

## Order-of-Addition Modeling

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### Supplementary Material

This file contains three designs discussed in Section 4 of the article.

The design in Table 1 is a strength-2 OofA-OA. It is fully efficient for the main effect PWO model, and its foldover contains 48 distinct orderings. This foldover design is strength-3, and permits estimation of the triplet model, with D-efficiency of 83.76% relative to the full  $5!$  design; the maximum VIF is 9.47.

The minimal point design for the triplet model in Table 2 is believed to be D-optimal, with D-efficiency of 79.4% relative to the full  $5!$  design. Its foldover contains 62 distinct orderings and has D-efficiency of 92.1%; the maximum VIF is 3.176.

The 36-run design in Table 3 is believed to be a D-optimal design for the triplet model; its D-efficiency = 87.2% relative to the full  $5!$  design. Its foldover contains 72 distinct orderings and has D-efficiency of 95.4%; the

maximum VIF is 2.574.

Table 1: Strength-2 OofA-OA, with foldover ( $n = 48$ ) that supports estimation of the triplet model

Sequence	$X_{1.2}$	$X_{1.3}$	$X_{1.4}$	$X_{1.5}$	$X_{2.3}$	$X_{2.4}$	$X_{2.5}$	$X_{3.4}$	$X_{3.5}$	$X_{4.5}$
12354	1	1	1	1	1	1	1	1	1	-1
12453	1	1	1	1	1	1	1	-1	-1	1
13524	1	1	1	1	-1	1	-1	1	1	-1
14352	1	1	1	1	-1	-1	-1	-1	1	1
21543	-1	1	1	1	1	1	1	-1	-1	-1
23415	-1	-1	-1	1	1	1	1	1	1	1
24351	-1	-1	-1	-1	1	1	1	-1	1	1
25134	-1	1	1	-1	1	1	1	1	-1	-1
31425	1	-1	1	1	-1	-1	1	1	1	1
31542	1	-1	1	1	-1	-1	-1	1	1	-1
32145	-1	-1	1	1	-1	1	1	1	1	1
32541	-1	-1	-1	-1	-1	1	1	1	1	-1
34512	1	-1	-1	-1	-1	-1	-1	1	1	1
41253	1	1	-1	1	1	-1	1	-1	-1	1
41532	1	1	-1	1	-1	-1	-1	-1	-1	1
42135	-1	1	-1	1	1	-1	1	-1	1	1
43251	-1	-1	-1	-1	-1	-1	1	-1	1	1
45213	-1	1	-1	-1	1	-1	-1	-1	-1	1
51243	1	1	1	-1	1	1	-1	-1	-1	-1
51342	1	1	1	-1	-1	-1	-1	1	-1	-1
52314	-1	-1	1	-1	1	1	-1	1	-1	-1
53241	-1	-1	-1	-1	-1	1	-1	1	-1	-1
54231	-1	-1	-1	-1	1	-1	-1	-1	-1	-1
54312	1	-1	-1	-1	-1	-1	-1	-1	-1	-1

