Z+3)| - 3E|h(Z+1)| + 2E|h(Z)|\}.$$

Remarks. 1. These theorems allow a very wide choice of possible functions h. No smoothness or positivity condition is assumed, and the growth condition $EZ^l|h(Z)| < \infty$ ($l = 2$ or 4), which ensures that $E|h(Z+k)| < \infty$ for $0 \le k \le l$, is

hardly restrictive at all. If h is such that $E|h(Z)|$ is small (for example, $h = I_{[z,\infty)}$ for large z), then the smallness is also reflected in the error bounds.
2. By taking h to be such that $|h(z)| = 1$ for all z, Theorem 3.1 yields a total variation bound of the right order.
3. The form of the factors R_1 and R_2 in the error in Theorem 3.2 seems rather complicated. However, their behaviour for large λ is exactly right. To see this, note that a simple calculation gives, for any $\lambda \geq 1$,

$$\lambda^l Ef(Z+l) = EZ_{(l)}f(Z), \quad \text{where} \quad Z_{(l)} = Z(Z-1)\cdots(Z-l+1).$$

Thus $R_1 = Ep(Z)|h(Z)|$, where

$$\begin{aligned} p(z) &= 12(1 \wedge \lambda^{-2})z_{(2)}\lambda^{-2} + \frac{1}{3}(1 \wedge \lambda^{-1})p_1(z) + \frac{1}{8}p_2(z), \\ p_1(z) &= 5z_{(3)}\lambda^{-3} - 9z_{(2)}\lambda^{-2} + 3z\lambda^{-1} + 1, \\ p_2(z) &= z_{(4)}\lambda^{-4} - 4z_{(3)}\lambda^{-3} + 6z_{(2)}\lambda^{-2} - 4z\lambda^{-1} + 1. \end{aligned}$$

Now write $z = \lambda + x\sqrt{\lambda}$. Then

$$\begin{aligned} \lambda^{-2}z_{(2)} &= 1 + \frac{2x}{\sqrt{\lambda}} + \frac{x^2}{\lambda} - \frac{1}{\lambda} - \frac{x}{\lambda\sqrt{\lambda}}; \\ p_1(z) &= \lambda^{-1}\left\{\frac{5x^3}{\sqrt{\lambda}} + x^2(6 - \frac{15}{\lambda}) - \frac{21x}{\sqrt{\lambda}} - 6 + \frac{10}{\lambda} + \frac{10x}{\lambda\sqrt{\lambda}}\right\}, \end{aligned}$$

and

$$p_2(z) = \lambda^{-2}\Big\{x^4 - 6x^2(1 + \frac{x}{\sqrt{\lambda}}) + 3 + \frac{14x}{\sqrt{\lambda}} + \frac{11x^2}{\lambda} - \frac{6}{\lambda} - \frac{6x}{\lambda\sqrt{\lambda}}\Big\}.$$

Thus, for $\lambda \geq 1$ and $x \geq -\sqrt{\lambda}$,

$$|p(z)| \leq \lambda^{-2}K_1(1 + x^4)$$

for a suitable constant K_1. Similarly, $R_2 = Eq(Z)|h(Z)|$, where

$$|q(z)| \leq \lambda^{-1}K_2(1 + x^2 + |x|^3/\sqrt{\lambda}).$$

More precisely, we can take the following expressions to define R_1 and R_2:

For $\lambda \geq 1$,

$$R_1 = \frac{1}{24\lambda^2}E\beta_1(\frac{Z-\lambda}{\sqrt{\lambda}}, \lambda)|h(Z)| \quad \text{and} \quad R_2 = \frac{1}{3\lambda}E\beta_2(\frac{Z-\lambda}{\sqrt{\lambda}}, \lambda)|h(Z)|, \quad (3.1)$$

where

$$\beta_1(x,\lambda) = 3x^4 + \frac{22}{\sqrt{\lambda}}x^3 + (30+\frac{201}{\lambda})x^2 + \frac{1}{\sqrt{\lambda}}(450-\frac{226}{\lambda})x + 249 - \frac{226}{\lambda} \quad (3.2)$$

and

$$\beta_2(x,\lambda) = \frac{1}{\sqrt{\lambda}}x^3 + 3(1+\frac{1}{\lambda})x^2 + \frac{4}{\sqrt{\lambda}}(3-\frac{1}{\lambda})x + 9 - \frac{4}{\lambda}. \quad (3.3)$$

For $0<\lambda<1$, we have

$$R_1 = \{E|h(Z+4)| + 28E|h(Z+3)| + 234E|h(Z+2)| + 12E|h(Z+1)| + 11E|h(Z)|\}/24 \quad (3.4)$$

and

$$R_2 = \{E|h(Z+3)| + 6E|h(Z+2)| + 3E|h(Z+1)| + 2E|h(Z)|\}/3. \quad (3.5)$$

By noting that the supremum of $|Eh(W)-Eh(Z)+\frac{1}{2}\sum_{i=1}^n p_i^2 E\triangle^2 h(Z)|$ over all h such that $|h(z)|\le 1$ for all z is attained by a function h satisfying $|h(z)|=1$ for all z, we have

