On a measure of lack of fit in nonlinear cointegrating regression with endogeneity

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Supplementary Material

This supplementary material provides some additional simulation results for the paper. In Sections S.1-S.3, we consider the case that the error term u_t follows an AR(1) model with a GARCH$(1,1)$ innovation. In Sections S.4, we consider the case that y_t follows the additive model in Chang et al. (2001), which allows the time trend and additional integrable or non-integrable regression functions.

S1 Integrable regression function

We generate 5000 replications of sample size $n = 100, 200$, or 500 from the following data-generating models:

$$y_t = \exp(-\theta_0|x_t|) + u_t; \tag{S1.1}$$

$$y_t = \exp(-\theta_0|x_t|) + 0.5|x_t|^2 I(|x_t| \le 10) + u_t; \tag{S1.2}$$

$$y_t = \exp(-\theta_0|x_t|) + 20\exp(-|x_t|^2) + u_t; \tag{S1.3}$$

$$y_t = \exp(-\theta_0|x_t|) + 0.1|x_t| + u_t; \tag{S1.4}$$

$$y_t = \exp(-\theta_0|x_t|) + 0.1|x_t|^2 + u_t, \tag{S1.5}$$

where $\theta_0 = 1$, $x_t = x_{t-1} + \xi_t$ with $(1-0.8B)(1-B)^d\xi_t = (1+0.3B)\epsilon_t$, and

$$u_t = \rho u_{t-1} + \nu_t,\ \nu_t = \varsigma_t\sigma_t,\ \sigma_t^2 = 0.01 + 0.01\nu_{t-1}^2 + 0.9\sigma_{t-1}^2$$

with $\rho = \pm 0.5$ and

$$(\epsilon_t, \varsigma_t) \sim \text{i.i.d. } N\left(0, \begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix}\right).$$

In all calculations, we compute $\widehat{\theta}_n$ as the non-linear LSE of θ_0 based on model (S1.1), and it is expected that

$$\sqrt{n}(\widehat{a}_1, \widehat{a}_2, \cdots, \widehat{a}_M) \to_D N(0, \Omega),$$

where

$$\widehat{a}_k = \frac{\sum_{t=k+1}^n \widehat{\nu}_t\widehat{\nu}_{t-k}}{\sum_{t=1}^n \widehat{\nu}_t^2} \text{ and } \Omega = \frac{1}{[E(\nu_t^2)]^2}(I_M, -Z)E(\nu_t^2 s_t s_t')(I_M, -Z)'$$

with $s_t = (\nu_{t-1}, \nu_{t-2}, \cdots, \nu_{t-M}, u_{t-1})'$, $Z = (Z_1, Z_2, \cdots, Z_M)'$, and $Z_k = E[\nu_{t-k}u_{t-1}]/E(u_t^2)$. Hence, the portmanteau test statistic in this case is

$$\widehat{U}_n(M) = n\left(\sqrt{\frac{n+2}{n-1}}\widehat{a}_1, \sqrt{\frac{n+2}{n-2}}\widehat{a}_2, \cdots, \sqrt{\frac{n+2}{n-M}}\widehat{a}_M\right)\widehat{\Omega}_n^{-1}$$
$$\times\left(\sqrt{\frac{n+2}{n-1}}\widehat{a}_1, \sqrt{\frac{n+2}{n-2}}\widehat{a}_2, \cdots, \sqrt{\frac{n+2}{n-M}}\widehat{a}_M\right)$$

$$\to_D \chi^2_M, \tag{S1.6}$$

where $\widehat{\Omega}_n$ is the sample counterpart of Ω.

Table S.1 reports the size and power of $\widehat{U}_n(M)$ for $M = 6, 12$ and 18 at the 5% significance level, where the size of $\widehat{U}_n(M)$ corresponds to the case of $y_t \sim$ model (S1.1). From this table, our findings are similar to those for Table 1 in the paper, except that the size of $\widehat{U}_n(M)$ tends to be conservative especially for small n.

Please insert Table S.1 about here.

S2 Nonintegrable regression function

We generate 5000 replications of sample size $n = 100, 200$, or 500 from the following data-generating models:

$$y_t = \theta_{10} + \theta_{20}x_t + u_t; \tag{S2.1}$$

$$y_t = \theta_{10} + \theta_{20}x_t + 0.5|x_t|^2 I(|x_t| \leq 10) + u_t; \tag{S2.2}$$

$$y_t = \theta_{10} + \theta_{20}x_t + 20\exp(-|x_t|^2) + u_t; \tag{S2.3}$$

$$y_t = \theta_{10} + \theta_{20}x_t + 0.1|x_t| + u_t; \tag{S2.4}$$

$$y_t = \theta_{10} + \theta_{20}x_t + 0.1|x_t|^2 + u_t, \tag{S2.5}$$

where $(\theta_{10}, \theta_{20}) = (0, 1)$ and the remaining set-ups inherit from models (S1.1)-(S1.5). In all calculations, we compute $(\widehat{\theta}_{0n}, \widehat{\theta}_{1n})$ as the non-linear LSE of $(\theta_{10}, \theta_{20})$ based on model (S2.1), and the portmanteau test $\widehat{U}_n(M)$ is calculated in the same way as (S1.6).

Table S.2 reports the size and power of $\widehat{U}_n(M)$ at the significance level 5%, and the size of $\widehat{U}_n(M)$ is corresponding to the case that $y_t \sim$ model (S2.1). From this table, our findings are similar to those in Table 2 in the paper, except that the size of $\widehat{U}_n(M)$ tends to be conservative especially for small n.

Please insert Table S.2 about here.

