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TWO-LEVEL NONREGULAR DESIGNS FROM QUATERNARY LINEAR CODES

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Abstract: A quaternary linear code is a linear space over the ring of integers modulo 4. Recent research in coding theory shows that many famous nonlinear codes such as the Nordstrom and Robinson (1967) code and its generalizations can be simply constructed from quaternary linear codes. This paper explores the use of quaternary codes to construct two-level nonregular designs. A general construction of nonregular designs is described, and some theoretic results are obtained. Many nonregular designs constructed by this method have better statistical properties than regular designs of the same size in terms of resolution and aberration. A systematic construction procedure is proposed and a collection of nonregular designs with 16, 32, 64, 128, 256 runs and up to 64 factors is presented.

Key words and phrases: Fractional factorial design, generalized minimum aberration, generalized resolution, MacWilliams identity, quaternary code.

1. Introduction

Fractional factorial designs with factors at two levels are among the most widely used experimental designs. Designs that can be constructed through defining relations among factors are called *regular* designs. Any two factorial effects in a regular design are either mutually orthogonal or fully aliased with each other. All other designs that do not possess this kind of defining relationship are called *nonregular* designs.

Regular designs are typically chosen by the *maximum resolution* criterion (Box and Hunter (1961)) and its refinement — the *minimum aberration* criterion (Fries and Hunter (1980)). Research on minimum aberration designs has been very active in the last 10-15 years. The reader is referred to Wu and Hamada (2000) for rich results and extensive references.

The concepts of resolution and aberration for regular designs have recently been extended to nonregular designs; see Deng and Tang (1999), Tang and Deng (1999) and Ye (2003). Tang and Deng (1999) showed that generalized minimum aberration designs tend to minimize the contamination of non-negligible two-factor and higher-order interactions on the estimation of the main effects. Tang (2001) provided a projection justification of the generalized minimum aberration criterion, and Cheng, Deng and Tang (2002) showed that the generalized minimum aberration criterion is connected with some traditional model-dependent efficiency criteria. For extensions to multi-level nonregular designs, see Xu and Wu (2001) and Cheng and Ye (2004).

With the generalized resolution and aberration criteria, it is now possible to systematically compare the statistical properties of nonregular designs. The construction of good nonregular designs, however, remains challenging. Deng and Tang (2002) constructed generalized minimum aberration designs from Hadamard matrices of order 12, 16, 20 and 24. Tang and Deng (2003) constructed generalized minimum aberration designs for 3, 4 and 5 factors and any run size. Li, Deng and Tang (2004) constructed designs with 20, 24, 28, 32 and 36 runs and up to 6 factors. Xu and Deng (2005) introduced the concept of moment aberration projection and further studied nonregular designs with 16 and 20 runs. Sun, Li and Ye (2002) proposed a sequential algorithm and completely enumerated all 16 and 20-run orthogonal arrays. All these algorithmic constructions are limited to small run sizes (< 32) or small number of factors, due to the existence of a large number of designs.

Butler (2003b, 2004) developed some theoretical results, and constructed some special generalized minimum aberration designs over all possible designs without computer search. Xu (2005a) constructed several nonregular designs with 32, 64, 128 and 256 runs and 7-16 factors from the Nordstrom and Robinson (1967) code, a well-known nonlinear code in coding theory. These nonregular designs are better than regular designs of the same size in terms of both generalized resolution and aberration.

This paper considers the construction of two-level nonregular designs and proposes the use of quaternary codes to derive nonregular designs. The study of quaternary codes started in the early 1990s when it was discovered that many famous nonlinear binary codes (such as the Nordstrom and Robinson code and its generalizations) can be viewed as linear codes over $Z_4 = \{0, 1, 2, 3\} \pmod 4$, the ring of integers modulo 4; see Hammons, Kumar, Calderbank, Sloane and Sole (1994).

The obvious advantages of using quaternary codes to construct nonregular designs are that the construction method is relatively straightforward, and that designs can be presented and described in a simple manner. Like most papers on regular designs, we use column indexes to describe these designs, because a linear code is a linear space and can be completely specified by a basis. More importantly, many nonregular designs constructed by this method have better statistical properties than regular designs of the same size in terms of resolution and aberration.

Background information, notation and definitions are presented in Section 2. Examples of quaternary codes and nonregular designs are given in Section 3.

Section 4 presents some theoretical results, and Section 5 describes a systematic construction procedure. A collection of nonregular designs with 16, 32, 64, 128, 256 runs and up to 64 factors is presented in Section 6. Concluding remarks are given in Section 7.

2. Background Information, Notation and Definitions

A design D of N runs and n factors is represented by an $N \times n$ matrix, where each row corresponds to a run and each column to a factor. A two-level design takes on only two symbols, say -1 or $+1$. For $s = \{c_1, \ldots, c_k\}$, a subset of k columns of D, define

$$J_k(s) = \Big| \sum_{i=1}^{N} c_{i1} \cdots c_{ik} \Big|, \tag{1}$$

where c_{ij} is the ith component of column c_j. The J_k values are called the *J-characteristics* of design D. When D is a regular design, $J_k(s)$ can take on only two values: 0 or N. In general, $0 \leq J_k(s) \leq N$. If $J_k(s) = N$, these k columns in s form a word of length k.

Suppose that r is the smallest integer such that $\max_{|s|=r} J_r(s) > 0$, where the maximization is over all subsets of r columns of D. The *generalized resolution* (Deng and Tang (1999)) of D is defined as $R(D) = r + [1 - \max_{|s|=r} J_r(s)/N]$. Let

$$A_k(D) = N^{-2} \sum_{|s|=k} [J_k(s)]^2. \tag{2}$$

The vector $(A_1(D), \ldots, A_n(D))$ is called the *generalized wordlength pattern.* The *generalized minimum aberration* criterion, called *minimum G_2-aberration* by Tang and Deng (1999), is to sequentially minimize $A_1(D), A_2(D), \ldots, A_n(D)$. When restricted to regular designs, generalized resolution, generalized wordlength pattern and generalized minimum aberration reduce to the traditional resolution, wordlength pattern and minimum aberration, respectively. In the rest of the paper, we use resolution, wordlength pattern and minimum aberration for both regular and nonregular designs.

There is another version of the generalized aberration criterion, based on the frequencies of J-characteristics. The *confounding frequency vector* of design D with run size N and n factors is

$$\mathrm{CFV}(D) = [(f_{11}, \ldots, f_{1N}); (f_{21}, \ldots, f_{2N}); \ldots; (f_{n1}, \ldots, f_{nN})],$$

where f_{kj} denotes the frequency of k-column combinations s with $J_k(s) = N + 1 - j$. The *minimum G-aberration* criterion (Deng and Tang (1999)) is to sequentially minimize the components in the confounding frequency vector.

Note that minimum aberration (MA) regular designs always have maximum resolution among all regular designs. The situation is more complicated for nonregular designs. Nonregular designs having minimum G_2-aberration may not have maximum resolution. However, nonregular designs having minimum G-aberration always have maximum resolution. Throughout the paper, aberration means G_2-aberration, unless otherwise specified.

A two-level design D of N runs and n factors is an *orthogonal array* of strength t, denoted by $OA(N, n, 2, t)$, if all possible 2^t level combinations for any t factors appear equally often. Deng and Tang (1999) showed that a design has resolution $r \leq R < r + 1$ if and only if it is an orthogonal array of strength $t = r - 1$.

A two-level design is said to have *projectivity* p (Box and Tyssedal (1996)) if any p-factor projection contains a complete 2^p factorial design, possibly with some points replicated, and p is the largest integer having that property. A regular design with resolution $R = r$ is an orthogonal array of strength $r - 1$ and hence has projectivity $r - 1$. Deng and Tang (1999) showed that a design with resolution $R > r$ has projectivity $p \geq r$.

Two designs are said to be *isomorphic* if one can be obtained from the other by permuting the rows, the columns, or the symbols of each column.

2.1. Connection with coding theory

The connection between factorial designs and linear codes was first observed by Bose (1961). For an introduction to coding theory, see Hedayat, Sloane and Stufken (1999, Chap. 4), MacWilliams and Sloane (1977) and van Lint (1999).

A two-level design is also called a *binary code* in coding theory. From now on, a two-level design takes on values from $Z_2 = \{0, 1\}$ (mod 2). For any row vector x in D, the Hamming weight is the number of non-zero elements in x. Let $W_i(D)$ be the number of row vectors of D with Hamming weight i. The vector $(W_0(D), \ldots, W_n(D))$ is called the *weight distribution* of D.

