Frequentist Model Averaging for the Nonparametric Additive Model

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Supplementary Material

Section S1 presents the additional simulation results on the averaged squared (prediction) errors.

Section S2 provides the simulation results on the average weights (selection frequencies).

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S1 Averaged squared (prediction) errors

Table S.1: Averaged squared errors under independent data when covariate selection is subject to uncertainty and $\sigma = 1$

	n	AIC	BIC	SAIC	SBIC	AMAH	AMA
$k = 0$	50	0.677	0.625	0.605	0.528	0.375	0.366
	70	0.473	0.589	0.410	0.509	0.311	0.313
	100	0.341	0.526	0.304	0.453	0.250	0.253
	150	0.251	0.452	0.230	0.400	0.194	0.196
	200	0.188	0.341	0.179	0.308	0.160	0.161
$k = 1$	50	0.625	0.469	0.556	0.384	0.312	0.285
	70	0.412	0.413	0.346	0.346	0.245	0.235
	100	0.297	0.338	0.255	0.293	0.202	0.198
	150	0.227	0.285	0.200	0.260	0.163	0.162
	200	0.183	0.240	0.164	0.218	0.139	0.138
$k = 1.5$	50	0.615	0.437	0.542	0.349	0.296	0.263
	70	0.409	0.365	0.341	0.305	0.238	0.223
	100	0.286	0.291	0.243	0.251	0.188	0.183
	150	0.225	0.243	0.195	0.220	0.154	0.152
	200	0.179	0.217	0.157	0.199	0.130	0.129

Table S.2: Averaged squared errors under independent data when covariate selection is subject to uncertainty and $\sigma = 1.5$

	n	AIC	BIC	SAIC	SBIC	AMAH	AMA
$k = 0$	50	1.363	0.740	1.177	0.655	0.641	0.576
	70	0.902	0.698	0.738	0.633	0.516	0.493
	100	0.658	0.674	0.545	0.619	0.417	0.411
	150	0.520	0.655	0.444	0.617	0.341	0.337
	200	0.402	0.622	0.355	0.575	0.285	0.285
$k = 1$	50	1.284	0.633	1.106	0.538	0.565	0.487
	70	0.790	0.569	0.637	0.495	0.427	0.389
	100	0.569	0.529	0.464	0.462	0.343	0.322
	150	0.436	0.503	0.367	0.453	0.281	0.270
	200	0.345	0.438	0.294	0.390	0.232	0.226
$k = 1.5$	50	1.274	0.661	1.092	0.538	0.550	0.466
	70	0.785	0.589	0.634	0.484	0.414	0.368
	100	0.550	0.510	0.437	0.425	0.322	0.300
	150	0.426	0.455	0.350	0.396	0.259	0.246
	200	0.336	0.371	0.284	0.325	0.216	0.209

Table S.3: Averaged squared errors ($\times 10^{-1}$) under independent data when the degree of smoothing is subject to uncertainty and $\sigma = 0.4$

	n	AIC	BIC	SAIC	SBIC	AMAH	AMA
$k = 0$	50	0.941	0.707	0.887	0.660	0.638	0.635
	70	0.601	0.485	0.562	0.464	0.465	0.451
	100	0.411	0.342	0.385	0.334	0.345	0.335
	150	0.287	0.234	0.269	0.231	0.241	0.234
	200	0.225	0.184	0.208	0.183	0.188	0.185
$k = 1$	50	0.946	0.738	0.885	0.682	0.642	0.648
	70	0.634	0.538	0.583	0.502	0.483	0.474
	100	0.433	0.374	0.396	0.354	0.343	0.336
	150	0.315	0.266	0.285	0.255	0.246	0.241
	200	0.249	0.214	0.224	0.206	0.192	0.189
$k = 1.5$	50	0.953	0.734	0.888	0.682	0.639	0.638
	70	0.629	0.528	0.579	0.494	0.476	0.467
	100	0.434	0.368	0.395	0.351	0.344	0.337
	150	0.317	0.268	0.285	0.258	0.247	0.242
	200	0.245	0.214	0.220	0.207	0.190	0.188