S3 Additive regression function

We generate 5000 replications of sample size $n = 100, 200$, or 500 from the following data-generating models:

$$y_t = \theta_{10} + \theta_{20}x_t + \exp(-\eta_0|z_t|) + u_t; \tag{S3.1}$$

$$y_t = \theta_{10} + \theta_{20}x_t + \exp(-\eta_0|z_t|) + 0.5|\kappa_t|^2 I(|\kappa_t| \leq 10) + u_t; \tag{S3.2}$$

$$y_t = \theta_{10} + \theta_{20}x_t + \exp(-\eta_0|z_t|) + 20\exp(-|\kappa_t|^2) + u_t; \tag{S3.3}$$

$$y_t = \theta_{10} + \theta_{20}x_t + \exp(-\eta_0|z_t|) + 0.1|\kappa_t| + u_t; \tag{S3.4}$$

$$y_t = \theta_{10} + \theta_{20}x_t + \exp(-\eta_0|z_t|) + 0.1|\kappa_t|^2 + u_t, \tag{S3.5}$$

where $\kappa_t = \max(x_t, z_t)$, $(\theta_{10}, \theta_{20}, \eta_0) = (0, 1, 1)$, $z_t = z_{t-1} + \zeta_t$ with $(1 - 0.8B)(1-B)^d \zeta_t = (1 + 0.3B)\epsilon_t^*$,

$$(\epsilon_t, \epsilon_t^*, \varsigma_t) \sim \text{i.i.d. } N\left(0, \begin{pmatrix} 1 & 0.5 & r \\ 0.5 & 1 & 0.5 \\ r & 0.5 & 1 \end{pmatrix}\right),$$

and the remaining set-ups inherit from models (S1.1)-(S1.5). In all calculations, we compute $(\widehat{\theta}_{0n}, \widehat{\theta}_{1n}, \widehat{\eta}_n)$ as the two-step non-linear LSE of $(\theta_{10}, \theta_{20}, \eta_0)$ based on model (S3.1), and the portmanteau test $\widehat{U}_n(M)$ is calculated in the same way as (S1.6).

Table S.3 reports the size and power of $\widehat{U}_n(M)$ at the significance level 5%, and the size of $\widehat{U}_n(M)$ is corresponding to the case that $y_t \sim$ model (S3.1). From this table, our findings are similar to those in Table 3 in the paper, except that the size of $\widehat{U}_n(M)$ tends to be conservative especially for small n.

Please insert Table S.3 about here.

S4 Additive model in Chang et al. (2001)

First, we consider the additive model in Chang et al. (2001) with two non-integrable functions. We generate 5000 replications of sample size

$n = 100, 200$, or 500 from the following data-generating models:

$$y_t = \theta_{10} + \theta_{20}x_{1t} + \theta_{30}x_{2t} + u_t; \tag{S4.1}$$

$$y_t = \theta_{10} + \theta_{20}x_{1t} + \theta_{30}x_{2t} + 0.5|\kappa_t|^2 I(|\kappa_t| \le 10) + u_t; \tag{S4.2}$$

$$y_t = \theta_{10} + \theta_{20}x_{1t} + \theta_{30}x_{2t} + 20\exp(-|\kappa_t|^2) + u_t; \tag{S4.3}$$

$$y_t = \theta_{10} + \theta_{20}x_{1t} + \theta_{30}x_{2t} + 0.1|\kappa_t| + u_t; \tag{S4.4}$$

$$y_t = \theta_{10} + \theta_{20}x_{1t} + \theta_{30}x_{2t} + 0.1|\kappa_t|^2 + u_t, \tag{S4.5}$$

where $\kappa_t = \max(x_{1t}, x_{2t})$, $(\theta_{10}, \theta_{20}, \theta_{30}) = (0, 1, 1)$, $x_{1t} = x_{1t-1} + \zeta_{1t}$ with $(1-0.8B)(1-B)^d\zeta_{1t} = (1+0.3B)\epsilon_t$, $x_{2t} = x_{2t-1} + \zeta_{2t}$ with $(1-0.8B)(1-B)^d\zeta_{2t} = (1+0.3B)\epsilon_t^*$, and u_t is generated by one of the error generating models (EGMs):

EGM 1 (AR(1) + i.i.d. innovations):

$$u_t = \rho u_{t-1} + \nu_t,$$

EGM 2: (AR(1) + GARCH(1, 1) innovations):

$$u_t = \rho u_{t-1} + \nu_t,\ \nu_t = \varsigma_t\sigma_t,\ \sigma_t^2 = 0.01 + 0.01\nu_{t-1}^2 + 0.9\sigma_{t-1}^2,$$

with $\rho = \pm 0.5$, where

$$(\epsilon_t, \epsilon_t^*, c_t) \sim \text{i.i.d. } N\left(0, \begin{pmatrix} 1 & 0.5 & r \\ 0.5 & 1 & 0.5 \\ r & 0.5 & 1 \end{pmatrix}\right),$$

with $c_t = \nu_t$ for EGM 1 and $c_t = \varsigma_t$ for EGM 2. In all calculations, we compute $(\widehat{\theta}_{1n}, \widehat{\theta}_{2n}, \widehat{\theta}_{3n})$ as the one-step non-linear LSE of $(\theta_{10}, \theta_{20}, \theta_{30})$ based on model (S4.1), and the portmanteau test $\widehat{U}_n(M)$ is calculated as the one in the paper for EGM 1 or the one in (S1.6) for EGM 2.

Tables S.4–S.5 report the size and power of $\widehat{U}_n(M)$ at the 5% significance level, where the size of $\widehat{U}_n(M)$ corresponds to the case of $y_t \sim$ model (S4.1). From this table, our findings are similar to those in Table 3 in the paper for EGM 1 or Table S.3 above for EGM 2.

Please insert Tables S.4–S.5 about here.