For two row vectors a and b, the *Hamming distance* $d_H(a, b)$ is the number of places where they differ. Let

$$B_i(D) = N^{-1} \left|\{(a, b) : a, b \text{ are row vectors of } D, \text{ and } d_H(a, b) = i\}\right|.$$

The vector $(B_0(D), B_1(D), \ldots, B_n(D))$ is called the *distance distribution* of D.

A binary code D is said to be *distance invariant* if the weight distributions of its translators $u + D$ are the same for all $u \in D$, where $u + D = \{u + x \pmod 2 : x \in D\}$. Essentially, a distance invariant code has the special characteristics that its distance distribution and weight distribution are the same, assuming that it contains the null vector (i.e., the row with all zeros). Clearly, binary linear codes (i.e., regular designs) are distance invariant.

Xu and Wu (2001) showed that the wordlength pattern is the *MacWilliams transform* of the distance distribution, i.e.,

$$A_j(D) = N^{-1} \sum_{i=0}^{n} P_j(i;n) B_i(D) \text{ for } j = 0, \ldots, n, \tag{3}$$

where $P_j(x;n) = \sum_{i=0}^{j} (-1)^i \binom{x}{i}\binom{n-x}{j-i}$ are the *Krawtchouk polynomials* and $A_0(D) = 1$. By the orthogonality of the Krawtchouk polynomials, it is easy to show that

$$B_j(D) = N\, 2^{-n} \sum_{i=0}^{n} P_j(i;n) A_i(D) \text{ for } j = 0, \ldots, n. \tag{4}$$

The equations (3) and (4) are known as the generalized *MacWilliams identities.*

3. Quaternary Codes and Nonregular Designs

Let G be a $k \times n$ matrix over Z_4. All possible linear combinations of the rows in G over Z_4 form a quaternary linear code, denoted by C. We can write C as a $4^k \times n$ matrix, possibly with duplicated rows. To obtain a two-level design, apply the so-called *Gray map*

$$\phi : 0 \to (0,0), \quad 1 \to (0,1), \quad 2 \to (1,1), \quad 3 \to (1,0).$$

That is, each element in Z_4 is replaced with a pair from 0 and 1. The resulting two-level design, a $4^k \times 2n$ matrix over Z_2, is called the *binary image* of C and denoted by $D = \phi(C)$.

Consider another matrix

$$G' = \begin{pmatrix} G & G \\ 0_n & 2_n \end{pmatrix}, \tag{5}$$

where 0_n and 2_n are row vectors of n 0's and 2's, respectively. Although G' has $k+1$ rows, the quaternary linear code C' generated by G' does not have 4^{k+1} distinct rows, because G' contains a row with only 0 and 2. If C has 4^k distinct rows, C' has 2^{2k+1} distinct rows, each duplicated once. Without loss of generality, we can ignore the duplicated rows and write C' as

$$C' = \begin{pmatrix} C & C \\ C & C+2 \end{pmatrix} \pmod 4.$$

Then its binary image is a $2^{2k+1} \times 4n$ design as follows:

$$D' = \phi(C') = \begin{pmatrix} D & D \\ D & D+1 \end{pmatrix} \pmod 2.$$

Although C and C' are linear over Z_4, D and D' are not necessarily linear over Z_2. Indeed, most of the designs generated are nonlinear and nonregular, because the Gray map ϕ is *not* an additive group homomorphism from Z_4 to Z_2^2. The gray map, originally introduced in communication systems involving four phases, is pivotal in the construction and has some unique properties (e.g., Theorem 3 below).

Example 1. Consider a 2×6 matrix

$$G = \begin{bmatrix} 1 & 0 & 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 1 & 3 \end{bmatrix}.$$

All linear combinations of the two rows of G form a 16×6 linear code C over Z_4. Applying the Gray map, we obtain a 16×12 design $D = \phi(C)$. See Table 1 for the C and D matrices. It is straightforward to verify that D has resolution 3.5; therefore, it is a nonregular design. Moreover, the binary image $D' = \phi(C')$ generated by G' defined in (5) is a 32×24 design with resolution 3.5. For comparison, regular designs of the same sizes have resolution 3 in both cases.

Table 1. An example of quaternary code and nonregular design.

(a) Quaternary code C

Run	1	2	3	4	5	6
1	0	0	0	0	0	0
2	0	1	1	2	1	3
3	0	2	2	0	2	2
4	0	3	3	2	3	1
5	1	0	2	1	1	1
6	1	1	3	3	2	0
7	1	2	0	1	3	3
8	1	3	1	3	0	2
9	2	0	0	2	2	2
10	2	1	1	0	3	1
11	2	2	2	2	0	0
12	2	3	3	0	1	3
13	3	0	2	3	3	3
14	3	1	3	1	0	2
15	3	2	0	3	1	1
16	3	3	1	1	2	0

(b) Nonregular design D

Run	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	1	0	1	1	1	0	1	1	0
3	0	0	1	1	1	1	0	0	1	1	1	1
4	0	0	1	0	1	0	1	1	1	0	0	1
5	0	1	0	0	1	1	0	1	0	1	0	1
6	0	1	0	1	1	0	1	0	1	1	0	0
7	0	1	1	1	0	0	0	1	1	0	1	0
8	0	1	1	0	0	1	1	0	0	0	1	1
9	1	1	0	0	0	0	1	1	1	1	1	1
10	1	1	0	1	0	1	0	0	1	0	0	1
11	1	1	1	1	1	1	1	1	0	0	0	0
12	1	1	1	0	1	0	0	0	0	1	1	0
13	1	0	0	0	1	1	1	0	1	0	1	0
14	1	0	0	1	1	0	0	1	0	0	1	1
15	1	0	1	1	0	0	1	0	0	1	0	1
16	1	0	1	0	0	1	0	1	1	1	0	0

Example 2. Consider a 4×8 matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 3 & 1 & 2 \\ 0 & 0 & 1 & 0 & 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 2 & 3 \end{bmatrix}.$$

All linear combinations of the rows of G over Z_4 form a 256×8 quaternary linear code C. Applying the Gray map, we obtain a 256×16 design $D = \phi(C)$, which is isomorphic to the (extended) Nordstrom-Robinson code. The resulting design D is an $OA(256, 16, 2, 5)$ with many remarkable properties. Xu (2005a) showed that it has resolution 6.5 and projectivity 7. For comparison, for a regular design to achieve the same resolution and projectivity, it would require at least 512 runs. For more statistical properties and results from the Nordstrom-Robinson code, see Xu (2005a).

The corresponding C and D matrices are too large and therefore are not presented. For other forms of generator matrices of the Nordstrom-Robinson code, see Hammons et al. (1994) and Hedayat, Sloane and Stufken (1999, Sec. 5.10).

4. Some Theoretic Results

We first study when a binary image is a useful two-level design. The following lemma gives necessary and sufficient conditions on the generator matrix.

Lemma 1. *Let G be a $k \times n$ matrix over Z_4, C be the quaternary linear code generated by G, and $D = \phi(C)$ be the binary image. Then D is an orthogonal array of strength two if and only if G satisfies the following conditions:*
(i) *it does not have any column containing entries* 0 *and* 2 *only*;
(ii) *none of its column is a multiple of another column over Z_4.*

Proof. *Necessity.* If x is a column of G containing entries 0 and 2 only, then any linear combination of its elements is 0 or 2 over Z_4. A column with entries 0 and 2 only generates two identical columns after applying the Gray map. For any column x, its multiples are λx with $\lambda = 0, 1, 2, 3$ over Z_4. When $\lambda = 0, 2$, λx contains entries 0 and 2 only. When $\lambda = 3$, λx and x generate two identical pairs of columns after applying the Gray map. This proves that the conditions are necessary.

Sufficiency. First, consider the special case when $k = n = 2$. Let G be

$$G = \begin{pmatrix} a & c \\ b & d \end{pmatrix}.$$

Without loss of generality, assume that $a = 1$. Clearly

$$G_1 = \begin{pmatrix} 1 & c \\ b & d \end{pmatrix} \text{ and } G_2 = \begin{pmatrix} 1 & c \\ 0 & d - bc \end{pmatrix} \pmod 4$$

generate the same linear code over Z_4. Because (c, d) is not a multiple of $(a, b) = (1, b)$ over Z_4, $d - bc \neq 0 \pmod 4$. If $d - bc = 1$ or $3 \pmod 4$, then G_2 becomes

$$\begin{pmatrix} 1 & c \\ 0 & 3 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & c \\ 0 & 1 \end{pmatrix}.$$

Both matrices generate the same linear code over Z_4 with 16 distinct runs, regardless of c. The corresponding binary image is a 2^4 full factorial design. If $d - bc = 2 \pmod 4$, c must be 1 or 3 by (i) (otherwise, both c and d are 0 or 2, which violates condition (i)). Then G_2 becomes

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}.$$

Both matrices generate the same linear code over Z_4 with eight distinct runs, each duplicated once. The corresponding binary image is a duplicated 2^{4-1} design with resolution 4.