Table S.4: Averaged squared errors under independent data when the degree of smoothing is subject to uncertainty and $\sigma = 1$

	n	AIC	BIC	SAIC	SBIC	AMAH	AMA
$k = 0$	50	0.552	0.347	0.509	0.331	0.339	0.309
	70	0.336	0.259	0.306	0.246	0.248	0.236
	100	0.233	0.201	0.212	0.189	0.183	0.174
	150	0.155	0.153	0.142	0.141	0.128	0.123
	200	0.114	0.123	0.106	0.112	0.097	0.093
$k = 1$	50	0.561	0.354	0.519	0.335	0.347	0.313
	70	0.352	0.258	0.316	0.245	0.250	0.234
	100	0.231	0.199	0.208	0.187	0.179	0.171
	150	0.156	0.155	0.141	0.144	0.128	0.123
	200	0.116	0.126	0.107	0.114	0.099	0.096
$k = 1.5$	50	0.552	0.348	0.512	0.331	0.342	0.310
	70	0.349	0.258	0.317	0.245	0.250	0.234
	100	0.236	0.199	0.213	0.188	0.183	0.174
	150	0.157	0.153	0.143	0.141	0.128	0.122
	200	0.115	0.125	0.106	0.113	0.098	0.094

Table S.5: Averaged squared errors under independent data when the degree of smoothing is subject to uncertainty and $\sigma = 1.5$

	n	AIC	BIC	SAIC	SBIC	AMAH	AMA
$k = 0$	50	1.225	0.727	1.134	0.703	0.741	0.658
	70	0.729	0.498	0.667	0.486	0.524	0.478
	100	0.489	0.373	0.442	0.364	0.381	0.357
	150	0.317	0.264	0.285	0.256	0.258	0.244
	200	0.248	0.226	0.224	0.218	0.207	0.199
$k = 1$	50	1.188	0.692	1.111	0.671	0.717	0.631
	70	0.720	0.495	0.658	0.484	0.520	0.477
	100	0.477	0.366	0.430	0.356	0.372	0.350
	150	0.325	0.275	0.294	0.265	0.264	0.252
	200	0.249	0.223	0.223	0.214	0.204	0.197
$k = 1.5$	50	1.215	0.699	1.125	0.682	0.725	0.639
	70	0.748	0.499	0.676	0.489	0.526	0.481
	100	0.496	0.368	0.450	0.359	0.382	0.357
	150	0.330	0.271	0.296	0.263	0.265	0.251
	200	0.249	0.225	0.223	0.217	0.206	0.199

Table S.6: Averaged squared prediction errors under dependent data when covariate selection is subject to uncertainty and $\alpha = 0.4$

	n	AIC	BIC	SAIC	SBIC	AMAH	AMA
$\rho = 0$	80	0.258	0.235	0.252	0.227	0.223	0.218
	100	0.198	0.201	0.196	0.194	0.182	0.181
	200	0.162	0.171	0.162	0.167	0.155	0.154
$\rho = 0.2$	80	0.248	0.233	0.244	0.226	0.214	0.210
	100	0.214	0.213	0.212	0.207	0.198	0.197
	200	0.155	0.166	0.155	0.163	0.150	0.149
$\rho = 0.4$	80	0.273	0.252	0.266	0.242	0.233	0.227
	100	0.212	0.209	0.210	0.203	0.193	0.192
	200	0.169	0.180	0.169	0.177	0.163	0.162

Table S.7: Averaged squared prediction errors ($\times 10^{-1}$) under dependent data when the degree of smoothing is subject to uncertainty

	n	AIC	BIC	SAIC	SBIC	AMAH	AMA
$\rho = 0$	80	0.153	0.135	0.140	0.128	0.127	0.122
	100	0.152	0.130	0.137	0.124	0.125	0.121
	200	0.121	0.118	0.118	0.115	0.112	0.111
$\rho = 0.2$	80	0.162	0.140	0.147	0.133	0.132	0.127
	100	0.155	0.136	0.141	0.130	0.129	0.125
	200	0.124	0.120	0.118	0.118	0.115	0.114
$\rho = 0.4$	80	0.186	0.154	0.166	0.147	0.151	0.143
	100	0.168	0.148	0.154	0.142	0.142	0.138
	200	0.138	0.134	0.134	0.131	0.129	0.128

S2 Average weights (selection frequencies)

Here, we compute the average weights and selection frequencies of models by each method, based on the same setup as in Section 4.1 of the paper with $\alpha = 0$, $k = 0$, and the covariate choice being subject to uncertainty. In Table S.8, which describes the candidate models, "1" and "2-8" under the panel "Covariate" denote the intercept and covariates X_1–X_7 respectively. The combination of these covariates yields $2^7 = 128$ candidate models. The true model is Model 65, which includes X_1–X_4 in addition to the intercept. Some representative results are reported in Tables S.9–S.14. For brevity, we present only the results for $n = 50, 100, 200$, $\sigma = 0.4, 1$ and models with an average weight of no smaller than 0.01 or selected with at least 1% frequency.