Second, we consider the additive model in Chang et al. (2001) with time trend. We generate 5000 replications of sample size $n = 100, 200$, or 500 from the following data-generating models:

$$y_t = \theta_{10} + \theta_{20}t + \theta_{30}x_{1t} + \theta_{40}x_{2t} + u_t; \tag{S4.6}$$

$$y_t = \theta_{10} + \theta_{20}t + \theta_{30}x_{1t} + \theta_{40}x_{2t} + 0.5|\kappa_t|^2 I(|\kappa_t| \leq 10) + u_t; \tag{S4.7}$$

$$y_t = \theta_{10} + \theta_{20}t + \theta_{30}x_{1t} + \theta_{40}x_{2t} + 20\exp(-|\kappa_t|^2) + u_t; \tag{S4.8}$$

$$y_t = \theta_{10} + \theta_{20}t + \theta_{30}x_{1t} + \theta_{40}x_{2t} + 0.1|\kappa_t| + u_t; \tag{S4.9}$$

$$y_t = \theta_{10} + \theta_{20}t + \theta_{30}x_{1t} + \theta_{40}x_{2t} + 0.1|\kappa_t|^2 + u_t, \tag{S4.10}$$

where $(\theta_{10}, \theta_{20}, \theta_{30}, \theta_{40}) = (0, 1, 1, 1)$ and the remaining set-ups inherit from models (S4.1)-(S4.5). In all calculations, we compute $(\widehat{\theta}_{1n}, \widehat{\theta}_{2n}, \widehat{\theta}_{3n}, \widehat{\theta}_{4n})$ as the one-step non-linear LSE of $(\theta_{10}, \theta_{20}, \theta_{30}, \theta_{40})$ based on model (S4.6), and the portmanteau test $\widehat{U}_n(M)$ is calculated as the one in the paper for EGM 1 or the one in (S1.6) for EGM 2.

Tables S.6-S.7 report the size and power of $\widehat{U}_n(M)$ at the 5% significance level, where the size of $\widehat{U}_n(M)$ corresponds to the case of $y_t \sim$ model (S4.6). From this table, our findings are similar to those in Table 3 in the paper for EGM 1 or Table S.3 above for EGM 2.

Please insert Tables S.6-S.7 about here.

In summary, regardless of the type of regression function with either endogenous or exogenous regressor driven by either long or short memory innovations, our portmanteau test has good finite sample performance in all examined cases above, although its size may be conservative when the error generating model for u_t has the ARCH-type structure.

Bibliography

Chang, Y., Park, J.Y. and Phillips, P.C.B. (2001). Nonlinear econometric models with cointegrated and deterministically trending regressors. *Econometrics Journal*, **4**, 1–36.

Table S.1: Size and power ($\times 100$) of $\widehat{U}_n(M)$ for models (S1.1)–(S1.5)

				$M=6$			$M=12$			$M=18$		
Model	ρ	d	r \ n	100	200	500	100	200	500	100	200	500
(S1.1)	0.5	0.0	0.0	2.1	2.8	3.8	1.2	2.2	3.1	1.1	2.0	2.7
			0.5	1.4	2.9	3.7	1.2	2.0	2.6	1.2	1.6	2.5
			0.8	2.3	3.1	4.1	1.3	1.9	2.8	1.2	1.3	2.8
		0.2	0.0	2.1	2.9	3.5	1.0	1.9	2.4	1.0	1.4	2.5
			0.5	2.1	2.9	4.2	1.1	2.3	2.7	1.1	1.5	2.6
			0.8	1.9	2.6	4.2	1.4	2.3	3.0	1.0	1.5	3.3
	-0.5	0.0	0.0	2.5	3.0	4.0	1.4	2.5	2.8	1.2	1.9	2.7
			0.5	2.7	2.7	3.8	1.4	2.3	2.4	1.1	1.7	2.4