In general, for a $k \times n$ matrix G, consider any pair of columns. By the assumption on G, we can always choose two rows of G such that the resulting 2×2 submatrix satisfies conditions (i) and (ii). Then the binary image of the linear code corresponding to this pair of columns is either a 2^4 full factorial design, each run being repeated 4^{k-2} times, or a 2^{4-1} design with resolution 4, each run being repeated $2 \times 4^{k-2}$ times. Therefore, any two columns of the binary image D are orthogonal to each other.

Lemma 1 implies that the resulting design D has resolution at least 3. The next result shows that the resolution is indeed at least 3.5.

Lemma 2. *If G satisfies the conditions in Lemma* 1*, then $D = \phi(C)$ has resolution at least* 3.5*.*

Proof. As in the proof of Lemma 1, it is sufficient to look at all possible 3×3 generator matrices. It can be verified that under elementary row and column operations, the generator matrix G satisfying the conditions is equivalent to one of the following

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The first matrix generates a replicated 16×6 design with resolution 3.5, the second generates a replicated 16×6 design with resolution 4, the third generates a replicated 32×6 design with resolution 4, the fourth generates a replicated 32×6 design with resolution 6, and the fifth generates a full 2^6 design. Therefore, the binary image D has resolution at least 3.5.

Lemma 3. *If G satisfies the conditions in Lemma* 1*, then it has a maximum of $(4^k - 2^k)/2$ columns.*

Proof. There are 4^k vectors with k elements over Z_4, among which are 2^k vectors containing 0 and 2 only. If a vector x contains 1 or 3, so does its multiple $3x$

(mod 4). Note that $3x$ (mod 4) is also a multiple of x over Z_4. Therefore, we can only include either x or $3x$ in the generator matrix G as a column. Because there are $4^k - 2^k$ vectors containing 1 or 3, the generator matrix G has a maximum of $(4^k - 2^k)/2$ columns.

The proof of Lemma 3 implies that there exists a $k \times n$ generator matrix with $n = (4^k - 2^k)/2$ satisfying the conditions in Lemma 1. To be specific, such a matrix can be constructed as follows.

1. Write down all possible columns of k elements over Z_4.
2. Delete columns that do not contain any 1's.
3. Delete columns whose first non-zero and non-two entries are 3's.

Combining Lemmas 2, and 3, we have the following result.

Theorem 1. *For an integer $k > 1$, let G be the generator matrix obtained from the above procedure. Then the binary image D generated by G is a $4^k \times (4^k - 2^k)$ design with resolution* 3.5*.*

Theorem 1 shows that as long as $n \leq 4^k - 2^k$, we can always construct a $4^k \times n$ design with resolution 3.5 or higher. The condition $n \leq 4^k - 2^k$ is, however, not necessary for the existence of resolution 3.5 designs. For example, there exists a 16×14 design with resolution 3.5; see Deng and Tang (2002).

The nonregular design constructed in Theorem 1 has $4^k = 2^{2k}$ runs. We can construct designs with 2^{2k+1} runs using the generator matrix G' in (5). Clearly, if G satisfies the conditions in Lemma 1, so does G'. Combining Lemma 2 and Theorem 1, we have the following result.

Theorem 2. *For an integer $k > 1$, let G be the generator matrix in Theorem* 1 *and define G' by* (5)*. Then the binary image D' generated by G' is a $2^{2k+1} \times (2^{2k+1} - 2^{k+1})$ design with resolution* 3.5*.*

Note that the nonregular designs constructed in Theorems 1 and 2 have resolution 3.5. It is well known that for $n > 2^{k-1}$, a regular design with 2^k runs and n factors has resolution at most 3. Therefore, nonregular designs constructed from quaternary codes have higher resolution than corresponding regular designs when resolution 4 designs do not exist.

Now consider some computation issues. Note that the calculation of the wordlength pattern can be cumbersome according to definition (2), especially when n is large. An alternative is to compute the distance distribution and then apply the MacWilliams transform (3) to obtain the wordlength pattern. However, the calculation of the distance distribution can also be cumbersome, especially when the run sizes become large. The next theorem, Theorem 2 of Hammons et al. (1994), shows that binary images of quaternary codes are distance invariant. As a result, we can use the weight distribution instead of the distance distribution. The weight distribution is substantially easier to compute than the distance distribution and a large amount of computing time can be saved.

Theorem 3. *For any quaternary linear code C, its binary image $D = \phi(C)$ is distance invariant.*

5. A Systematic Construction Procedure

To obtain a collection of useful nonregular designs, we take a sequential approach as done by Chen, Sun and Wu (1993) and Xu (2005b). Specifically, assume that we have a set of quaternary codes with n columns. We construct a set of quaternary codes with $(n+1)$ columns by adding a column to the generator matrices from the unused columns. Two-level designs are then obtained as binary images of quaternary codes. To eliminate redundant designs, all designs are divided into different categories according to their weight distributions and moment projection patterns. The moment projection pattern counts the frequency of the values of moments of projection designs; see Xu (2005b) for more details. Designs in different categories are nonisomorphic. Whether designs in the same category are isomorphic can be determined by performing a time consuming isomorphism check. We do not perform isomorphism checks since empirical study of regular designs suggests that they are usually not necessary for designs with 16, 32 and 64 runs. Also note that it is impractical to perform isomorphism checks for designs with 128 runs and beyond because of the huge numbers of designs encountered. Indeed, we have to limit the number of designs generated for 128 and 256 runs. We keep a maximum of 120,000 designs for each n and rank them by the minimum G_2-aberration criterion; however, only the top 40,000 designs are used to construct new designs for the next n. These numbers are chosen arbitrarily.

To build a catalog, we choose two best designs among all designs according to the minimum G_2 and G-aberration criteria. It should be noted, however, that the two designs could be the same, in which case, the catalog only includes one design. To save computation time, we use a weak version of the minimum G-aberration criterion, so for designs with resolution $r \le R < r+1$, we only compute and compare the frequency of $J_r(s)$ values.

The above procedure generates designs with even numbers of columns. To obtain designs with odd numbers of columns, we simply delete one column. When doing so, we limit to the one or two designs of the same run size that are already included in the catalog. We try all possible deletions and choose two best designs according to the minimum G_2 and G-aberration criteria.

Some designs with 32 and 128 runs in the catalog are constructed as follows. We observe that sometimes better designs can be derived from other designs via the half fraction method. For example, from a $(2N) \times n$ design, we obtain an $N \times (n-1)$ designs by taking half of the rows whose components are 0 for any particular column and deleting that column. When doing so, we again limit to the one or two designs of run size $2N$ that are already included in the catalog. We try all possible fractions and choose two best designs of N runs according to the minimum G_2 and G-aberration criteria. After fractionation, we further consider deleting one or more columns from these N-run designs.

It should be noted that when deleting columns or taking fractions, wordlength patterns have to be calculated using the distance distributions, instead of the weight distributions.

6. Tables of Designs

With the construction method described in the last section, we obtain a collection of designs for 16, 32, 64, 128, 256 runs and up to 64 factors; see Tables 2−6.

The first column of these tables is the name of the designs. Designs with n factors and 2^{n-m} runs are labeled as n-m.a, n-m.c or n-m.ac. An "a" designation corresponds to designs identified by the minimum G_2-aberration criterion, a "c" designation by the minimum G-aberration criterion and an "ac" designation by both criteria.

The second and the third columns are the wordlength pattern (WLP) and the resolution (R) of the designs, respectively. Because all designs have resolution between 3 and 8, we only present A_3 up to A_8 for wordlength patterns.

The fourth column is the simplified confounding frequency vector (CFV). For a design D with resolution $r \leq R < r+1$, all possible nonzero J_r values and their frequencies are given as $J : f$, where J is the J_r value and f is the frequency.

The last column shows how the design can be constructed. If the design is a binary image of a quaternary linear code, the generator matrix is given in terms of column indexes, where a column $u = (u_0, \ldots, u_{k-1})$ is represented by its index $\sum_{i=0}^{k-1} 4^i u_i$. For example, design 12-8.ac in Table 2 has column indexes: 1, 4, 6, 9, 5 and 13. The corresponding generator matrix is presented in Example 1 and the design is given in Table 1(b). As another example, design 16-8.ac in Table 6 has column indexes: 1, 4, 16, 64, 84, 109, 181 and 217. The corresponding generator matrix is presented in Example 2, and the design is isomorphic to the Nordstrom-Robinson code.