Corollary 3.3.

$$\sup_{|h|\le 1} |Eh(W) - Eh(Z) + \frac{1}{2}\sum_{i=1}^n p_i^2 E\triangle^2 h(Z)| \le 4C^*\{3(1\wedge\lambda^{-2})(\sum_{i=1}^n p_i^2)^2 + (1\wedge\lambda^{-1})(\sum_{i=1}^n p_i^3)\}.$$

For $\lambda\ge 1$, the order of the bound in Corollary 3.3 is the same as that in Theorem 5.1 of Chen (1975a), whereas for $\lambda<1$, it is an improvement.

In order to prove Theorems 3.1 and 3.2, we need the following lemmas for bounding $E|V_\lambda^2 h(Z)|$ and $E|V_\lambda h(Z+k)|$ for $k=0,1$.

Lemma 3.4. *For $k\ge 0$ and any real-valued function h such that $EZ^{k+2}|h(Z)|<\infty$, we have*

$$EV_\lambda h(Z+k) = \frac{-1}{(k+1)(k+2)}\{(k+1)Eh(Z+k+2) - (k+2)Eh(Z+k+1) + Eh(Z)\}.$$

Proof. Direct computation based on the explicit form of $U_\lambda h$ shows that

$$lEU_\lambda h(Z+l) = Eh(Z) - Eh(Z+l) \quad (3.6)$$

if $EZ^l|h(Z)|<\infty$, the condition ensuring that all expectations exist. The proof is now immediate.

Lemma 3.5. *For nonnegative integers k and r, we have*

$$EV_\lambda I_r(Z+k) = \frac{-(k+1)P(Z=r-k-2) + (k+2)P(Z=r-k-1) - P(Z=r)}{(k+1)(k+2)}$$

and

$$E|V_\lambda I_r(Z+k)| \le \frac{(k+1)P(Z=r-k-2) - (k+2)P(Z=r-k-1) + P(Z=r)}{(k+1)(k+2)} + 2(1\wedge\lambda^{-1})P(Z=r-k-1).$$

Proof. Letting $h = I_r$ in Lemma 3.4 we get the first statement. From the fact that $V_\lambda I_r(w) > 0$ if and only if $w = r-1$ (Equation (37) in Stein (1986, page 88)), we have

$$\begin{aligned} E|V_\lambda I_r(Z+k)| &= E\{V_\lambda I_r(r-1)I(Z+k=r-1) - V_\lambda I_r(Z+k)I(Z+k\ne r-1)\} \\ &= E\{2V_\lambda I_r(r-1)I(Z+k=r-1) - V_\lambda I_r(Z+k)\} \\ &\le 2(1\wedge\lambda^{-1})P(Z+k=r-1) - EV_\lambda I_r(Z+k), \end{aligned}$$

where we have used the fact that

$$V_\lambda I_{r+1}(r) \le (1\wedge\lambda^{-1}). \tag{3.7}$$

(See Equation (41) in Stein (1986, page 88).)

The second statement in Lemma 3.5 then follows from this inequality and the first statement.

Lemma 3.6. *Let k, m be nonnegative integers and h a nonnegative function such that $EZ^{k+m+2}h(Z) < \infty$. Then we have*

$$\sum_{r=0}^{\infty} h(r+m)E|V_\lambda I_r(Z+k)| \le 2(1\wedge\lambda^{-1})Eh(Z+k+m+1) + \frac{Eh(Z+k+m+2)}{k+2} - \frac{Eh(Z+k+m+1)}{k+1} + \frac{Eh(Z+m)}{(k+1)(k+2)}.$$

Proof. Apply Lemma 3.5 and sum over r.

Lemma 3.7. *Let k be a nonnegative integer and h a real-valued function with $EZ^{k+2}|h(Z)| < \infty$. We have*

$$E|V_\lambda h(Z+k)| \le 2(1\wedge\lambda^{-1})E|h(Z+k+1)| + \frac{E|h(Z+k+2)|}{k+2} - \frac{E|h(Z+k+1)|}{k+1} + \frac{E|h(Z)|}{(k+1)(k+2)}.$$

Proof. Since

$$\begin{aligned} E|V_\lambda h(Z+k)| &= E|V_\lambda(\sum_{r=0}^\infty h(r)I_r)(Z+k)| \\ &= E|\sum_0^\infty h(r)V_\lambda I_r(Z+k)| \\ &\le \sum_0^\infty |h(r)|E|V_\lambda I_r(Z+k)|, \end{aligned}$$

Lemma 3.7 follows from Lemma 3.6 with $m=0$.