			0.8	2.1	3.1	3.6	1.2	2.0	2.6	1.0	1.4	2.4
		0.2	0.0	1.9	2.8	4.2	1.2	2.0	2.8	1.1	1.6	3.1
			0.5	2.1	3.2	3.6	1.2	2.6	2.6	1.4	1.6	2.4
			0.8	1.8	2.7	4.2	1.6	2.3	2.8	1.3	2.0	2.6
(S1.2)	0.5	0.0	0.0	17.4	13.1	12.3	46.0	34.8	26.3	67.1	54.3	40.8
			0.5	17.6	14.6	11.6	46.1	37.2	25.7	67.5	55.1	40.5
			0.8	16.4	13.1	11.9	45.8	34.4	27.0	67.2	53.3	40.8
		0.2	0.0	27.0	22.6	15.7	62.6	53.1	40.0	79.3	70.0	58.6
			0.5	27.0	20.9	16.2	62.1	52.6	40.3	79.6	70.4	58.6
			0.8	26.7	21.8	16.7	61.4	51.7	42.1	79.9	68.9	59.3
	-0.5	0.0	0.0	16.1	13.4	14.0	44.8	34.6	26.8	66.4	53.6	40.3
			0.5	16.5	12.7	15.4	45.6	34.9	27.5	67.3	53.1	41.6
			0.8	17.0	13.8	15.1	45.0	34.0	26.4	66.2	54.2	41.1
		0.2	0.0	27.1	22.2	19.2	59.6	52.3	41.9	78.0	69.2	59.5
			0.5	26.3	21.3	18.8	61.6	51.5	41.4	79.4	69.4	58.5
			0.8	26.2	22.2	18.8	61.4	52.0	41.3	79.9	70.2	57.7
(S1.3)	0.5	0.0	0.0	32.3	25.7	21.8	50.2	44.2	36.7	58.9	54.0	45.6
			0.5	31.8	26.5	21.3	50.1	43.4	35.7	60.7	52.5	44.8
			0.8	30.4	27.4	21.2	49.4	45.2	36.1	59.7	54.8	45.2
		0.2	0.0	33.7	29.7	29.0	49.4	45.6	42.1	57.1	52.9	49.2
			0.5	35.1	30.4	27.5	50.9	46.2	40.9	58.4	53.5	48.2
			0.8	34.0	30.7	25.8	50.4	45.7	39.4	58.6	53.4	46.9
	-0.5	0.0	0.0	33.1	30.2	32.1	50.8	46.3	45.2	59.8	55.1	51.8
			0.5	33.5	29.0	30.1	50.7	44.9	43.4	59.5	53.7	50.2
			0.8	34.0	29.4	29.6	51.5	45.9	43.1	59.7	53.6	50.9
		0.2	0.0	36.0	33.9	36.8	51.1	47.1	47.5	58.2	54.6	52.4
			0.5	36.9	34.6	36.1	52.7	48.3	47.6	59.0	55.6	53.4
			0.8	36.9	34.0	36.6	51.7	47.9	47.9	58.9	53.8	53.8
(S1.4)	0.5	0.0	0.0	6.2	15.4	46.7	3.5	9.3	35.4	4.2	6.0	27.6
			0.5	6.7	17.3	49.0	3.8	10.2	37.9	4.4	6.5	29.5
			0.8	6.7	16.6	50.8	4.1	9.6	39.5	4.9	6.9	31.9
		0.2	0.0	27.6	63.9	95.7	12.9	47.4	92.1	7.0	33.5	87.6
			0.5	27.5	63.1	96.1	12.5	48.5	92.5	6.3	33.9	88.4
			0.8	28.2	64.3	95.2	13.4	47.8	92.3	7.5	35.2	87.8
	-0.5	0.0	0.0	99.5	100	100	93.9	100	100	74.7	100	100
			0.5	99.2	100	100	93.2	100	100	74.8	100	100
			0.8	99.0	100	100	92.8	100	100	74.9	100	100
		0.2	0.0	99.5	100	100	89.4	100	100	62.2	100	100
			0.5	99.0	100	100	89.8	100	100	62.8	100	100
			0.8	98.9	100	100	88.0	100	100	62.6	100	100
(S1.5)	0.5	0.0	0.0	37.8	71.1	94.0	17.0	46.4	84.1	13.2	33.7	76.3
			0.5	36.9	69.9	94.9	17.2	45.6	84.3	13.8	32.4	76.9
			0.8	37.7	70.4	94.4	16.0	46.2	84.2	11.9	32.9	77.1
		0.2	0.0	62.9	90.2	99.4	42.9	79.3	98.4	33.4	71.4	97.0
			0.5	63.2	89.5	99.3	44.4	79.0	97.4	32.7	72.4	95.9
			0.8	61.9	89.8	99.7	43.6	78.9	98.0	33.8	71.6	96.5
	-0.5	0.0	0.0	35.5	69.4	94.2	15.2	45.5	83.7	11.7	32.5	76.8
			0.5	36.7	70.2	93.8	15.1	45.4	83.1	12.2	33.7	76.5
			0.8	34.8	71.2	94.0	15.6	45.2	83.2	12.0	32.6	75.4
		0.2	0.0	61.4	89.3	99.2	43.1	78.1	97.3	32.4	71.0	96.0
			0.5	62.1	89.7	99.3	43.5	78.1	97.7	32.6	71.1	96.4
			0.8	62.4	90.0	99.2	41.9	79.1	97.7	31.9	71.9	96.3