If a design is derived from another design, the original design is given with the column number that is being deleted or fractionated. Whether a design is obtained by deletion or fractionation should be clear from the labeling of the designs. For example, for design 11-7.ac in Table 2, the column index is 12-8.ac(1). This means that the 11-7.ac design is obtained by deleting the first column of the 12-8.ac design, i.e., the design in Table 1(b) without the first column. As another example, for design 15-8.ac in Table 5, the column index is 16-8.ac(1). Note that 15-8.ac has $2^{15-8} = 128$ runs while 16-8.ac has $2^{16-8} = 256$ runs. This means that the former is obtained by taking the half fraction of 16-8.ac whose first components are 0 and deleting the first column.

Table 2. 16-Run Designs.

Design	WLP	R	CFV	Column Indexes
6-2.ac**	0 3 0 0	4.0	16:3	1 4 6
7-3.ac**	0 7 0 0 0	4.0	16:7	8-4.ac(1)
8-4.ac**	0 14 0 0 0 1	4.0	16:14	1 4 6 9
9-5.ac**	4 14 8 0 4 1	3.5	8:16	10-6.ac(9)
10-6.ac**	8 18 16 8 8 5	3.5	8:32	1 4 6 9 5
11-7.ac**	12 26 28 24 20 13	3.5	8:48	12-8.ac(1)
12-8.ac**	16 39 48 48 48 39	3.5	8:64	1 4 6 9 5 13

Chen, Sun and Wu (1993) gave MA regular designs of 16, 32 and 64 runs up to 32 factors. MA regular designs with 64 runs and more than 32 factors can be obtained by the complementary design technique; see Chen and Hedayat (1996), Tang and Wu (1996) and Butler (2003a). Based on a conjecture, Block and Mee (2005) gave MA regular designs of 128 runs up to 64 factors. With computer random search, Block (2003) gave some 256-run designs up to 80 factors. These are the best known regular designs in terms of aberration in the literature.

We compare our "a" and "ac" designs with MA or best regular designs of the same size in terms of aberration using wordlength patterns. The results are denoted with different number of asterisks after the name of the design. Our design may have more aberration (*) than, the same aberration (**) as , or less aberration (***) than the MA regular design.

We also compare our designs with regular designs in terms of resolution and G-aberration. With the exception of 17-9.a and 17-9.c in Table 6, all designs in this catalog have the same resolution as, or larger resolution than, the corresponding MA regular designs; with the exception of 17-9.c, all of the "c" designs have less G-aberration than MA regular designs.

6.1. Designs of 16 runs

By Theorem 1, we can construct 16-run nonregular designs up to 12 columns with resolution at least 3.5. Table 2 shows the best designs for 6 to 12 columns. All designs are labeled with two asterisks, implying that they have the same aberration as competing MA regular designs. Designs with 6−8 columns in

Table 2 have resolution 4, which is the same as MA regular designs. In fact, these designs are isomorphic to MA regular designs. Designs with 9−12 columns in Table 2 have resolution 3.5, whereas regular designs of the same size have resolution 3.

Deng and Tang (2002) studied nonregular designs from five Hadamard matrices of order 16. Sun, Li and Ye (2002) showed that all 16-run designs with resolution 3 or higher are projection designs of these five Hadamard matrices. Therefore, the designs in Table 2 are not new; indeed, design 12-8.ac is isomorphic to design 16.12.3 in Deng and Tang (2002). It is interesting to note that all designs in Table 2 have minimum G_2-aberration and maximum resolution among all possible designs.

6.2. Designs of 32 runs

By Theorem 2, we can construct 32-run nonregular designs up to 24 columns with resolution at least 3.5. Table 3 shows the best designs for 7 to 24 columns. All designs are nonregular and have less G-aberration than MA regular designs.

Designs with 7 to 9 columns have higher resolution than and the same aberration as MA regular designs. These nonregular designs have resolution 4.5 whereas MA regular designs have resolution 4. Designs with 10 to 16 columns have the same resolution as MA regular designs. All but one "a" or "ac" designs have the same aberration as MA regular designs. Design 10-5.a has more aberration than MA regular design. Designs with 17 to 24 columns have the same aberration as MA regular designs, with the exception of the 20 and 21-column designs, which have slightly more aberration (same A_3 but larger A_4). These designs, however, have resolution 3.5 whereas MA regular designs have resolution 3.

Note that designs 7-2.ac and 9-4.ac are half fractions of the 64-run designs 8-2.ac and 10-4.ac given in Table 4. The 10 to 16-column designs are all generated from one single 64-run design, 18-12.c, via fractionation and deletion.

It is of interest to compare designs in Table 3 with other nonregular designs, for example, those derived from Hadamard matrices of order 32. Unlike the 16-run case, best 32-run designs from Hadamard matrices are still unknown. It is beyond the scope of this paper to fully investigate best nonregular designs of 32 runs. Here we consider only six Hadamard matrices of order 32 from Sloane's web site (http://www.research.att.com/∼njas/hadamard/). To obtain the "best" projection designs, we use a naive sequential search algorithm that keeps only one design at each step for each criterion. We find that all designs labeled by two asterisks in Table 3 are still the best in terms of aberration. Indeed, according to Butler (2003b, 2004) and Xu (2005a), for 7, 8, 11−18, 23 and 24 columns, designs in Table 3 have minimum G_2-aberration among *all* possible designs. In terms of resolution, designs in Table 3 are the best for 7 to 16 columns, but not for 17 to 24 columns. In particular, all projection designs from the Paley-type Hadamard matrix have resolution 3.75 for 17 to 24 columns; however, these designs are not competitive in terms of aberration.

Table 3. 32-Run Designs.

Design	WLP	R	CFV	Column Indexes
7-2.ac**	0 1 2 0 0	4.5	16:4	8-2.ac(3)
8-3.ac**	0 3 4 0 0 0	4.5	16:12	9-4.ac(1)
9-4.ac**	0 6 8 0 0 1	4.5	16:24	10-4.ac(5)
10-5.a*	0 15.75 0 12.75 0 2.25	4.0	32:5 16:43	11-6.c(5)
10-5.c	0 16 0 12 0 3	4.0	32:3 16:52	11-6.c(11)
11-6.a**	0 25 0 27 0 10	4.0	32:8 16:68	12-7.ac(8)
11-6.c	0 25.5 0 25.5 0 11.5	4.0	32:6 16:78	12-7.ac(12)
12-7.ac**	0 38 0 52 0 33	4.0	32:10 16:112	13-8.ac(13)
13-8.ac**	0 55 0 96 0 87	4.0	32:16 16:156	14-9.ac(14)
14-9.ac**	0 77 0 168 0 203	4.0	32:23 16:216	15-10.ac(14)
15-10.ac**	0 105 0 280 0 435	4.0	32:33 16:288	16-11.ac(16)
16-11.ac**	0 140 0 448 0 870	4.0	32:44 16:384	Δ
17-12.ac**	8 140 112 448 504	3.5	16:32	18-13.ac(17)
18-13.ac**	16 148 224 560 1008	3.5	16:64	1 4 33 9 36 6 38 41 5
19-14.ac**	24 164 344 784 1624	3.5	16:96	20-15.ac(17)
20-15.ac*	32 189 480 1120 2464	3.5	16:128	1 4 33 9 36 6 38 41 5 13
21-16.ac*	40 221 640 1600 3648	3.5	16:160	22-17.ac(17)
22-17.ac**	48 263 832 2224 5312	3.5	16:192	1 4 33 9 36 6 38 41 5 13 37
23-18.ac**	56 315 1064 3024 7616	3.5	16:224	24-19.ac(1)
24-19.ac**	64 378 1344 4032 10752	3.5	16:256	1 4 33 9 36 6 38 41 5 13 37 45

Δ: Obtained by taking half of the runs of 18-12.c whose fifth column is 0 and omitting the fifth and sixth columns.

6.3. Designs of 64 runs

Table 4 shows the best designs of 64 runs for 8 to 56 columns with resolution 3.5 or higher. Designs with 8−14 columns have higher resolution than MA regular designs. The MA regular design with 8 columns has resolution 5, while our design has resolution 5.5. MA regular designs with 9−14 columns have resolution 4, while our designs have resolution 4.5. Designs with 8−12 columns have the same aberration as MA regular designs. Designs with 13 and 14 columns have less aberration than MA regular designs. According to Xu (2005a), designs with 8, 9 and 12−14 columns in Table 4 have minimum G_2-aberration among *all* possible designs.

Designs with 15−32 columns have the same resolution as MA regular designs. Most of these designs have the same aberration as MA regular designs, except for a few designs (with 15, 16, 21 and 22 columns) having slightly more aberration. Designs with 33−56 columns have resolution 3.5 whereas MA regular designs have resolution 3.

Table 4. 64-Run Designs.