Lemma 3.8. *Let h be a real-valued function such that $EZ^4|h(Z)|<\infty$. Then*

$$\begin{aligned} E|V_\lambda^2 h(Z)| \le\ & 12(1\wedge\lambda^{-2})E|h(Z+2)| \\ &+(1\wedge\lambda^{-1})\{5E|h(Z+3)|-9E|h(Z+2)|+3E|h(Z+1)|+E|h(Z)|\}/3 \\ &+\{E|h(Z+4)|-4E|h(Z+3)|+6E|h(Z+2)| \\ &-4E|h(Z+1)|+E|h(Z)|\}/8. \end{aligned}$$

Proof. The estimate we obtain is better than the one that results from writing $E|V_\lambda^2 h(Z)| = E|V_\lambda(V_\lambda h)(Z)|$ and applying Lemma 3.7 twice, but the proof is trickier. We have

$$\begin{aligned} V_\lambda^2 I_r = V_\lambda(V_\lambda I_r) &= V_\lambda(\sum_{s=0}^\infty V_\lambda I_r(s)I_s) \\ &= \sum_{s=0}^\infty V_\lambda I_r(s)V_\lambda I_s = \sum_{s=0}^\infty\sum_{t=0}^\infty V_\lambda I_r(s)V_\lambda I_s(t)I_t. \end{aligned}$$

Since $V_\lambda I_r(s)>0$ if and only if $s=r-1$, we have pointwise

$$\begin{aligned} |V_\lambda^2 I_r| \le\ & V_\lambda I_r(r-1)V_\lambda I_{r-1}(r-2)I_{r-2} + \sum_{s\ne r-1}\sum_{t\ne s-1} V_\lambda I_r(s)V_\lambda I_s(t)I_t \\ &- V_\lambda I_r(r-1)\sum_{t\ne r-2} V_\lambda I_{r-1}(t)I_t - \sum_{s\ne r-1} V_\lambda I_r(s)V_\lambda I_s(s-1)I_{s-1} \\ =\ & 2V_\lambda I_r(r-1)V_\lambda I_{r-1}(r-2)I_{r-2} + 2\sum_{s\ne r-1}\sum_{t\ne s-1} V_\lambda I_r(s)V_\lambda I_s(t)I_t - V_\lambda^2 I_r. \end{aligned}$$

Therefore

$$|V_\lambda^2 h| = |\sum_{r=0}^\infty h(r)V_\lambda^2 I_r| \le \sum_{r=0}^\infty |h(r)||V_\lambda^2 I_r| \le a+b-c, \tag{3.8}$$

where a, b and c are functions given by

$$a = 2\sum_{r=2}^{\infty} |h(r)| V_\lambda I_r (r-1) V_\lambda I_{r-1} (r-2) I_{r-2},$$

$$b = 2\sum_{r=0}^{\infty} |h(r)| \sum_{s\neq r-1} \sum_{t\neq s-1} V_\lambda I_r(s) V_\lambda I_s(t) I_t,$$

$$c = \sum_{r=0}^{\infty} |h(r)| V_\lambda^2 I_r = V_\lambda^2 |h|.$$

By (3.7), $a \le 2(1\wedge\lambda^{-1})^2 \sum_{r=2}^{\infty} |h(r)| I_{r-2} = 2(1\wedge\lambda^{-2}) \sum_{r=0}^{\infty} |h(r+2)| I_r$ and

$$\begin{aligned}
b &= 2\sum_{s=0}^{\infty} \sum_{t\neq s-1} \sum_{r\neq s+1} |h(r)| V_\lambda I_r(s) V_\lambda I_s(t) I_t \\
&= 2\sum_{s=0}^{\infty} \sum_{t\neq s-1} V_\lambda |h|(s) V_\lambda I_s(t) I_t - 2\sum_{s=0}^{\infty} \sum_{t\neq s-1} |h(s+1)| V_\lambda I_{s+1}(s) V_\lambda I_s(t) I_t \\
&= 2\sum_{s=0}^{\infty} \sum_{t=0}^{\infty} V_\lambda |h|(s) V_\lambda I_s(t) I_t - 2\sum_{s=1}^{\infty} V_\lambda |h|(s) V_\lambda I_s(s-1) I_{s-1} \\
&\quad - 2\sum_{s=0}^{\infty} \sum_{t=0}^{\infty} |h(s+1)| V_\lambda I_{s+1}(s) V_\lambda I_s(t) I_t \\
&\quad + 2\sum_{s=1}^{\infty} |h(s+1)| V_\lambda I_{s+1}(s) V_\lambda I_s(s-1) I_{s-1} \\
&\le 2V_\lambda^2 |h| + 2(1\wedge\lambda^{-1}) \left[\sum_{s=0}^{\infty} |V_\lambda |h|(s+1)| I_s + \sum_{s=0}^{\infty} |h(s+1)| |V_\lambda I_s|\right] \\
&\quad + 2(1\wedge\lambda^{-2}) \sum_{s=0}^{\infty} |h(s+2)| I_s.
\end{aligned}$$