Table S.2: Size and power of $\widehat{U}_n(M)$ for models (S2.1)–(S2.5)

Model	ρ	d	r \ n	$M=6$			$M=12$			$M=18$		
				100	200	500	100	200	500	100	200	500
(S2.1)	0.5	0.0	0.0	2.6	2.8	3.5	2.0	2.2	3.0	1.5	2.1	2.9
			0.5	2.5	3.0	4.0	1.5	2.5	3.2	1.3	1.9	3.0
			0.8	2.2	2.5	3.9	1.5	2.2	2.9	1.3	1.6	2.8
		0.2	0.0	2.3	3.0	4.5	1.7	2.4	3.5	1.4	2.1	3.1
			0.5	2.5	3.1	4.4	1.5	2.1	3.0	1.4	2.0	2.8
			0.8	2.7	3.0	4.0	1.7	2.5	2.7	1.1	2.1	3.2
	-0.5	0.0	0.0	2.5	3.5	4.1	1.7	2.9	3.0	1.4	2.1	2.4
			0.5	2.6	2.7	3.6	1.6	2.2	2.7	1.3	2.0	2.6
			0.8	2.2	3.2	4.0	1.3	2.1	2.7	1.3	1.8	2.8
		0.2	0.0	2.6	3.5	4.0	1.4	2.8	2.6	1.6	1.9	2.6
			0.5	2.6	2.8	4.6	1.8	2.5	3.5	1.4	1.9	3.3
			0.8	2.7	3.6	4.6	1.6	2.4	3.2	1.6	2.2	3.1
(S2.2)	0.5	0.0	0.0	10.5	9.2	7.9	35.2	26.4	20.4	54.8	41.2	32.4
			0.5	11.0	8.5	9.2	35.5	25.4	21.4	54.4	40.4	33.1
			0.8	10.4	8.5	8.0	34.9	26.1	20.3	55.3	40.8	32.0
		0.2	0.0	15.5	15.7	14.5	47.0	39.7	35.6	65.1	55.6	50.0
			0.5	15.4	15.2	14.0	46.3	40.0	34.2	64.7	56.1	48.8
			0.8	15.1	14.6	14.4	45.9	39.6	34.9	64.4	55.5	49.2
	-0.5	0.0	0.0	11.0	9.2	9.8	35.8	28.3	20.7	55.1	42.8	32.8
			0.5	9.9	8.8	10.4	35.5	26.7	21.2	54.7	41.4	33.2
			0.8	10.1	8.4	9.9	35.2	26.6	21.6	54.6	41.3	33.2
		0.2	0.0	14.5	13.5	16.2	45.9	40.8	34.5	65.4	55.4	48.5
			0.5	14.7	14.4	16.9	47.1	39.7	36.4	64.8	55.9	50.8
			0.8	15.1	13.5	16.1	46.8	39.9	34.5	65.8	55.2	48.1
(S2.3)	0.5	0.0	0.0	25.4	24.1	21.8	42.8	38.4	33.9	54.0	47.0	42.9
			0.5	27.3	23.4	20.9	43.7	37.8	33.8	54.5	47.6	42.1
			0.8	27.5	23.2	20.9	43.7	38.2	33.8	55.2	48.1	41.8
		0.2	0.0	29.7	28.9	29.1	45.3	42.9	40.0	54.6	50.4	46.8
			0.5	30.4	28.4	28.1	45.3	42.6	39.5	56.2	50.5	46.6
			0.8	29.5	28.2	29.5	44.7	41.2	41.0	54.2	49.2	47.7
	-0.5	0.0	0.0	33.5	29.0	28.9	51.2	44.1	41.3	63.0	52.3	48.6
			0.5	32.6	29.1	28.4	48.8	43.6	40.5	60.2	53.1	48.1
			0.8	32.6	28.0	27.9	50.3	43.1	40.4	61.9	52.8	47.7
		0.2	0.0	40.6	37.7	40.2	56.2	51.4	50.3	65.9	58.4	56.3
			0.5	40.4	37.3	39.7	55.5	51.5	50.0	65.6	60.0	56.4
			0.8	41.4	36.1	38.9	56.6	50.5	49.5	66.1	57.9	55.5
(S2.4)	0.5	0.0	0.0	5.5	9.9	30.8	4.1	8.1	27.1	6.7	7.9	24.5
			0.5	5.2	9.8	30.4	4.6	8.7	26.4	7.3	8.5	24.3
			0.8	5.2	10.8	31.1	4.6	8.9	27.7	6.9	8.4	25.6
		0.2	0.0	13.9	26.8	53.4	11.9	23.9	51.5	15.9	24.2	50.0
			0.5	13.1	27.1	52.8	11.7	23.7	51.0	15.4	23.3	49.7
			0.8	12.5	27.4	53.6	11.0	24.1	51.3	15.6	23.9	50.4
	-0.5	0.0	0.0	52.7	67.8	80.4	47.6	67.5	80.3	41.0	67.2	80.4
			0.5	53.7	68.9	79.7	48.8	68.3	79.7	41.5	68.1	79.4
			0.8	54.6	69.1	79.5	48.6	68.9	79.3	41.3	68.4	79.2
		0.2	0.0	53.1	66.5	77.0	48.9	66.0	77.0	42.6	66.1	76.9
			0.5	52.7	65.4	78.2	48.6	65.5	77.8	42.7	65.7	78.0
			0.8	52.4	66.5	77.1	47.8	66.2	77.1	42.0	66.2	77.1
(S2.5)	0.5	0.0	0.0	99.3	100	100	96.6	100	100	88.6	99.7	100
			0.5	99.3	100	100	96.2	100	100	88.2	99.8	100
			0.8	99.4	99.9	100	96.7	100	100	88.4	99.7	100
		0.2	0.0	99.7	100	100	99.1	100	100	95.6	100	100
			0.5	99.8	100	100	99.0	100	100	95.3	99.9	100
			0.8	99.8	100	100	98.7	100	100	95.2	100	100
	-0.5	0.0	0.0	99.0	99.9	100	96.0	99.9	100	87.7	99.7	100
			0.5	99.1	100	100	95.6	100	100	88.2	99.7	100
			0.8	99.1	99.9	100	96.2	100	100	88.1	99.6	100
		0.2	0.0	99.8	100	100	98.8	100	100	95.1	100	100
			0.5	99.8	100	100	97.2	100	100	94.9	100	100
			0.8	99.7	100	100	99.0	100	100	95.2	100	100

Table S.3: Size and power of $\widehat{U}_n(M)$ for models (S3.1)–(S3.5)