Design	WLP	R	CFV	Column Indexes
8-2.ac**	0 0 2 1 0 0	5.5	32:8	1 4 16 22
9-3.ac**	0 1 4 2 0 0	4.5	32:4	10-4.ac(1)
10-4.ac**	0 2 8 4 0 1	4.5	32:8	1 4 16 22 25
11-5.ac**	0 4 14 8 0 3	4.5	32:16	12-6.ac(1)
12-6.ac**	0 6 24 16 0 9	4.5	32:24	1 4 16 22 25 45
13-7.ac***	0 10 36 28 8 21	4.5	32:40	14-8.ac(1)
14-8.ac***	0 14 56 49 16 49	4.5	32:56	1 4 16 22 25 45 53
15-9.ac*	0 33 54 60 108	4.0	64:21 32:48	16-10.a(3)
16-10.a*	0 47 72 98 192	4.0	64:31 32:64	1 4 16 22 25 33 36 54
16-10.c	0 60 0 256 0	4.0	64:28 32:128	1 4 16 6 24 33 21 29
17-11.a**	0 59 108 150 324	4.0	64:59	18-12.a(3)
17-11.c	0 64 96 156 320	4.0	64:40 32:96	18-12.c(1)
18-12.a**	0 78 144 228 528	4.0	64:78	1 4 16 22 9 33 24 36 54
18-12.c	0 84 128 240 512	4.0	64:52 32:128	1 4 16 22 25 33 36 54 57
19-13.a**	0 100 192 336	4.0	64:100	20-14.a(1)
19-13.c	0 131 0 847	4.0	64:71 32:240	20-14.c(19)
20-14.a**	0 125 256 480	4.0	64:125	1 4 16 22 9 33 24 36 54 41
20-14.c	0 166 0 1194	4.0	64:94 32:288	1 4 16 6 24 33 21 29 53 9
21-15.ac*	0 205 0 1672	4.0	64:115 32:360	22-16.c(7)
22-16.a*	0 251 0 2296	4.0	64:155 32:384	1 4 16 6 24 33 21 29 9 41 18
22-16.c	0 252 0 2288	4.0	64:144 32:432	1 4 16 6 24 33 21 9 18 36 29
23-17.a**	0 304 0 3105	4.0	64:178 32:504	24-18.a(1)
23-17.c	0 305 0 3096	4.0	64:170 32:540	24-18.c(1)
24-18.a**	0 365 0 4138	4.0	64:221 32:576	1 4 16 6 24 33 21 29 9 41 18 53
24-18.c	0 366 0 4128	4.0	64:204 32:648	1 4 16 6 24 33 21 9 18 36 29 53
25-19.a**	0 435 0 5440	4.0	64:255 32:720	26-20.ac(9)
25-19.c	0 436 0 5430	4.0	64:247 32:756	26-20.1(1)
26-20.ac**	0 515 0 7062	4.0	64:299 32:864	1 4 16 6 24 33 21 29 9 41 18 53 36
27-21.ac**	0 605 0 9075	4.0	64:353 32:1008	28-22.ac(1)
28-22.ac**	0 706 0 11548	4.0	64:418 32:1152	1 4 16 6 24 33 21 29 9 41 18 53 36 26
29-23.ac**	0 819 0 14560	4.0	64:483 32:1344	30-24.ac(1)
30-24.ac**	0 945 0 18200	4.0	64:561 32:1536	1 4 16 6 24 33 21 29 9 41 18 53 36 26 38
31-25.ac**	0 1085 0 22568	4.0	64:637 32:1792	32-26.ac(1)
32-26.ac**	0 1240 0 27776	4.0	64:728 32:2048	1 4 16 6 24 33 21 29 9 41 18 53 36 26 38 61
33-27.ac**	16 1240 1120	3.5	32:64	34-28.ac(33)
34-28.ac**	32 1256 2240	3.5	32:128	1 4 16 6 24 33 21 29 9 41 18 53 36 26 38 61 5
35-29.ac**	48 1288 3376	3.5	32:192	36-30.ac(33)
36-30.ac**	64 1336 4544	3.5	32:256	1 4 16 6 24 33 21 29 9 41 18 53 36 26 38 61 5 17
37-31.ac**	80 1400 5760	3.5	32:320	38-32.ac(33)
38-32.ac**	96 1480 7040	3.5	32:384	1 4 16 6 24 33 21 29 9 41 18 53 36 26 38 61 5 17 20
39-33.ac*	112 1578 8400	3.5	32:448	40-34.ac(33)
40-34.ac*	128 1693 9856	3.5	32:512	1 4 16 6 24 33 21 29 9 41 18 53 36 26 38 61 5 17 20 45
41-35.ac*	144 1825 11424	3.5	32:576	42-36.ac(35)
42-36.ac*	160 1976 13120	3.5	32:640	1 4 16 6 24 33 21 29 9 41 18 53 36 26 38 61 5 17 13 37 25
43-37.ac**	176 2145 14960	3.5	32:704	44-38.ac(37)
44-38.ac**	192 2334 16960	3.5	32:768	1 4 16 6 24 33 21 29 9 41 18 53 36 26 38 61 5 17 13 37 25 49
45-39.ac**	208 2543 19136	3.5	32:832	46-40.ac(33)
46-40.ac**	224 2773 21504	3.5	32:896	1 4 16 6 24 33 21 29 9 41 18 53 36 26 38 61 5 17 13 37 25 49 45
47-41.ac**	240 3025 24080	3.5	32:960	48-42.ac(1)
48-42.ac**	256 3300 26880	3.5	32:1024	1 4 16 6 24 33 21 29 9 41 18 53 36 26 38 61 5 17 13 37 25 49 45 57
49-43.ac**	280 3556 29904	3.5	32:1120	50-44.ac(49)
50-44.ac**	304 3836 33184	3.5	32:1216	1 4 16 6 24 33 21 29 9 41 18 53 36 26 38 61 5 17 13 37 25 49 45 57 20
51-45.ac**	328 4140 36744	3.5	32:1312	52-46.ac(49)
52-46.ac*	352 4469 40608	3.5	32:1408	1 4 16 6 24 33 21 29 9 41 18 53 36 26 38 61 5 17 13 37 25 49 45 57 20 22
53-47.ac*	376 4821 44800	3.5	32:1504	54-48.ac(49)
54-48.ac**	400 5199 49344	3.5	32:1600	1 4 16 6 24 33 21 29 9 41 18 53 36 26 38 61 5 17 13 37 25 49 45 57 20 22 52
55-49.ac**	424 5603 54264	3.5	32:1696	56-50.ac(1)
56-50.ac**	448 6034 59584	3.5	32:1792	1 4 16 6 24 33 21 29 9 41 18 53 36 26 38 61 5 17 13 37 25 49 45 57 20 22 52 54

Using the doubling technique (Chen and Cheng (2006)), one can construct nonregular designs with resolution 3.75 for 33 to 56 columns; however, these designs are less competitive in terms of aberration. According to Butler (2003b, 2004), the designs with 24, 28−34, 47−50 and 56 columns in Table 4 have minimum G_2-aberration among *all* possible designs.

6.4. Designs of 128 runs

Table 5 shows the best designs of 128 runs for 9 to 64 factors with resolution 4 or higher. Designs with 10−15 columns have resolution 5.5 whereas MA regular designs have resolution 5 for 10−11 columns and resolution 4 for 12−15 columns. Designs with 12−15 columns also have less aberration than MA regular designs. According to Xu (2005a), designs with 9 and 13−15 columns in Table 5 have minimum G_2-aberration among all possible designs.

For 16−19 columns, all of the "c" designs have resolution 4.5 whereas MA regular designs (and all of the "a" designs) have resolution 4. Designs with 20−64 columns have resolution 4, the same as MA regular designs. For 19−28 columns, all of the "a" and "ac" designs have less aberration than MA regular designs, with the exception of 23-16.ac, which has slightly more aberration. For example, design 19-12.a has wordlength pattern $(0, 25, 132, \ldots)$, while the MA regular design given by Block and Mee (2005) has wordlength pattern $(0, 27, 120, \ldots)$. Designs with 29−64 columns either have the same aberration as, or more aberration than, MA regular designs. According to Butler (2004), designs with 60−64 columns in Table 5 have minimum G_2-aberration among all possible designs.

Note that designs with 9, 11, 13, 15 and 17 columns are half fractions of 256-run designs given in Table 6. Designs 18-11.a and 19-12.c are also derived from 256-run designs.

6.5. Designs of 256 runs

Table 6 shows the best designs of 256 runs for 10 to 64 factors with resolution 4 or higher. Designs with 11−16 columns and 10-2.c have resolution 6.5. All of the "c" designs with 17−30 columns have resolution 4.5. All designs with 31−64 columns have resolution 4. In comparison, MA regular designs have resolution 6 for 10−12 columns, resolution 5 for 13−17 columns and resolution 4 for 18−64 columns. Note that designs 17-9.a and 17-9.c have smaller resolution than the MA regular design and therefore are not recommended.