Therefore from (3.8)

$$\begin{aligned}
|V_\lambda^2 h| &\le 4(1\wedge\lambda^{-2}) \sum_{r=0}^{\infty} |h(r+2)| I_r \\
&\quad + 2(1\wedge\lambda^{-1}) \left[\sum_{r=0}^{\infty} |V_\lambda |h|(r+1)| I_r + \sum_{r=0}^{\infty} |h(r+1)| |V_\lambda I_r|\right] + V_\lambda^2 |h|.
\end{aligned}$$

From this we have

$$\begin{aligned}
E|V_\lambda^2 h(Z)| &\le 4(1\wedge\lambda^{-2}) E|h(Z+2)| + 2(1\wedge\lambda^{-1}) E|V_\lambda |h|(Z+1)| \\
&\quad + 2(1\wedge\lambda^{-1}) \sum_{r=0}^{\infty} |h(r+1)| E|V_\lambda I_r(Z)| + EV_\lambda^2 |h|(Z).
\end{aligned}$$

Applying Lemma 3.7 with $k = 1$ to the second term in the bound on $E|V_\lambda^2 h(Z)|$, Lemma 3.6 with $k = 0, m = 1$ to the third term, and Lemma 3.4 twice to the last term, we complete the proof of Lemma 3.8.

In the proofs below, note that all the W-expectations exist because W takes only a finite number of values. Also note that Proposition 2.1 allows us to make convenient estimates of the remainder terms, even for fast growing functions h.

Proof of Theorem 3.1. We begin with the following identity (Stein (1986, page 86)):

$$Eh(W) - Eh(Z) = \sum_{i=1}^{n} p_i^2 EV_\lambda h(W^{(i)}). \tag{3.9}$$

Note that this equation holds not only for bounded h but also for any h with $E|h(Z)| < \infty$. Since $P(W^{(i)} = r) \le C^* P(Z = r)$ for $r \ge 0$ and $1 \le i \le n$, we have

$$|Eh(W) - Eh(Z)| \le C^* (\sum_{i=1}^{n} p_i^2) E|V_\lambda h(Z)|.$$

The theorem then follows from this and Lemma 3.7.

Proof of Theorem 3.2. Rewriting (3.9), we get

$$\begin{aligned} Eh(W) - Eh(Z) &= (\sum_{i=1}^{n} p_i^2) EV_\lambda h(W) + \sum_{i=1}^{n} p_i^2 \{EV_\lambda h(W^{(i)}) - EV_\lambda h(W)\} \\ &= (\sum_{i=1}^{n} p_i^2) EV_\lambda h(W) \\ &\quad + \sum_{i=1}^{n} p_i^3 \{EV_\lambda h(W^{(i)}) - EV_\lambda h(W^{(i)} + 1)\}. \end{aligned} \tag{3.10}$$

Replacing h in (3.9) by $V_\lambda h$ and substituting the equation in (3.10), we obtain

$$\begin{aligned} Eh(W) - Eh(Z) = (\sum_{i=1}^{n} p_i^2) EV_\lambda h(Z) + (\sum_{i=1}^{n} p_i^2) \sum_{j=1}^{n} p_j^2 EV_\lambda^2 h(W^{(j)}) \\ + \sum_{i=1}^{n} p_i^3 \left\{ EV_\lambda h(W^{(i)}) - EV_\lambda h(W^{(i)} + 1) \right\}. \end{aligned}$$

Since $P(W^{(i)} = r) \le C^* P(Z = r)$ for $r \ge 0$ and $1 \le i \le n$, we have

$$|(\sum_{i=1}^{n} p_i^2) \sum_{j=1}^{n} p_j^2 EV_\lambda^2 h(W^{(j)})| \le C^* (\sum_{i=1}^{n} p_i^2)^2 E|V_\lambda^2 h(Z)|$$

and

$$|\sum_{i=1}^{n} p_i^3 \left\{ EV_\lambda h(W^{(i)}) - EV_\lambda h(W^{(i)}+1) \right\}| \le C^* (\sum_{i=1}^{n} p_i^3) E\left\{ |V_\lambda h(Z)| + |V_\lambda h(Z+1)| \right\}.$$

Applying Lemmas 3.7 and 3.8 and using the fact that $EV_\lambda h(Z) = -E\triangle^2 h(Z)/2$ (which follows from (3.6)), we complete the proof of Theorem 3.2.

4. Large Deviations and Asymptotics

Let $\mathcal{A}$ be the class of real-valued functions defined on $[0, \infty)$ and satisfying the following condition:

There exist constants $c \ge 0$ and $u_0 > 0$ (depending on h) such that for all $u \ge u_0$ and $\epsilon \in (0, 1]$, $|h(u + \epsilon) - h(u)|/\epsilon \le ch(u)$.