				$M=6$			$M=12$			$M=18$		
Model	ρ	d	r \ n	100	200	500	100	200	500	100	200	500
(S3.1)	0.5	0.0	0.0	2.1	3.0	4.3	1.4	2.1	3.1	1.4	1.9	2.9
			0.5	2.6	2.7	4.1	1.6	1.9	3.1	1.2	1.7	2.8
			0.8	2.2	2.8	3.8	1.5	2.2	2.7	1.6	1.7	2.6
		0.2	0.0	2.1	3.3	4.1	1.5	2.3	2.8	1.2	2.0	2.9
			0.5	2.1	2.7	4.1	1.1	2.1	3.0	1.2	1.4	2.7
			0.8	2.7	2.9	3.9	1.3	2.1	2.7	1.4	1.9	2.5
	-0.5	0.0	0.0	3.6	3.7	4.2	2.1	2.6	3.6	2.2	2.0	3.2
			0.5	3.8	3.8	4.9	2.4	2.7	3.3	1.9	2.7	3.0
			0.8	4.3	4.2	4.2	2.6	3.4	3.2	2.2	2.4	3.3
		0.2	0.0	2.9	3.0	4.1	1.8	2.3	3.0	1.9	1.8	3.0
			0.5	2.8	3.3	3.4	2.2	2.9	2.8	1.8	2.0	2.8
			0.8	2.8	3.3	4.1	1.6	2.2	2.7	2.0	1.6	2.6
(S3.2)	0.5	0.0	0.0	11.2	10.5	9.6	35.6	28.0	22.7	54.0	43.4	34.9
			0.5	12.5	10.0	9.7	35.9	27.7	21.7	54.6	42.6	33.0
			0.8	11.6	9.9	11.0	35.5	28.3	24.0	54.7	42.7	35.9
		0.2	0.0	18.6	16.5	14.7	47.8	41.6	34.5	65.2	57.1	48.5
			0.5	17.4	17.0	14.6	47.7	41.6	35.1	65.5	57.1	49.7
			0.8	17.2	15.6	14.6	47.9	41.4	35.2	66.1	57.2	50.5
	-0.5	0.0	0.0	12.0	9.9	11.7	36.3	28.3	23.3	55.4	43.8	34.3
			0.5	12.2	9.9	11.2	37.2	28.3	23.4	56.0	43.0	34.7
			0.8	10.8	9.8	11.1	36.2	30.0	23.7	55.5	43.7	35.5
		0.2	0.0	16.9	16.0	16.6	48.8	43.2	35.6	66.9	58.5	50.0
			0.5	17.2	16.4	16.6	48.1	42.4	36.5	66.3	57.6	50.2
			0.8	16.3	15.6	16.4	49.2	42.4	36.8	65.9	56.9	50.2
(S3.3)	0.5	0.0	0.0	26.6	23.5	19.4	43.2	38.5	33.2	54.4	47.9	41.3
			0.5	26.1	22.6	19.7	42.3	38.2	33.2	53.4	47.3	41.2
			0.8	26.4	24.1	18.7	42.0	39.4	31.4	52.7	48.9	40.7
		0.2	0.0	29.8	30.0	26.3	45.4	43.9	38.7	55.2	51.6	45.9
			0.5	30.1	29.6	25.9	45.4	43.5	39.2	54.9	51.8	46.6
			0.8	29.6	27.7	26.6	45.4	42.8	38.3	55.2	50.1	45.7
	-0.5	0.0	0.0	35.1	29.4	25.4	51.4	45.2	38.5	60.8	54.5	46.5
			0.5	33.8	28.2	27.4	50.3	43.3	40.1	60.2	52.7	47.8
			0.8	34.4	29.3	26.5	51.3	44.7	38.3	61.4	54.1	46.8
		0.2	0.0	40.8	37.2	34.8	56.0	51.9	47.4	64.8	59.1	54.8
			0.5	41.7	38.2	35.3	57.5	52.6	48.2	66.4	60.9	54.8
			0.8	42.4	37.2	35.1	58.0	52.8	47.3	67.5	60.0	54.7
(S3.4)	0.5	0.0	0.0	9.4	18.2	48.1	9.2	16.3	43.8	13.7	15.8	39.5
			0.5	10.3	17.0	45.2	10.0	15.5	41.3	14.0	16.1	37.7
			0.8	10.0	18.5	45.8	9.1	15.6	42.1	13.6	15.8	38.4
		0.2	0.0	31.8	55.8	84.3	27.5	51.0	82.8	31.4	49.1	81.0
			0.5	31.6	55.5	82.8	27.4	50.8	82.2	30.0	48.3	80.1
			0.8	31.7	56.7	82.8	27.4	51.9	81.4	30.5	49.9	79.9
	-0.5	0.0	0.0	86.1	92.9	96.5	77.1	92.6	96.4	62.2	92.4	96.3
			0.5	84.7	93.5	96.3	75.9	93.0	96.3	61.4	92.7	96.3
			0.8	85.3	92.6	96.1	76.1	92.0	96.0	61.6	91.7	96.0
		0.2	0.0	84.1	91.4	95.6	76.9	91.3	95.6	63.5	91.3	95.7
			0.5	85.9	91.6	95.4	79.4	91.6	95.5	66.2	91.5	95.5
			0.8	85.2	91.6	95.9	78.3	91.5	95.8	64.7	91.5	95.8
(S3.5)	0.5	0.0	0.0	98.9	100	100	92.7	99.9	100	80.9	98.9	100
			0.5	98.9	100	100	92.9	99.9	100	81.4	99.1	100
			0.8	99.0	100	100	92.8	99.7	100	80.7	99.1	100
		0.2	0.0	99.5	100	100	96.4	99.9	100	88.8	99.6	100
			0.5	99.4	100	100	96.5	99.9	100	89.6	99.6	100
			0.8	99.7	100	100	96.1	99.8	100	89.0	99.5	100
	-0.5	0.0	0.0	98.5	100	100	90.9	99.8	100	79.0	99.0	100
			0.5	98.5	100	100	91.7	99.8	99.9	79.7	99.1	100
			0.8	98.2	100	100	91.4	99.9	100	79.2	99.2	100
		0.2	0.0	99.5	100	100	96.5	100	100	89.6	99.6	100
			0.5	99.5	99.9	100	96.4	99.9	100	88.5	99.7	100
			0.8	99.6	100	100	96.4	100	100	88.9	99.8	100

Table S.4: Size and power of $\widehat{U}_n(M)$ for models (S4.1)–(S4.5) with $u_t \sim$ EGM 1