Results under $\sigma = 0.4$ (the small noise scenario) are reported in Tables S.9–S.11, which show that except for the AIC and SAIC methods, all methods generally favour smaller models when the sample is of small to moderate sizes ($n = 50, 100$) and larger models when the sample size is large ($n = 200$). Further, Model 65 (i.e., the true model) is the most frequently selected or most highly weighted model for the large sample size (e.g., $n = 200$). These findings generally carry over to situations of $\sigma = 1$ (the large noise scenario), except that an increase in sample size is required

to produce the same result observed under $\sigma = 0.4$, as reported in Tables S.12–S.14. The larger sample size neutralises the bigger uncertainty caused by the higher noise level. For example, by the BIC or SBIC methods, the small weights are assigned to the true model for small and moderate sample sizes ($n = 50, 100$) when $\sigma = 1$, but to produce selection frequencies or average weights similar to those reported in Table S.11 for $n = 200$ and $\sigma = 0.4$, a larger sample size (e.g., $n = 800$) is needed when $\sigma = 1$; see Table S.15 for the results with $n = 800$ and $\sigma = 1$.

Table S.8: The candidate model set for the covariate selection

Model	Covariate				Model	Covariate							
1	1				65	1	2	3	4	5			
2	1	2			66	1	2	3	4	6			
3	1	3			67	1	2	3	4	7			
4	1	4			68	1	2	3	4	8			
5	1	5			69	1	2	3	5	6			
6	1	6			70	1	2	3	5	7			
7	1	7			71	1	2	3	5	8			
8	1	8			72	1	2	3	6	7			
9	1	2	3		73	1	2	3	6	8			
10	1	2	4		74	1	2	3	7	8			
11	1	2	5		75	1	2	4	5	6			
12	1	2	6		76	1	2	4	5	7			
13	1	2	7		77	1	2	4	5	8			
14	1	2	8		78	1	2	4	6	7			
15	1	3	4		79	1	2	4	6	8			
16	1	3	5		80	1	2	4	7	8			
17	1	3	6		81	1	2	5	6	7			
18	1	3	7		82	1	2	5	6	8			
19	1	3	8		83	1	2	5	7	8			
20	1	4	5		84	1	2	6	7	8			
21	1	4	6		85	1	3	4	5	6			
22	1	4	7		86	1	3	4	5	7			
23	1	4	8		87	1	3	4	5	8			
24	1	5	6		88	1	3	4	6	7			
25	1	5	7		89	1	3	4	6	8			
26	1	5	8		90	1	3	4	7	8			
27	1	6	7		91	1	3	5	6	7			
28	1	6	8		92	1	3	5	6	8			
29	1	7	8		93	1	3	5	7	8			
30	1	2	3	4	94	1	3	6	7	8			
31	1	2	3	5	95	1	4	5	6	7			
32	1	2	3	6	96	1	4	5	6	8			
33	1	2	3	7	97	1	4	5	7	8			
34	1	2	3	8	98	1	4	6	7	8			
35	1	2	4	5	99	1	5	6	7	8			
36	1	2	4	6	100	1	2	3	4	5	6		
37	1	2	4	7	101	1	2	3	4	5	7		
38	1	2	4	8	102	1	2	3	4	5	8		
39	1	2	5	6	103	1	2	3	4	6	7		
40	1	2	5	7	104	1	2	3	4	6	8		
41	1	2	5	8	105	1	2	3	4	7	8		
42	1	2	6	7	106	1	2	3	5	6	7		
43	1	2	6	8	107	1	2	3	5	6	8		
44	1	2	7	8	108	1	2	3	5	7	8		
45	1	3	4	5	109	1	2	3	6	7	8		
46	1	3	4	6	110	1	2	4	5	6	7		
47	1	3	4	7	111	1	2	4	5	6	8		
48	1	3	4	8	112	1	2	4	5	7	8		
49	1	3	5	6	113	1	2	4	6	7	8		
50	1	3	5	7	114	1	2	5	6	7	8		
51	1	3	5	8	115	1	3	4	5	6	7		
52	1	3	6	7	116	1	3	4	5	6	8		
53	1	3	6	8	117	1	3	4	5	7	8		
54	1	3	7	8	118	1	3	4	6	7	8		
55	1	4	5	6	119	1	3	5	6	7	8		
56	1	4	5	7	120	1	4	5	6	7	8		
57	1	4	5	8	121	1	2	3	4	5	6	7	
58	1	4	6	7	122	1	2	3	4	5	6	8	
59	1	4	6	8	123	1	2	3	4	5	7	8	
60	1	4	7	8	124	1	2	3	4	6	7	8	
61	1	5	6	7	125	1	2	3	5	6	7	8	
62	1	5	6	8	126	1	2	4	5	6	7	8	
63	1	5	7	8	127	1	3	4	5	6	7	8	
64	1	6	7	8	128	1	2	3	4	5	6	7	8