It is not difficult to observe

Proposition 4.1. (a) *$h \in \mathcal{A}$ if and only if $\log h$ is Lipschitz on $[v_0, \infty)$ for some $v_0 > 0$ (depending on h).*
(b) *The class of functions, $\mathcal{A}$, contains polynomials with positive leading coefficient and exponential functions. It is closed under addition, nonnegative scalar multiplication and multiplication of functions (in the sense that, if $f, g \in \mathcal{A}$, then $fg \in \mathcal{A}$).*

For the rest of this section, z is taken to be a positive integer.

Theorem 4.2. *Let $z = \lambda + \xi\sqrt{\lambda}$.*
(a) *For $\lambda \ge 1$, let $V = (W - \lambda)/\sqrt{\lambda}$ and $U = (Z - \lambda)/\sqrt{\lambda}$ and let $h \in \mathcal{A}$ such that $EU^4 h(U) I(U \ge u_0) < \infty$. Suppose $\tilde{p} \to 0$ and $\xi = o([\lambda/\sum_{i=1}^n p_i^2]^{1/2})$ as $n \to \infty$. Then as $n, \xi \to \infty$,*

$$\frac{Eh(V)I(V \ge \xi)}{Eh(U)I(U \ge \xi)} - 1 \sim -\frac{\xi^2}{2\lambda} \sum_{i=1}^{n} p_i^2.$$

(b) *For $0 < \lambda < 1$, let $h \in \mathcal{A}$ such that $EZ^4 h(Z) I(Z \ge u_0) < \infty$. Suppose $\tilde{p} \to 0$ and $\xi = o([\lambda/\sum_{i=1}^n p_i^2]^{1/2})$ as $n \to \infty$. Then as $n, z \to \infty$,*

$$\frac{Eh(W)I(W \ge z)}{Eh(Z)I(Z \ge z)} - 1 \sim -\frac{\xi^2}{2\lambda} \sum_{i=1}^{n} p_i^2.$$

Letting $h \equiv 1$, we obtain the following corollary in which there is no restriction on λ.

Corollary 4.3. *Let $z = \lambda + \xi\sqrt{\lambda}$. Suppose $\tilde{p} \to 0$ and $\xi = o([\lambda/\sum_{i=1}^n p_i^2]^{1/2})$ as $n \to \infty$. Then as n, z and $\xi \to \infty$,*

$$\frac{P(W \ge z)}{P(Z \ge z)} - 1 \sim -\frac{\xi^2}{2\lambda} \sum_{i=1}^{n} p_i^2.$$

Theorem 4.2 and Corollary 4.3 generalize Theorem 2.3 and Corollary 2.4 of Chen and Choi (1992) respectively. Corollary 4.3 is also essentially contained in Theorem 9.D of Barbour, Holst and Janson (1992, page 188).

Theorem 4.4. *Let N be a standard normal random variable. Let h be a nonnegative function defined on $\mathbf{R}$ which is continuous almost everywhere and not identically zero. Suppose $\left\{\left(\frac{Z-\lambda}{\sqrt{\lambda}}\right)^4 h\left(\frac{Z-\lambda}{\sqrt{\lambda}}\right) : \lambda \geq 1\right\}$ is uniformly integrable. Then as $\lambda \to \infty$ such that $\tilde{p} \to 0$,*

$$\sum_{r=0}^{\infty} h(\frac{r-\lambda}{\sqrt{\lambda}})|P(W=r)-P(Z=r)| \sim \frac{1}{2\lambda}(\sum_{i=1}^{n} p_i^2)E|N^2-1|h(N).$$

By letting $h \equiv 1$, $E|N^2-1|h(N) = E|N^2-1| = 2\sqrt{2/(\pi e)}$, and Theorem 4.4 yields a result of Barbour and Hall (1984, page 477) and Theorem 1.2 of Deheuvels and Pfeifer (1986).

Before we prove Theorems 4.2 and 4.4, we need some preliminary results which are also of independent interest.

Lemma 4.5. *Let $z = \lambda + \xi\sqrt{\lambda}$, $h \in \mathcal{A}$, and let c and u_0 be the constants associated with h.*
(a) *For $\lambda \geq 1$, let $U = (Z-\lambda)/\sqrt{\lambda}$. Then for $\xi > \max\{c, u_0\}$,*

$$(1+\frac{c}{\xi}+\frac{1}{\xi^2})^{-1} \leq \frac{Eh(U)I(U \geq \xi)}{\frac{z}{z-\lambda}h(\xi)P(Z=z)} \leq (1-\frac{c}{\xi})^{-1}, \tag{4.1}$$

provided $Eh(U)I(U \geq u_0) < \infty$.
(b) *For $\lambda > 0$ and $z+1 > \max\{(c+1)\lambda, u_0\}$,*

$$1 \leq \frac{Eh(Z)I(Z \geq z)}{h(z)P(Z=z)} \leq (1-\frac{(c+1)\lambda}{z+1})^{-1}, \tag{4.2}$$

provided $Eh(Z)I(Z \geq u_0) < \infty$.