				$M=6$			$M=12$			$M=18$		
Model	ρ	d	r \ n	100	200	500	100	200	500	100	200	500
(S4.1)	0.5	0.0	0.0	6.7	5.6	5.1	7.0	6.1	5.2	7.3	6.2	5.6
			0.5	5.9	5.6	5.6	6.7	6.3	5.9	7.4	6.2	6.2
			0.8	6.2	5.7	5.4	7.1	6.5	5.3	7.5	6.7	5.8
		0.2	0.0	6.0	5.4	5.2	7.0	6.3	5.4	7.5	6.5	5.4
			0.5	7.1	5.9	5.1	8.1	6.2	5.7	8.0	6.8	5.8
			0.8	6.6	5.4	6.1	6.9	6.2	5.5	7.6	6.8	6.2
	-0.5	0.0	0.0	6.1	5.9	5.9	6.7	5.8	5.4	7.3	6.1	5.7
			0.5	6.9	5.1	5.4	6.9	6.0	5.5	7.5	5.9	5.6
			0.8	6.4	6.3	5.4	6.3	6.3	5.0	7.2	6.1	5.2
		0.2	0.0	6.6	5.8	5.5	6.8	6.5	5.0	7.4	6.7	5.8
			0.5	6.3	6.1	5.5	6.9	6.2	5.7	7.6	6.5	6.2
			0.8	6.8	6.5	6.0	7.2	6.6	5.3	7.5	6.6	5.3
(S4.2)	0.5	0.0	0.0	15.1	27.2	48.4	15.8	32.0	61.6	14.6	31.7	64.6
			0.5	15.6	27.4	50.4	16.1	31.3	62.5	14.1	31.0	65.5
			0.8	15.7	26.7	49.7	15.5	31.3	63.6	14.2	30.3	66.2
		0.2	0.0	15.8	31.8	59.8	15.5	33.5	65.8	14.6	31.5	64.5
			0.5	15.6	30.4	60.4	15.4	32.4	66.9	13.7	30.3	65.9
			0.8	15.0	31.2	59.0	13.8	32.4	65.4	13.0	29.6	64.4
	-0.5	0.0	0.0	14.3	27.8	57.3	13.8	31.9	66.6	13.5	31.2	69.2
			0.5	14.2	28.6	59.0	14.4	32.1	68.0	12.9	30.8	69.2
			0.8	13.7	27.1	59.8	14.4	31.4	68.7	12.9	30.9	70.1
		0.2	0.0	13.8	32.1	71.8	14.2	32.4	73.8	12.8	29.9	72.0
			0.5	12.9	32.2	73.0	13.1	33.4	75.0	12.0	30.4	72.2
			0.8	13.5	32.4	72.3	13.7	33.3	74.3	12.4	31.1	72.1
(S4.3)	0.5	0.0	0.0	16.9	25.1	35.2	13.2	24.3	38.3	9.2	21.5	37.4
			0.5	15.9	24.1	35.5	12.1	22.7	37.8	8.8	20.0	37.1
			0.8	17.0	24.2	34.1	12.8	23.8	37.7	9.9	21.0	37.8
		0.2	0.0	13.5	19.6	27.0	10.3	18.5	27.8	7.2	16.2	27.3
			0.5	13.5	20.4	26.5	9.6	18.5	27.6	6.8	16.5	27.3
			0.8	14.3	19.6	25.8	10.6	18.9	28.2	7.1	16.3	27.5
	-0.5	0.0	0.0	22.3	31.6	59.4	19.7	30.8	57.7	15.1	27.9	57.2
			0.5	21.4	32.3	59.2	17.8	30.3	58.8	14.4	27.7	57.7
			0.8	21.8	32.5	59.0	19.1	31.5	57.8	14.8	29.6	57.0
		0.2	0.0	20.8	27.9	43.8	18.1	26.9	43.0	13.9	25.0	40.5
			0.5	22.5	28.6	44.9	19.6	28.0	43.6	15.3	25.6	42.3
			0.8	21.8	28.1	44.1	18.5	27.6	42.9	14.8	25.0	41.8
(S4.4)	0.5	0.0	0.0	5.3	10.8	53.2	6.3	10.2	58.2	6.7	9.4	55.0
			0.5	6.9	10.5	53.6	7.4	9.9	57.5	7.9	9.4	54.9
			0.8	6.1	10.6	54.1	6.9	10.0	58.3	7.2	9.5	55.0
		0.2	0.0	6.9	15.7	65.2	6.8	13.7	64.0	7.5	11.9	58.7
			0.5	6.4	15.9	65.6	6.4	14.0	64.2	7.5	12.1	58.7
			0.8	6.4	16.8	65.8	6.4	13.5	64.6	7.6	12.2	59.4
	-0.5	0.0	0.0	47.1	76.3	89.6	44.4	75.2	89.4	44.1	74.3	88.9
			0.5	47.2	75.7	90.5	44.9	74.9	89.8	44.7	74.2	89.7
			0.8	48.9	75.7	88.7	45.6	74.7	88.0	46.0	74.1	88.0
		0.2	0.0	57.7	78.2	89.8	56.6	77.9	89.7	56.5	77.0	89.3
			0.5	56.8	77.2	89.6	55.7	76.3	89.4	55.6	76.0	89.0
			0.8	57.8	78.0	89.0	55.9	76.9	88.8	56.1	77.0	88.5
(S4.5)	0.5	0.0	0.0	92.0	97.0	98.8	88.9	94.8	97.8	87.2	93.8	97.1
			0.5	92.6	96.4	98.9	89.6	94.5	98.1	87.5	93.3	97.4
			0.8	92.4	96.7	99.0	89.7	94.5	98.1	88.2	93.3	97.5
		0.2	0.0	90.8	94.3	97.2	86.5	90.9	95.5	84.4	89.1	94.1
			0.5	90.7	95.0	97.4	87.2	91.5	96.0	85.2	89.7	94.4
			0.8	91.3	94.7	97.1	87.9	91.4	95.8	85.9	89.7	94.2
	-0.5	0.0	0.0	90.8	96.8	99.0	87.9	94.9	98.0	86.1	93.5	97.5
			0.5	91.8	96.8	99.1	88.5	94.8	98.4	86.4	93.9	97.5
			0.8	92.0	96.4	99.1	88.7	94.6	98.4	86.0	93.4	97.8
		0.2	0.0	91.0	94.5	97.0	87.5	91.3	95.1	85.0	89.3	93.8
			0.5	90.9	94.4	97.2	87.4	91.8	95.4	85.5	89.7	94.2
			0.8	90.1	94.4	96.9	86.4	91.3	94.8	84.9	89.7	93.4

Table S.5: Size and power of $\widehat{U}_n(M)$ for models (S4.1)–(S4.5) with $u_t \sim$ EGM 2