Table 2: Minimal point design for the triplet model;  $n = 31$ ,  $m = 5$

Sequence	$X_{1.2}$	$X_{1.3}$	$X_{1.4}$	$X_{1.5}$	$X_{2.3}$	$X_{2.4}$	$X_{2.5}$	$X_{3.4}$	$X_{3.5}$	$X_{4.5}$
12354	1	1	1	1	1	1	1	1	1	-1
12453	1	1	1	1	1	1	1	-1	-1	1
13524	1	1	1	1	-1	1	-1	1	1	-1
14325	1	1	1	1	-1	-1	1	-1	1	1
15234	1	1	1	1	1	1	-1	1	-1	-1
15423	1	1	1	1	1	-1	-1	-1	-1	-1
21345	-1	1	1	1	1	1	1	1	1	1
21543	-1	1	1	1	1	1	1	-1	-1	-1
23415	-1	-1	-1	1	1	1	1	1	1	1
24135	-1	1	-1	1	1	1	1	-1	1	1
24531	-1	-1	-1	-1	1	1	1	-1	-1	1
25314	-1	-1	1	-1	1	1	1	1	-1	-1
31245	1	-1	1	1	-1	1	1	1	1	1
31452	1	-1	1	1	-1	-1	-1	1	1	1
32154	-1	-1	1	1	-1	1	1	1	1	-1
32541	-1	-1	-1	-1	-1	1	1	1	1	-1
34215	-1	-1	-1	1	-1	-1	1	1	1	1
34521	-1	-1	-1	-1	-1	-1	-1	1	1	1
35142	1	-1	1	-1	-1	-1	-1	1	1	-1
41253	1	1	-1	1	1	-1	1	-1	-1	1
41532	1	1	-1	1	-1	-1	-1	-1	-1	1
42351	-1	-1	-1	-1	1	-1	1	-1	1	1
42513	-1	1	-1	-1	1	-1	1	-1	-1	1
43152	1	-1	-1	1	-1	-1	-1	-1	1	1
45231	-1	-1	-1	-1	1	-1	-1	-1	-1	1
51342	1	1	1	-1	-1	-1	-1	1	-1	-1
52143	-1	1	1	-1	1	1	-1	-1	-1	-1
53124	1	-1	1	-1	-1	1	-1	1	-1	-1
53412	1	-1	-1	-1	-1	-1	-1	1	-1	-1
54123	1	1	-1	-1	1	-1	-1	-1	-1	-1
54321	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

Table 3: D-efficient design for triplet model;  $n = 36, m = 5$

Sequence	$X_{1.2}$	$X_{1.3}$	$X_{1.4}$	$X_{1.5}$	$X_{2.3}$	$X_{2.4}$	$X_{2.5}$	$X_{3.4}$	$X_{3.5}$	$X_{4.5}$
12345	1	1	1	1	1	1	1	1	1	1
12543	1	1	1	1	1	1	1	-1	-1	-1
13254	1	1	1	1	-1	1	1	1	1	-1
13425	1	1	1	1	-1	-1	1	1	1	1
14523	1	1	1	1	1	-1	-1	-1	-1	1
15243	1	1	1	1	1	1	-1	-1	-1	-1
15324	1	1	1	1	-1	1	-1	1	-1	-1
21354	-1	1	1	1	1	1	1	1	1	-1
21435	-1	1	1	1	1	1	1	-1	1	1
21534	-1	1	1	1	1	1	1	1	-1	-1
23514	-1	-1	1	-1	1	1	1	1	1	-1
24153	-1	1	-1	1	1	1	1	-1	-1	1
24315	-1	-1	-1	1	1	1	1	-1	1	1
25413	-1	1	-1	-1	1	1	1	-1	-1	-1
31542	1	-1	1	1	-1	-1	-1	1	1	-1
32145	-1	-1	1	1	-1	1	1	1	1	1
32451	-1	-1	-1	-1	-1	1	1	1	1	1
34152	1	-1	-1	1	-1	-1	-1	1	1	1
34215	-1	-1	-1	1	-1	-1	1	1	1	1
34512	1	-1	-1	-1	-1	-1	-1	1	1	1
35241	-1	-1	-1	-1	-1	1	-1	1	1	-1
35412	1	-1	-1	-1	-1	-1	-1	1	1	-1
41235	1	1	-1	1	1	-1	1	-1	1	1
41352	1	1	-1	1	-1	-1	-1	-1	1	1
42531	-1	-1	-1	-1	1	-1	1	-1	-1	1
43125	1	-1	-1	1	-1	-1	1	-1	1	1
43521	-1	-1	-1	-1	-1	-1	-1	-1	1	1
45132	1	1	-1	-1	-1	-1	-1	-1	-1	1
51234	1	1	1	-1	1	1	-1	1	-1	-1
51432	1	1	1	-1	-1	-1	-1	-1	-1	-1
52143	-1	1	1	-1	1	1	-1	-1	-1	-1
52341	-1	-1	-1	-1	1	1	-1	1	-1	-1
53124	1	-1	1	-1	-1	1	-1	1	-1	-1
53421	-1	-1	-1	-1	-1	-1	-1	1	-1	-1
54213	-1	1	-1	-1	1	-1	-1	-1	-1	-1
54312	1	-1	-1	-1	-1	-1	-1	-1	-1	-1