Letting $h \equiv 1$, in which case $c = u_0 = 0$, we have the following corollary, which is an improvement of Propositions A.2.1 (ii) and A.2.3 (ii) of Barbour, Holst and Janson (1992).

Corcollary 4.6. *Let $z = \lambda + \xi\sqrt{\lambda}$ where $\xi > 0$.*
(a) *For $\lambda \geq 1$,*

$$(1+\frac{1}{\xi^2})^{-1}\frac{z}{z-\lambda}P(Z=z) \leq P(Z \geq z) \leq \frac{z}{z-\lambda}P(Z=z).$$

(b) *For* $\lambda > 0$ *and* $z+1 > \lambda$,

$$P(Z = z) \le P(Z \ge z) \le \frac{z+1}{z+1-\lambda} P(Z = z).$$

Note that Lemma 4.5 and Corollary 4.6 imply that $Eh(U)I(U \ge \xi) \sim (z/(z-\lambda))h(\xi)P(Z = z)$ as $\xi \to \infty$, and that in the case $h \equiv 1, P(Z \ge z) \sim (z/(z-\lambda))P(Z = z)$ as $\xi \to \infty$, both uniformly in $\lambda > 0$.

Proof of Lemma 4.5. (a) Recall the following identity (see Stein (1986, page 81, Theorem 1)):

$$EZf(Z) = \lambda Ef(Z+1). \tag{4.3}$$

It is not difficult to show that for $U = (Z-\lambda)/\sqrt{\lambda}$,

$$EUf(U) = \sqrt{\lambda} E\left[f(U + \frac{1}{\sqrt{\lambda}}) - f(U)\right].$$

Let $f(x) = h(x)I(x \ge \xi)/x$. Then

$$\begin{aligned}
&Eh(U)I(U \ge \xi) \\
&= \sqrt{\lambda} E\left[(U + \frac{1}{\sqrt{\lambda}})^{-1} h(U + \frac{1}{\sqrt{\lambda}}) I(U + \frac{1}{\sqrt{\lambda}} \ge \xi) - U^{-1} h(U) I(U \ge \xi)\right] \\
&= \sqrt{\lambda} E(U + \frac{1}{\sqrt{\lambda}})^{-1} h(U + \frac{1}{\sqrt{\lambda}}) I(U + \frac{1}{\sqrt{\lambda}} = \xi) \\
&\quad + E(U + \frac{1}{\sqrt{\lambda}})^{-1} \sqrt{\lambda} \left[h(U + \frac{1}{\sqrt{\lambda}}) - h(U)\right] I(U \ge \xi) \\
&\quad - E[U(U + \frac{1}{\sqrt{\lambda}})]^{-1} h(U) I(U \ge \xi).
\end{aligned} \tag{4.4}$$

The first term in the right hand side of the second equality in (4.4) is

$$\frac{\sqrt{\lambda}}{\xi} h(\xi) P(Z = z-1) = \frac{z}{z-\lambda} h(\xi) P(Z = z).$$

Since $h \in \mathcal{A}$ and $\lambda \ge 1$,

$$|E(U + \frac{1}{\sqrt{\lambda}})^{-1} \sqrt{\lambda} \left[h(U + \frac{1}{\sqrt{\lambda}}) - h(U)\right] I(U \ge \xi)| \le \frac{c}{\xi} Eh(U) I(U \ge \xi).$$

Therefore from (4.4) we obtain

$$Eh(U)I(U \ge \xi) \le \frac{z}{z-\lambda} h(\xi) P(Z = z) + \frac{c}{\xi} Eh(U) I(U \ge \xi).$$

This implies the second inequality in (4.1). The first inequality in (4.1) follows in a similar fashion from (4.4).

(b) The first inequality in (4.2) is trivial. Using the identity (4.3) again, we obtain

$$\begin{aligned} Eh(Z)I(Z \ge z) &= \lambda E \frac{h(Z+1)}{Z+1} I(Z \ge z-1) \\ &= \lambda \frac{h(z)}{z} P(Z = z-1) + \lambda E \frac{h(Z+1)}{Z+1} I(Z \ge z) \\ &= h(z)P(Z=z) + \lambda E \frac{h(Z)}{Z+1} I(Z \ge z) \\ &\quad + \lambda E \frac{h(Z+1)-h(Z)}{Z+1} I(Z \ge z) \\ &\le h(z)P(Z=z) + \frac{(c+1)\lambda}{z+1} Eh(Z)I(Z \ge z). \end{aligned}$$

This implies the second inequality in (4.2) and completes the proof of Lemma 4.5.