Table S.9: Average weights (selection frequencies) of models by model averaging (selection) methods when $n = 50$ and $\sigma = 0.4$

Model	AIC	BIC	SAIC	SBIC	AMAH	AMA
1	0.000	0.012	0.000	0.014	0.002	0.000
5	0.000	0.022	0.000	0.023	0.009	0.011
9	0.000	0.000	0.000	0.000	0.007	0.023
10	0.000	0.005	0.000	0.004	0.013	0.042
11	0.000	0.009	0.000	0.009	0.018	0.056
15	0.000	0.001	0.000	0.002	0.013	0.044
16	0.000	0.005	0.000	0.005	0.017	0.053
20	0.001	0.113	0.001	0.105	0.057	0.159
30	0.000	0.000	0.000	0.001	0.016	0.035
31	0.000	0.007	0.000	0.006	0.031	0.054
35	0.008	0.090	0.007	0.093	0.088	0.124
45	0.006	0.118	0.007	0.125	0.107	0.156
57	0.000	0.001	0.000	0.003	0.010	0.010
65	0.119	0.367	0.106	0.335	0.160	0.026
75	0.002	0.004	0.004	0.005	0.013	0.002
76	0.001	0.004	0.002	0.004	0.011	0.003
77	0.002	0.004	0.002	0.004	0.015	0.002
85	0.003	0.007	0.004	0.008	0.011	0.001
86	0.004	0.008	0.005	0.008	0.015	0.003
87	0.000	0.006	0.002	0.007	0.014	0.002
100	0.071	0.022	0.071	0.030	0.016	0.001
101	0.063	0.025	0.067	0.030	0.016	0.000
102	0.063	0.023	0.063	0.030	0.018	0.001
121	0.095	0.022	0.098	0.020	0.011	0.000
122	0.100	0.026	0.098	0.021	0.011	0.000
123	0.074	0.019	0.079	0.022	0.007	0.000
127	0.007	0.002	0.010	0.002	0.003	0.000
128	0.351	0.041	0.336	0.040	0.014	0.000

Table S.10: Average weights (selection frequencies) of models by model averaging (selection) methods when $n = 100$ and $\sigma = 0.4$

Model	AIC	BIC	SAIC	SBIC	AMAH	AMA
20	0.000	0.007	0.000	0.009	0.015	0.018
30	0.000	0.000	0.000	0.000	0.008	0.024
31	0.000	0.000	0.000	0.000	0.011	0.032
35	0.000	0.021	0.000	0.030	0.041	0.103
45	0.000	0.034	0.000	0.045	0.042	0.109
65	0.550	0.932	0.426	0.899	0.497	0.492
75	0.000	0.000	0.000	0.000	0.011	0.008
77	0.000	0.000	0.000	0.000	0.011	0.009
85	0.000	0.000	0.000	0.000	0.010	0.009
86	0.000	0.000	0.000	0.000	0.011	0.009
87	0.000	0.001	0.000	0.001	0.012	0.009
100	0.121	0.001	0.139	0.005	0.050	0.005
101	0.098	0.003	0.119	0.006	0.041	0.005
102	0.095	0.001	0.124	0.004	0.039	0.003
121	0.040	0.000	0.056	0.000	0.009	0.000
122	0.036	0.000	0.050	0.000	0.005	0.000
123	0.044	0.000	0.055	0.000	0.010	0.000
128	0.016	0.000	0.030	0.000	0.003	0.000

Table S.11: Average weights (selection frequencies) of models by model averaging (selection) methods when $n = 200$ and $\sigma = 0.4$

Model	AIC	BIC	SAIC	SBIC	AMAH	AMA
35	0.000	0.000	0.000	0.000	0.016	0.027
45	0.000	0.000	0.000	0.000	0.018	0.031
65	0.685	0.998	0.565	0.997	0.613	0.718
100	0.078	0.001	0.111	0.001	0.058	0.027
101	0.097	0.001	0.120	0.001	0.063	0.026
102	0.092	0.000	0.114	0.000	0.063	0.024
121	0.018	0.000	0.029	0.000	0.012	0.001
122	0.014	0.000	0.026	0.000	0.012	0.001
123	0.014	0.000	0.029	0.000	0.012	0.001