Compared to the best regular designs given by Block (2003) in terms of aberration, 22 designs in Table 6 (with 13−16, 24, 32−34, 41−46 and 49−56 columns) have less aberration, while other designs either have the same or more aberration. According to Xu (2005a), designs with 14−16 columns in Table 6 have minimum G_2-aberration among all possible designs.

Table 5. 128-Run Designs.

Design	WLP	R	CFV	Column Indexes
9-2.ac**	0 0 0 3 0 0	6.0	128:1 64:8	10-2.a(1)
10-3.ac**	0 0 3 3 1 0	5.5	64:12	11-4.ac(2)
11-4.ac**	0 0 6 6 2 1	5.5	64:24	12-4.ac(1)
12-5.ac***	0 0 11 13 2 1	5.5	64:44	13-6.ac(2)
13-6.ac***	0 0 18 24 4 3	5.5	64:72	14-6.ac(1)
14-7.ac***	0 0 28 42 8 7	5.5	64:112	15-8.ac(1)
15-8.ac***	0 0 42 70 15 15	5.5	64:168	16-8.ac(1)
16-9.a**	0 10 48 72 80 90	4.0	128:2 64:32	1 4 16 149 22 180 25 185
16-9.c	0 11 47.5 71 76.5	4.5	64:32 32:48	17-10.c(14)
17-10.a*	0 15 64 116 130	4.0	128:3 64:32 32:64	18-10.a(1)
17-10.c	0 16 65 105 135	4.5	64:48 32:64	18-10.c(13)
18-11.a**	0 20 80 200 192	4.0	128:4 32:256	Δ
18-11.c	0 24 88 142 228	4.5	64:64 32:128	19-12.c(7)
19-12.a***	0 25 132 223 308	4.0	128:15 64:40	20-13.a(19)
19-12.c	0 32 116 206 370	4.5	64:96 32:128	20-12.c(7)
20-13.a***	0 32 176 316 472	4.0	128:18 64:56	1 4 16 149 22 25 181 45 157 53
20-13.c	0 39 152 308 568	4.0	128:15 64:96	1 4 16 149 22 180 25 45 134 154
21-14.a***	0 42 224 434 744	4.0	128:28 64:56	22-15.a(7)
21-14.c	0 52 196 411 864	4.0	128:21 64:124	22-15.c(21)
22-15.a***	0 56 280 581 1136	4.0	128:42 64:56	1 4 16 149 22 25 181 45 157 53 189
22-15.c	0 66 254 544 1274	4.0	128:28 64:152	1 4 16 149 22 180 25 45 134 154 53
23-16.ac*	0 83 318 728	4.0	128:36 64:188	24-17.ac(1)
24-17.ac***	0 101 400 962	4.0	128:45 64:224	1 4 16 149 22 180 25 45 134 154 53 137
25-18.ac***	0 123 492 1264	4.0	128:55 64:272	26-19.ac(1)
26-19.ac***	0 146 608 1640	4.0	128:66 64:320	1 4 16 149 22 180 25 45 134 154 53 137 173
27-20.ac***	0 174 736 2112	4.0	128:78 64:384	28-21.ac(1)
28-21.ac***	0 203 896 2688	4.0	128:91 64:448	1 4 16 149 22 180 25 45 134 154 53 137 173 177
29-22.a*	0 290 810 3734	4.0	128:290	30-23.a(3)
29-22.c	0 315 608 4712	4.0	128:123 64:768	30-23.c(1)
30-23.a*	0 336 972 4651	4.0	128:336	1 4 16 133 37 146 164 24 26 161 6 144 45 169 152
30-23.c	0 369 704 5976	4.0	128:145 64:896	1 4 16 149 22 25 141 144 146 36 173 54 181 33 57
31-24.a*	0 391 1134 5827	4.0	128:391	32-25.a(5)
31-24.c	0 417 832 7576	4.0	128:161 64:1024	32-25.10(15)
32-25.a**	0 452 1322 7219	4.0	128:452	1 4 16 133 37 146 164 24 26 161 6 144 45 169 152 18
32-25.c	0 480 960 9440	4.0	128:192 64:1152	1 4 16 149 22 25 141 144 146 36 173 152 33 54 57 181
33-26.a**	0 518 1543 8863	4.0	128:518	34-27.a(1)
33-26.c	0 540 1120 11756	4.0	128:220 64:1280	34-27.10(15)
34-27.a*	0 597 1764 10882	4.0	128:597	1 4 16 133 37 146 164 24 26 161 6 144 45 169 152 18 9
34-27.c	0 616 1280 14432	4.0	128:264 64:1408	1 4 16 149 22 25 141 144 146 152 154 33 57 36 54 173 181
35-28.a*	0 674 2058 13140	4.0	128:674	36-29.a(1)
35-28.c	0 849 0 25358	4.0	128:321 64:2112	36-29.44(3)
36-29.a*	0 766 2352 15890	4.0	128:766	1 4 16 133 37 146 164 24 26 161 6 144 45 169 152 18 9 141
36-29.c	0 957 0 30403	4.0	128:369 64:2352	1 4 16 133 38 148 9 165 145 18 185 180 21 61 6 29 153 150
37-30.a**	0 854 2744 18886	4.0	128:854	38-31.a(1)
37-30.c	0 1075 0 36262	4.0	128:412 64:2652	38-31.c(31)
38-31.a**	0 959 3136 22512	4.0	128:959	1 4 16 133 37 146 164 24 26 161 6 144 45 169 152 18 9 141 166
38-31.c	0 1205 0 43016	4.0	128:467 64:2952	1 4 16 133 38 148 9 165 145 18 185 180 21 61 6 29 153 150 24
39-32.a**	0 1071 3584 26656	4.0	128:1071	40-33.a(1)
39-32.c	0 1342 0 50845	4.0	128:514 64:3312	40-33.c(19)
40-33.a**	0 1190 4096 31360	4.0	128:1190	1 4 16 133 37 146 164 24 26 161 6 144 45 169 152 18 9 141 166 154
40-33.c	0 1493 0 59790	4.0	128:575 64:3672	1 4 16 133 38 148 9 165 145 18 185 180 21 61 6 29 153 150 24 33
41-34.a**	0 1648 0 70146	4.0	128:1000 64:2592	42-35.a(41)
41-34.c	0 1653 0 70062	4.0	128:627 64:4104	42-35.c(1)
42-35.a*	0 1824 0 81792	4.0	128:1104 64:2880	1 4 16 129 26 164 18 152 21 149 33 36 181 6 132 9 161 134 169 189 29
42-35.c	0 1827 0 81739	4.0	128:693 64:4536	1 4 16 133 38 148 9 165 145 18 185 180 21 61 6 29 153 150 24 33 141
43-36.a*	0 2012 0 95040	4.0	128:1220 64:3168	44-37.a(7)
43-36.c	0 2017 0 94951	4.0	128:775 64:4968	44-37.c(5)

Δ: Obtained by taking half of the runs of 20-12.a whose first column is 0 and omitting the first two columns.

Table 5. 128-Run Designs (Continued).