In the proofs of Theorems 4.2 and 4.4, we let

$$\lambda_k = \sum_{i=1}^{n} p_i^k, \quad k = 2, 3, \ldots.$$

Proof of Theorem 4.2. (a) Assume $\lambda \ge 1$. First apply Theorem 3.2 to the function g where $g(x) = h((x-\lambda)/\sqrt{\lambda})I((x-\lambda)/\sqrt{\lambda} \ge \xi)$. Then part (a) of Theorem 4.2 is proved once the following three statements are proved:

$$\frac{E\triangle^2 g(Z)}{Eg(Z)} \sim \frac{\xi^2}{\lambda}, \tag{4.5}$$

$$\frac{\lambda_2 R_1}{E\triangle^2 g(Z)} \to 0, \tag{4.6}$$

$$\frac{\lambda_3 R_2}{\lambda_2 E\triangle^2 g(Z)} \to 0 \tag{4.7}$$

as $n, \xi \to \infty$.

First we prove (4.5). By (4.3), $E\triangle^2 g(Z) = E[Z^2 - (2\lambda+1)Z + \lambda^2]g(Z)/\lambda^2$. Observe that $x/(x-\lambda)^2$ is a decreasing function in $x > \lambda$, so on the set $\{U \ge \xi\} = \{Z \ge z\}$ we have

$$(1 - \frac{z}{(z-\lambda)^2})(Z-\lambda)^2 \le Z^2 - (2\lambda+1)Z + \lambda^2 \le (Z-\lambda)^2.$$

Since $z/(z-\lambda)^2 = 1/\xi^2 + 1/(\xi\sqrt{\lambda}) \to 0$, we have

$$\frac{E\triangle^2 g(Z)}{Eg(Z)} \sim \frac{E(Z-\lambda)^2 g(Z)}{\lambda^2 Eg(Z)} = \frac{EU^2 h(U)I(U \geq \xi)}{\lambda Eh(U)I(U \geq \xi)} \sim \frac{\xi^2}{\lambda},$$

where we have applied part (a) of Lemma 4.5 in the last step. This proves (4.5).

Next we prove (4.6). By (4.5) and (3.1), the left hand side of (4.6) is bounded above by

$$\begin{aligned}\frac{\lambda\lambda_2 R_1}{\xi^2 Eh(U)I(U \geq \xi)}[1+o(1)] &= \frac{\lambda_2 E\beta_1(U,\lambda)h(U)I(U \geq \xi)}{24\lambda\xi^2 Eh(U)I(U \geq \xi)}[1+o(1)] \\ &= \frac{\lambda_2 EU^4 h(U)I(U \geq \xi)}{8\lambda\xi^2 Eh(U)I(U \geq \xi)}[1+o(1)] \\ &= \frac{\lambda_2\xi^2}{8\lambda}[1+o(1)] \to 0\end{aligned}$$

as $\xi = o(\sqrt{\lambda/\lambda_2})$. Here we have used the fact that on $\{U \geq \xi\}, \beta_1(U,\lambda) = 3U^4[1+o(1)]$ and part (a) of Lemma 4.5. This proves (4.6).

To prove (4.7), we proceed similarly. The left hand side of (4.7) is bounded above by

$$\begin{aligned}&\frac{\lambda_3 E\beta_2(U,\lambda)h(U)I(U \geq \xi)}{3\lambda_2\xi^2 Eh(U)I(U \geq \xi)}[1+o(1)] \\ &= \frac{\lambda_3 E(U^3/\sqrt{\lambda} + 3U^2)h(U)I(U \geq \xi)}{3\lambda_2\xi^2 Eh(U)I(U \geq \xi)}[1+o(1)] \\ &= \frac{\lambda_3}{\lambda_2}(\frac{\xi}{3\sqrt{\lambda}} + 1)[1+o(1)] \\ &\leq (\frac{\xi}{3}\sqrt{\frac{\lambda_2}{\lambda}} + \tilde{p})[1+o(1)] \to 0\end{aligned}$$

as $\tilde{p} \to 0$ and $\xi = o(\sqrt{\lambda/\lambda_2})$. In the last inequality, we have used the fact that

$$\lambda_3^2 = (\sum_{i=1}^n p_i^3)^2 \leq (\sum_{i=1}^n p_i^4)(\sum_{i=1}^n p_i^2) \leq (\sum_{i=1}^n p_i^2)^3 = \lambda_2^3.$$

This proves (4.7) and hence completes the proof of part (a).

(b) Assume $0 < \lambda < 1$. First apply Theorem 3.2 to the function g where $g(x) = h(x)I(x \geq z)$. Then as in the proof of part (a), it suffices to prove the following three statements.

$$\frac{E\triangle^2 g(Z)}{Eg(Z)} \sim \frac{\xi^2}{\lambda}, \tag{4.8}$$

$$\frac{\lambda_2 R_1}{E\triangle^2 g(Z)} \to 0, \tag{4.9}$$

$$\frac{\lambda_3 R_2}{\lambda_2 E\triangle^2 g(Z)} \to 0 \tag{4.10}$$

as $n, z \to \infty$.