Table S.12: Average weights (selection frequencies) of models by model averaging (selection) methods when $n = 50$ and $\sigma = 1$

Model	AIC	BIC	SAIC	SBIC	AMAH	AMA
1	0.011	0.643	0.007	0.575	0.082	0.067
2	0.003	0.020	0.003	0.025	0.025	0.053
3	0.002	0.022	0.003	0.027	0.030	0.058
4	0.009	0.081	0.008	0.090	0.070	0.129
5	0.021	0.106	0.015	0.122	0.094	0.170
6	0.001	0.002	0.002	0.004	0.011	0.021
7	0.000	0.000	0.001	0.005	0.010	0.019
8	0.001	0.003	0.001	0.004	0.010	0.018
9	0.002	0.002	0.002	0.002	0.014	0.019
10	0.004	0.006	0.004	0.008	0.024	0.032
11	0.008	0.006	0.008	0.009	0.031	0.039
15	0.003	0.003	0.005	0.006	0.024	0.035
16	0.011	0.012	0.009	0.013	0.030	0.036
17	0.000	0.001	0.001	0.001	0.009	0.012
20	0.033	0.039	0.024	0.042	0.066	0.080
21	0.004	0.000	0.003	0.001	0.009	0.013
22	0.001	0.001	0.003	0.002	0.011	0.014
23	0.001	0.000	0.003	0.002	0.010	0.015
24	0.001	0.001	0.003	0.003	0.012	0.017
25	0.002	0.001	0.003	0.003	0.009	0.015
26	0.003	0.002	0.003	0.002	0.011	0.014
31	0.011	0.002	0.011	0.003	0.012	0.005
35	0.019	0.008	0.018	0.009	0.020	0.009
45	0.018	0.006	0.019	0.006	0.019	0.006
55	0.009	0.001	0.009	0.002	0.010	0.004
57	0.009	0.001	0.009	0.001	0.011	0.003
65	0.021	0.002	0.023	0.003	0.009	0.001
75	0.013	0.001	0.012	0.001	0.007	0.000
76	0.010	0.000	0.011	0.001	0.006	0.000
77	0.012	0.000	0.012	0.000	0.005	0.000
86	0.011	0.000	0.009	0.000	0.004	0.000
87	0.013	0.000	0.011	0.000	0.004	0.000
100	0.014	0.002	0.020	0.001	0.005	0.000
101	0.020	0.000	0.022	0.000	0.005	0.000
102	0.017	0.001	0.020	0.001	0.004	0.000
110	0.012	0.000	0.011	0.000	0.003	0.000
111	0.009	0.001	0.012	0.001	0.004	0.000
115	0.010	0.001	0.011	0.000	0.003	0.000
116	0.009	0.000	0.011	0.000	0.002	0.000
117	0.010	0.001	0.011	0.001	0.003	0.000
121	0.048	0.000	0.044	0.000	0.003	0.000
122	0.049	0.000	0.043	0.000	0.004	0.000
123	0.043	0.000	0.044	0.000	0.005	0.000
124	0.014	0.001	0.013	0.001	0.002	0.000
125	0.007	0.000	0.011	0.000	0.003	0.000
126	0.017	0.000	0.024	0.000	0.003	0.000
127	0.028	0.001	0.025	0.001	0.003	0.000
128	0.261	0.002	0.226	0.002	0.008	0.000

Table S.13: Average weights (selection frequencies) of models by model averaging (selection) methods when $n = 100$ and $\sigma = 1$