Design	WLP	R	CFV	Column Indexes
44-37.a*	0 2215 0 110016	4.0	128:1351 64:3456	1 4 16 129 26 164 18 152 21 149 33 36 181 6 132 9 161 134 169 189 29 53
44-37.c	0 2222 0 109888	4.0	128:872 64:5400	1 4 16 133 38 148 9 165 145 18 185 180 21 61 6 29 153 150 24 33 141 26
45-38.a*	0 2433 0 126902	4.0	128:1497 64:3744	46-39.a(21)
45-38.c	0 2441 0 126758	4.0	128:965 64:5904	46-39.c(3)
46-39.a*	0 2667 0 145892	4.0	128:1659 64:4032	1 4 16 129 26 164 18 152 21 149 33 36 181 6 132 9 161 134 169 189 29 53 41
46-39.c	0 2677 0 145716	4.0	128:1075 64:6408	1 4 16 133 38 148 9 165 145 18 185 180 21 61 6 29 153 150 24 33 141 26 36
47-40.a**	0 2915 0 167244	4.0	128:1727 64:4752	48-41.a(1)
47-40.c	0 2925 0 167052	4.0	128:1179 64:6984	48-41.c(3)
48-41.a**	0 3180 0 191136	4.0	128:1884 64:5184	1 4 16 129 26 164 18 152 21 149 33 36 181 6 132 9 161 134 169 189 29 61 144 154
48-41.c	0 3192 0 190896	4.0	128:1302 64:7560	1 4 16 133 38 148 9 165 145 18 185 180 21 61 6 29 153 150 24 33 141 26 36 182
49-42.a**	0 3466 0 217734	4.0	128:2062 64:5616	50-43.a(49)
49-42.c	0 3478 0 217494	4.0	128:1417 64:8244	50-43.c(1)
50-43.a**	0 3770 0 247368	4.0	128:2258 64:6048	1 4 16 129 26 164 18 152 21 149 33 36 181 6 132 9 161 134 169 189 29 61 144 154 24
50-43.c	0 3785 0 247074	4.0	128:1553 64:8928	1 4 16 133 38 148 9 165 145 18 185 180 21 61 6 29 153 150 24 33 141 26 36 182 41
51-44.a*	0 4092 0 280324	4.0	128:2508 64:6336	52-45.a(1)
51-44.c	0 4107 0 280023	4.0	128:1679 64:9712	52-45.c(1)
52-45.a**	0 4433 0 316888	4.0	128:2705 64:6912	1 4 16 129 26 164 18 152 21 149 33 36 181 6 132 9 161 134 169 189 29 61 144 154 24 146
52-45.c	0 4452 0 316504	4.0	128:1828 64:10496	1 4 16 133 38 148 9 165 145 18 185 180 21 61 6 29 153 150 24 33 141 26 36 182 41 53
53-46.a**	0 4797 0 357292	4.0	128:2925 64:7488	54-47.a(35)
53-46.c	0 4813 0 356952	4.0	128:1965 64:11392	54-47.c(1)
54-47.a*	0 5183 0 401900	4.0	128:3167 64:8064	1 4 16 129 26 164 18 152 21 149 33 36 181 6 132 9 161 134 169 189 29 61 144 154 24 146 38
54-47.c	0 5199 0 401552	4.0	128:2127 64:12288	1 4 16 133 38 148 9 165 145 18 185 180 21 61 6 29 153 150 24 33 141 26 36 182 41 53 173
55-48.a*	0 5590 0 451100	4.0	128:3361 64:8916	56-49.a(1)
55-48.c	0 5603 0 450800	4.0	128:2275 64:13312	56-49.c(1)
56-49.a**	0 6020 0 505232	4.0	128:3620 64:9600	1 4 16 129 26 164 18 152 21 149 33 36 181 6 132 9 161 134 169 189 29 61 144 154 24 38 41 157
56-49.c	0 6034 0 504896	4.0	128:2450 64:14336	1 4 16 133 38 148 9 165 145 18 185 180 21 61 6 29 153 150 24 33 141 26 36 182 41 53 173 177
57-50.a**	0 6475 0 564655	4.0	128:3927 64:10192	58-51.ac(1)
57-50.c	0 6475 0 564655	4.0	128:3903 64:10288	58-51.1(17)
58-51.ac**	0 6955 0 629798	4.0	128:4211 64:10976	1 4 16 129 26 164 18 152 21 149 33 36 181 6 132 9 161 134 169 189 29 61 144 154 24 38 41 157 53
59-52.ac**	0 7461 0 701091	4.0	128:4521 64:11760	60-53.ac(1)
60-53.ac**	0 7994 0 778988	4.0	128:4858 64:12544	1 4 16 129 26 164 18 152 21 149 33 36 181 6 132 9 161 134 169 189 29 61 144 154 24 38 41 157 53 137
61-54.ac**	0 8555 0 863968	4.0	128:5195 64:13440	62-55.ac(1)
62-55.ac**	0 9145 0 956536	4.0	128:5561 64:14336	1 4 16 129 26 164 18 152 21 149 33 36 181 6 132 9 161 134 169 189 29 61 144 154 24 38 41 157 53 137 146
63-56.ac**	0 9765 0 1057224	4.0	128:5925 64:15360	64-57.ac(1)
64-57.ac**	0 10416 0 1166592	4.0	128:6320 64:16384	1 4 16 129 26 164 18 152 21 149 33 36 181 6 132 9 161 134 169 189 29 61 144 154 24 38 41 157 53 137 146 166

Table 6. 256-Run Designs.

Design	WLP	R	CFV	Column Indexes
10-2.a**	0 0 0 1 2 0	6.0	256:1	1 4 16 64 90
10-2.c	0 0 0 2 0 1	6.5	128:8	1 4 16 64 86
11-3.ac**	0 0 0 6 0 1	6.5	128:24	12-4.ac(1)
12-4.ac**	0 0 0 12 0 3	6.5	128:48	1 4 16 64 86 109
13-5.ac***	0 0 0 24 0 3	6.5	128:96	14-6.ac(1)
14-6.ac***	0 0 0 42 0 7	6.5	128:168	1 4 16 64 86 109 181
15-7.ac***	0 0 0 70 0 15	6.5	128:280	16-8.ac(1)
16-8.ac***	0 0 0 112 0 30	6.5	128:448	1 4 16 64 86 109 181 217
17-9.a*	0 1 30 73 76	4.0	256:1	18-10.a(15)
17-9.c	0 2 31 67 73	4.5	128:8	18-10.c(15)
18-10.a*	0 3 40 104 113	4.0	256:3	1 4 16 64 86 109 181 25 153
18-10.c	0 4 44 92 116	4.5	128:16	1 4 16 64 86 109 181 25 37
19-11.a**	0 4 48 168 208	4.0	256:4	20-12.a(1)
19-11.c	0 7 59 126 184	4.5	128:28	20-12.c(15)
20-12.a**	0 5 64 240 320	4.0	256:5	1 4 16 64 85 26 98 125 137 164
20-12.c	0 10 80 172 276	4.5	128:40	1 4 16 64 86 109 25 133 53 180
21-13.a*	0 13 88 276	4.0	256:1 128:48	22-14.a(21)
21-13.c	0 14 94 254	4.5	128:56	22-14.a(13)
22-14.a*	0 17 120 356	4.0	256:1 128:64	1 4 16 64 86 109 25 185 53 209 141
22-14.c	0 22 122 315	4.5	128:88	1 4 16 64 90 97 118 133 253 22 198
23-15.a*	0 21 172 441	4.0	256:1 128:80	24-16.a(17)
23-15.c	0 30 156 399	4.5	128:120	24-16.c(23)
24-16.a***	0 26 216 584	4.0	256:2 128:96	1 4 16 64 86 109 25 133 54 180 100 198
24-16.c	0 38 192 533	4.5	128:152	1 4 16 64 86 109 25 133 54 249 157 210
25-17.a**	0 34 266 752	4.0	256:4 128:120	26-18.a(17)
25-17.c	0 48 237 689	4.5	128:192	26-18.c(13)
26-18.a*	0 43 326 960	4.0	256:7 128:144	1 4 16 64 86 109 25 133 54 180 100 198 37
26-18.c	0 58 296 880	4.5	128:232	1 4 16 64 86 109 25 133 54 249 157 210 198
27-19.a**	0 53 395 1224	4.0	256:11 128:168	28-20.a(13)
27-19.c	0 72 356 1124	4.5	128:288	28-20.c(27)
28-20.a**	0 64 476 1550	4.0	256:16 128:192	1 4 16 64 86 109 25 133 54 180 100 198 37 185
28-20.c	0 86 428 1432	4.5	128:344	1 4 16 64 86 109 25 133 54 249 157 210 198 213
29-21.a*	0 81 573 1884	4.0	256:20 128:244	30-22.a(17)
29-21.c	0 110 516 1756	4.5	128:440	30-22.c(1)
30-22.a**	0 95 686 2340	4.0	256:25 128:280	1 4 16 64 86 109 25 133 54 180 100 198 37 146 205
30-22.c	0 130 616 2185	4.5	128:520	1 4 16 64 86 109 25 133 54 249 117 100 61 225 218
31-23.a*	0 114 798 2906	4.0	256:33 128:324	32-24.a(1)
31-23.c	0 138 736 2785	4.0	256:2 128:544	32-24.c(7)
32-24.a***	0 131 944 3570	4.0	256:35 128:384	1 4 16 64 90 97 118 133 198 146 229 18 152 25 53 166
32-24.c	0 155 876 3458	4.0	256:5 128:600	1 4 16 64 86 109 25 133 54 100 66 189 117 88 81 225
33-25.a***	0 151 1108 4354	4.0	256:39 128:448	34-26.a(7)
33-25.c	0 181 1016 4236	4.0	256:7 128:696	34-26.c(33)
34-26.a***	0 174 1288 5280	4.0	256:46 128:512	1 4 16 64 86 109 25 133 54 180 100 33 106 161 169 88 113
34-26.c	0 210 1168 5172	4.0	256:12 128:792	1 4 16 64 86 109 25 133 54 100 66 189 117 88 81 225 73
35-27.a*	0 200 1496 6340	4.0	256:52 128:592	36-28.a(7)
35-27.c	0 239 1356 6269	4.0	256:16 128:892	36-28.c(31)
36-28.a*	0 229 1728 7576	4.0	256:61 128:672	1 4 16 64 86 109 25 133 54 180 100 33 106 161 169 88 113 212
36-28.c	0 273 1552 7569	4.0	256:23 128:1000	1 4 16 64 86 109 25 133 54 100 66 189 117 88 81 225 73 180
37-29.a**	0 264 2004 8928	4.0	256:92 128:688	38-30.a(1)
37-29.c	0 318 1750 9055	4.0	256:32 128:1144	38-30.c(37)
38-30.a**	0 297 2304 10592	4.0	256:105 128:768	1 4 16 64 86 109 25 133 54 180 100 198 37 146 205 106 161 185 166
38-30.c	0 366 1972 10806	4.0	256:44 128:1288	1 4 16 64 86 109 25 133 54 100 66 189 117 88 81 225 73 180 212
39-31.a**	0 333 2632 12512	4.0	256:117 128:864	40-32.a(1)
39-31.c	0 379 2328 13060	4.0	256:55 128:1296	40-32.c(9)
40-32.a**	0 370 3008 14720	4.0	256:130 128:960	1 4 16 64 86 109 25 133 54 180 100 198 37 146 205 106 161 185 166 212
40-32.c	0 426 2624 15488	4.0	256:66 128:1440	1 4 16 64 90 97 133 125 209 84 216 21 205 180 245 102 54 233 198 173
41-33.a***	0 468 3134 17401	4.0	256:138 128:1320	42-34.a(27)
41-33.c	0 511 2918 17602	4.0	256:92 128:1676	42-34.c(3)
42-34.a***	0 525 3516 20389	4.0	256:165 128:1440	1 4 16 64 86 109 181 25 37 96 148 216 205 169 129 218 246 137 6 18 157
42-34.c	0 568 3300 20546	4.0	256:104 128:1856	1 4 16 64 86 109 100 198 25 132 69 37 189 54 221 244 165 121 61 182 213