To prove (4.8), we use part (b) of Lemma 4.5, and as in the proof of (4.5),

$$\frac{E\triangle^2 g(Z)}{Eg(Z)} \sim \frac{E(Z-\lambda)^2 h(Z) I(Z \ge z)}{\lambda^2 Eh(Z) I(Z \ge z)} \sim \frac{(z-\lambda)^2}{\lambda^2} = \frac{\xi^2}{\lambda}.$$

This proves (4.8).

To prove (4.9), we use (3.4) and (4.8), so the left hand side of (4.9) is bounded above by

$$\begin{aligned}\frac{\lambda\lambda_2 EZ^4 h(Z) I(Z \ge z)}{24\xi^2\lambda^4 Eh(Z) I(Z \ge z)}[1+o(1)] &= \frac{z^4\lambda_2}{24\xi^2\lambda^3}[1+o(1)] \\ &= \frac{\lambda_2\xi^2}{24\lambda}[1+o(1)] \to 0\end{aligned}$$

as $\xi = o(\sqrt{\lambda/\lambda_2})$. This proves (4.9).

To prove (4.10), we use (3.5) and (4.8), so the left hand side of (4.10) is bounded above by

$$\begin{aligned}\frac{\lambda\lambda_3 EZ^3 h(Z) I(Z \ge z)}{3\lambda^3\lambda_2\xi^2 Eh(Z) I(Z \ge z)}[1+o(1)] &= \frac{z^3\lambda_3}{3\xi^2\lambda^2\lambda_2}[1+o(1)] \\ &= \frac{\xi\lambda_3}{3\lambda_2\sqrt{\lambda}}[1+o(1)] \\ &\le \frac{\xi}{3}\sqrt{\frac{\lambda_2}{\lambda}}[1+o(1)] \to 0.\end{aligned}$$

This proves (4.10) and the proof of Theorem 4.2 is completed.

Proof of Theorem 4.4. Since $\lambda \to \infty$, we may assume $\lambda \ge 1$. Letting $h = I_r$ in Theorem 3.2 and then applying the triangle inequality, we get

$$(A(r)-\sum_{i=1}^{2} B_i(r))P(Z=r) \le |P(W=r)-P(Z=r)| \le (A(r)+\sum_{i=1}^{2} B_i(r))P(Z=r),$$

where

$$\begin{aligned}A(r) &= \frac{\lambda_2|r^2-(2\lambda+1)r+\lambda^2|}{2\lambda^2}, \\ B_1(r) &= \frac{C^*\lambda_2^2}{24\lambda^2}\beta_1(\frac{r-\lambda}{\sqrt{\lambda}}, \lambda), \\ B_2(r) &= \frac{C^*\lambda_3}{3\lambda}\beta_2(\frac{r-\lambda}{\sqrt{\lambda}}, \lambda).\end{aligned}$$

Here we have used (3.1), (3.2), (3.3) and the fact that $E\triangle^2 h(Z) = E[Z^2 - (2\lambda + 1)Z + \lambda^2]h(Z)/\lambda^2$.

Recall that $U = (Z - \lambda)/\sqrt{\lambda}$ and that C^* is bounded by an absolute constant. It suffices to prove the following three statements:

$$EA(Z)h(U) \sim \frac{\lambda_2}{2\lambda} E|N^2 - 1|h(N), \tag{4.11}$$

$$\lambda_2 E\beta_1(U,\lambda)h(U)/\lambda \to 0, \tag{4.12}$$

$$\lambda_3 E\beta_2(U,\lambda)h(U)/\lambda_2 \to 0 \tag{4.13}$$

as $\lambda \to \infty$.

Since h is continuous a.e., $\{U^4 h(U) : \lambda \geq 1\}$ is uniformly integrable and U converges in distribution to N as $\lambda \to \infty$, we have

$$EA(Z)h(U) = \frac{\lambda_2}{2\lambda} E|U^2 - \frac{U}{\sqrt{\lambda}} - 1|h(U) \sim \frac{\lambda_2}{2\lambda} E|N^2 - 1|h(N)$$

as $\lambda \to \infty$. This proves (4.11).

Similarly,

$$E\beta_1(U,\lambda)h(U) \to E(3N^4 + 30N^2 + 249)h(N)$$

and

$$E\beta_2(U,\lambda)h(U) \to E(3N^2 + 9)h(N)$$

as $\lambda \to \infty$.

As $\lambda_2/\lambda \leq \tilde{p} \to 0$ and $\lambda_3/\lambda_2 \leq \tilde{p} \to 0$ as λ (and therefore n) $\to \infty$, this proves (4.12) and (4.13) and completes the proof of Theorem 4.4.

As a final remark, we mention that for the case $\lambda \to \infty$, results similar to those in Lemma 4.5 and Theorem 4.2 can also be obtained for the left tail by the present method.

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Institut für Angewandte Mathematik der Universität Zürich, Winterthurestrasse 190, CH-8057, Zürich, Switzerland.

Department of Mathematics, National University of Singapore, Singapore 0511, Singapore.

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