Model	AIC	BIC	SAIC	SBIC	AMAH	AMA
1	0.000	0.403	0.000	0.369	0.012	0.004
2	0.000	0.013	0.000	0.017	0.012	0.014
3	0.001	0.016	0.001	0.019	0.013	0.015
4	0.006	0.115	0.004	0.125	0.025	0.031
5	0.005	0.216	0.005	0.216	0.037	0.046
9	0.000	0.001	0.001	0.001	0.012	0.023
10	0.004	0.006	0.004	0.009	0.027	0.049
11	0.007	0.010	0.008	0.014	0.035	0.057
15	0.004	0.005	0.004	0.007	0.021	0.040
16	0.007	0.020	0.007	0.021	0.030	0.056
20	0.064	0.151	0.047	0.143	0.094	0.162
24	0.001	0.002	0.001	0.002	0.008	0.012
25	0.001	0.000	0.001	0.001	0.009	0.013
30	0.006	0.001	0.006	0.002	0.022	0.025
31	0.009	0.000	0.010	0.001	0.026	0.031
35	0.100	0.014	0.083	0.017	0.074	0.070
45	0.091	0.019	0.077	0.023	0.084	0.081
55	0.008	0.000	0.011	0.000	0.013	0.012
56	0.006	0.000	0.011	0.000	0.014	0.011
57	0.007	0.000	0.010	0.001	0.010	0.011
65	0.233	0.006	0.172	0.006	0.056	0.014
75	0.017	0.000	0.023	0.000	0.013	0.003
76	0.026	0.000	0.025	0.000	0.011	0.002
77	0.018	0.000	0.021	0.000	0.010	0.002
85	0.031	0.000	0.027	0.000	0.010	0.001
86	0.017	0.000	0.021	0.000	0.011	0.003
87	0.017	0.000	0.022	0.000	0.011	0.003
100	0.049	0.000	0.058	0.000	0.010	0.000
101	0.049	0.000	0.054	0.000	0.008	0.000
102	0.054	0.000	0.057	0.000	0.012	0.000
121	0.023	0.000	0.028	0.000	0.004	0.000
122	0.021	0.000	0.028	0.000	0.003	0.000
123	0.022	0.000	0.024	0.000	0.003	0.000
128	0.020	0.000	0.021	0.000	0.001	0.000

Table S.14: Average weights (selection frequencies) of models by model averaging (selection) methods when $n = 200$ and $\sigma = 1$

Model	AIC	BIC	SAIC	SBIC	AMAH	AMA
1	0.000	0.094	0.000	0.089	0.003	0.000
4	0.000	0.056	0.000	0.062	0.010	0.007
5	0.000	0.193	0.000	0.190	0.012	0.007
9	0.000	0.000	0.000	0.000	0.007	0.011
10	0.000	0.002	0.000	0.004	0.012	0.018
11	0.000	0.008	0.000	0.013	0.016	0.024
15	0.000	0.004	0.000	0.006	0.014	0.022
16	0.000	0.013	0.000	0.014	0.017	0.025
20	0.012	0.465	0.012	0.442	0.058	0.075
30	0.000	0.000	0.000	0.000	0.021	0.038
31	0.001	0.004	0.002	0.003	0.028	0.050
35	0.053	0.060	0.054	0.064	0.100	0.157
45	0.065	0.077	0.067	0.081	0.107	0.162
55	0.001	0.000	0.001	0.000	0.011	0.015
57	0.002	0.000	0.003	0.000	0.011	0.014
65	0.564	0.019	0.446	0.023	0.219	0.140
75	0.006	0.000	0.010	0.000	0.015	0.009
76	0.009	0.000	0.011	0.000	0.013	0.009
77	0.003	0.000	0.008	0.000	0.011	0.007
85	0.016	0.000	0.015	0.000	0.014	0.011
86	0.005	0.000	0.011	0.000	0.015	0.010
87	0.003	0.000	0.008	0.000	0.010	0.006
100	0.083	0.000	0.095	0.000	0.026	0.003
101	0.061	0.000	0.081	0.000	0.022	0.003
102	0.060	0.000	0.081	0.000	0.021	0.004
121	0.008	0.000	0.017	0.000	0.002	0.000
122	0.018	0.000	0.027	0.000	0.006	0.000
123	0.014	0.000	0.022	0.000	0.004	0.000

Table S.15: Average weights (selection frequencies) of models by model averaging (selection) methods when $n = 800$ and $\sigma = 1$

Model	AIC	BIC	SAIC	SBIC	AMAH	AMA
20	0.000	0.009	0.000	0.008	0.013	0.012
35	0.000	0.026	0.000	0.032	0.027	0.033
45	0.000	0.051	0.000	0.060	0.025	0.032
65	0.831	0.914	0.721	0.901	0.587	0.619
85	0.000	0.000	0.000	0.000	0.009	0.010
100	0.052	0.000	0.084	0.000	0.057	0.044
101	0.051	0.000	0.081	0.000	0.054	0.042
102	0.050	0.000	0.081	0.000	0.057	0.043
122	0.006	0.000	0.012	0.000	0.009	0.005
123	0.004	0.000	0.010	0.000	0.008	0.004