Table 6. 256-Run Designs (Continued).

Design	WLP	R	CFV	Column Indexes
43-35.a***	0 602 4032 22960	4.0	256:490 128:448	44-36.a(3)
43-35.c	0 626 3702 24067	4.0	256:114 128:2048	44-36.c(13)
44-36.a***	0 679 4480 26656	4.0	256:567 128:448	1 4 16 64 90 97 38 25 6 209 104 132 177 121 152 169 201 36 82 134 164 166
44-36.c	0 693 4120 28109	4.0	256:133 128:2240	1 4 16 64 86 109 25 185 53 100 141 197 144 246 33 81 73 149 241 38 98 88
45-37.a***	0 755 4728 31809	4.0	256:162 128:2372	46-38.a(13)
45-37.c	0 770 4556 32728	4.0	256:146 128:2496	46-38.c(13)
46-38.a***	0 830 5296 36553	4.0	256:192 128:2552	1 4 16 64 86 109 25 185 53 100 141 197 153 38 177 73 81 33 88 98 182 233 165
46-38.c	0 858 5008 37981	4.0	256:170 128:2752	1 4 16 64 86 109 25 185 53 100 141 197 144 246 33 81 73 149 241 38 98 88 218
47-39.ac*	0 939 5895 41162	4.0	256:199 128:2960	48-40.ac(7)
48-40.ac*	0 1030 6552 47096	4.0	256:222 128:3232	1 4 16 64 86 109 100 198 25 69 233 37 216 221 161 166 148 61 98 141 146 54 153 244
49-41.ac***	0 1131 7260 53689	4.0	256:244 128:3548	50-42.ac(15)
50-42.ac***	0 1235 8054 60970	4.0	256:269 128:3864	1 4 16 64 86 109 100 198 25 69 233 37 216 221 161 166 148 61 98 141 146 54 153 244 181
51-43.ac***	0 1348 8890 69172	4.0	256:293 128:4220	52-44.ac(5)
52-44.ac***	0 1464 9824 78188	4.0	256:320 128:4576	1 4 16 64 86 109 100 198 25 69 233 37 216 221 161 166 148 61 98 141 146 54 153 244 181 201
53-45.ac***	0 1590 10808 88274	4.0	256:346 128:4976	54-46.ac(3)
54-46.ac***	0 1719 11904 99312	4.0	256:375 128:5376	1 4 16 64 86 109 100 198 25 69 233 37 216 221 161 166 148 61 98 141 146 54 153 244 181 201 213
55-47.ac***	0 1859 13056 111600	4.0	256:403 128:5824	56-48.ac(1)
56-48.ac***	0 2002 14336 124992	4.0	256:434 128:6272	1 4 16 64 86 109 100 198 25 69 233 37 216 221 161 166 148 61 98 141 146 54 153 244 181 201 213 218
57-49.a*	0 2537 9562 191272	4.0	256:1190 128:5388	58-50.a(7)
57-49.c	0 2618 10960 171856	4.0	256:914 128:6816	58-50.c(7)
58-50.a*	0 2743 10298 214552	4.0	256:1291 128:5808	1 4 16 64 86 109 25 153 6 116 113 249 72 129 237 146 36 132 38 18 69 244 161 134 98 241 106 121 33
58-50.c	0 2858 11680 193976	4.0	256:1018 128:7360	1 4 16 64 86 109 181 25 37 104 116 146 161 148 61 237 54 166 141 153 81 121 209 249 69 197 74 98 214
59-51.a*	0 2956 11096 240123	4.0	256:1375 128:6324	60-52.a(7)
59-51.c	0 3118 12416 218496	4.0	256:1118 128:8000	60-52.c(7)
60-52.a*	0 3186 11920 268252	4.0	256:1482 128:6816	1 4 16 64 86 109 25 153 6 116 113 249 72 129 237 146 36 132 38 18 69 244 161 134 98 241 106 121 33 197
60-52.c	0 3395 13184 245696	4.0	256:1235 128:8640	1 4 16 64 86 109 181 25 37 104 116 146 161 148 61 237 54 166 141 153 81 121 209 249 69 197 74 98 214 244
61-53.a*	0 3428 12796 299074	4.0	256:1584 128:7376	62-54.a(1)
61-53.c	0 3467 12672 298744	4.0	256:1547 128:7680	62-54.c(3)
62-54.a*	0 3681 13728 332812	4.0	256:1697 128:7936	1 4 16 64 86 109 25 153 6 116 113 249 72 129 237 146 36 132 38 18 69 244 161 134 98 241 106 121 33 197 144
62-54.c	0 3711 13632 332568	4.0	256:1663 128:8192	1 4 16 64 86 109 25 153 6 116 113 249 72 129 237 146 36 132 38 18 69 244 161 121 241 98 33 106 197 144 214
63-55.a*	0 3948 14704 369729	4.0	256:1964 128:7936	64-56.a(3)
63-55.c	0 3963 14656 369592	4.0	256:1787 128:8704	64-56.c(3)
64-56.a*	0 4227 15744 409966	4.0	256:2147 128:8320	1 4 16 64 86 109 25 153 6 116 113 249 72 129 237 146 36 132 38 18 69 244 161 134 98 241 106 121 164 166 33 144
64-56.c	0 4228 15744 409936	4.0	256:1924 128:9216	1 4 16 64 86 109 25 153 6 116 113 249 72 129 237 146 36 132 38 18 69 244 161 134 98 241 106 121 33 197 144 214

7. Concluding Remarks

This paper uses quaternary codes to construct nonregular designs with 16, 32, 64, 128 and 256 runs. We observe that it is relatively easier to construct nonregular designs having higher resolution than regular designs, but it is more challenging to construct nonregular designs having less G_2-aberration than regular designs. With the quaternary method, we construct 37 nonregular designs with less G_2-aberration than MA or best regular designs. A limitation of this method is that it only produces designs whose run size is a power of two.

It is a challenging task to construct nonregular designs with good statistical properties. The main reason is that these designs do not have the aliasing structure of regular designs and, therefore, there are too many designs to consider, especially when the run size becomes large. We are able to keep all quaternary codes for 16, 32 and 64 runs. For 128 and 256 runs, however, the computation time becomes so long that it is necessary to put an upper limit to the maximum number of designs generated. Depending on the choice of limits, our algorithm ends with 50−68 columns for 256-run designs with resolution 4. It is apparent that we are missing some good designs because 256-run designs with resolution 4 can have up to 128 columns. An alternative to our forward addition approach is to use backward elimination in the sequential search. It would be interesting to see whether backward elimination can generate new good designs. Further research is needed for 256-run and larger designs.

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