				$M=6$			$M=12$			$M=18$		
Model	ρ	d	r \ n	100	200	500	100	200	500	100	200	500
(S4.1)	0.5	0.0	0.0	3.2	2.7	4.4	2.1	2.7	3.1	1.7	2.1	3.1
			0.5	2.7	3.4	3.8	1.6	3.0	2.7	1.5	2.4	3.1
			0.8	2.3	3.3	4.3	1.9	2.5	3.4	1.5	2.0	3.4
		0.2	0.0	3.2	2.9	4.0	2.3	2.8	3.1	1.8	1.9	2.9
			0.5	3.1	3.1	3.9	1.9	2.7	2.8	1.5	1.9	2.5
			0.8	3.1	3.2	4.3	2.2	2.8	3.5	1.6	2.2	3.4
	-0.5	0.0	0.0	3.4	3.2	4.1	2.0	2.4	2.8	1.4	2.5	2.4
			0.5	3.3	3.7	4.0	1.8	2.5	2.8	1.5	2.0	2.8
			0.8	3.1	3.7	4.4	2.0	2.5	2.9	1.5	2.0	3.2
		0.2	0.0	3.3	4.0	4.5	2.1	3.0	2.9	1.9	2.3	3.5
			0.5	3.1	4.0	4.4	2.0	3.1	3.1	1.6	2.1	3.0
			0.8	3.6	4.6	4.6	1.8	2.9	3.3	1.7	2.7	3.3
(S4.2)	0.5	0.0	0.0	11.7	10.0	9.8	37.5	27.7	21.7	56.6	42.5	33.2
			0.5	11.7	10.3	9.2	36.7	28.5	22.1	56.5	43.4	33.6
			0.8	12.9	10.4	10.3	37.5	28.5	22.9	56.4	43.3	33.9
		0.2	0.0	17.5	15.6	15.7	50.3	40.4	34.9	68.8	56.3	48.2
			0.5	17.1	16.2	15.4	49.8	41.2	34.0	68.2	56.7	47.2
			0.8	17.4	15.5	15.6	50.1	40.6	34.7	68.4	56.2	48.8
	-0.5	0.0	0.0	11.9	9.7	11.4	38.0	28.7	23.1	57.9	42.9	33.5
			0.5	11.8	10.1	10.5	36.7	28.4	21.7	56.6	42.6	32.7
			0.8	11.1	9.3	11.2	36.3	27.1	23.3	57.4	42.2	33.9
		0.2	0.0	15.5	14.5	17.1	49.1	40.1	34.8	67.8	56.5	48.6
			0.5	15.3	13.9	17.2	49.4	39.9	35.5	68.1	55.7	48.6
			0.8	16.1	14.1	17.5	48.4	40.5	35.1	68.1	56.4	48.6
(S4.3)	0.5	0.0	0.0	24.3	22.9	19.5	41.6	36.0	32.7	54.0	47.3	40.9
			0.5	23.6	22.5	20.4	42.1	36.9	33.0	54.1	46.7	40.6
			0.8	24.8	22.2	19.4	42.7	37.4	32.3	54.2	46.7	40.4
		0.2	0.0	29.6	28.0	27.5	46.1	40.7	39.1	56.5	49.3	45.8
			0.5	27.8	27.6	26.8	45.1	40.8	38.6	55.4	48.9	45.7
			0.8	29.5	28.6	26.8	46.4	42.0	38.9	56.3	49.4	44.8
	-0.5	0.0	0.0	32.0	29.0	28.3	50.9	44.1	40.0	61.9	53.4	47.1
			0.5	31.6	28.7	29.2	50.2	43.4	40.7	63.0	52.4	48.5
			0.8	32.8	28.9	27.5	53.2	44.1	38.9	65.0	54.2	47.0
		0.2	0.0	40.4	38.2	38.2	58.0	51.7	49.3	67.3	60.1	55.3
			0.5	42.3	38.4	38.3	58.5	52.1	48.7	68.3	60.6	54.8
			0.8	39.9	37.9	37.8	56.9	51.9	48.7	68.1	59.5	54.7
(S4.4)	0.5	0.0	0.0	6.1	12.3	41.6	4.3	9.5	36.4	6.4	8.5	30.8
			0.5	5.4	11.9	41.8	4.2	9.1	36.1	6.4	7.9	30.9
			0.8	5.5	11.9	41.6	4.6	10.1	35.7	6.5	8.0	30.7
		0.2	0.0	14.0	31.3	64.7	10.9	27.3	62.4	14.3	24.9	60.0
			0.5	14.2	31.1	64.3	10.8	26.7	62.3	13.2	24.3	60.1
			0.8	13.7	32.6	64.8	9.9	28.4	62.7	13.4	26.3	60.4
	-0.5	0.0	0.0	68.9	84.9	93.0	59.2	84.3	92.8	48.3	83.9	92.9
			0.5	68.6	84.2	93.1	58.2	83.9	93.0	47.5	83.2	92.9
			0.8	67.9	85.0	92.9	59.7	84.8	93.0	48.7	84.3	92.9
		0.2	0.0	69.0	82.7	91.2	62.0	83.1	91.2	51.2	83.2	91.3
			0.5	68.8	83.2	92.3	62.4	83.1	92.3	52.8	82.8	92.3
			0.8	67.4	84.2	91.4	60.8	84.1	91.7	50.6	84.0	91.6
(S4.5)	0.5	0.0	0.0	98.7	100	100	94.3	100	100	85.4	99.6	100
			0.5	99.0	100	100	94.8	100	100	84.7	99.8	100
			0.8	99.1	100	100	94.8	99.9	100	85.1	99.6	100
		0.2	0.0	99.3	100	100	97.2	99.9	100	91.2	99.8	100
			0.5	99.6	100	100	97.3	99.9	100	91.0	99.7	100
			0.8	99.5	100	100	97.2	99.9	100	91.2	99.8	100
	-0.5	0.0	0.0	98.7	100	100	93.1	99.9	100	84.3	99.6	100
			0.5	98.6	99.9	100	93.2	99.9	100	84.3	99.6	100
			0.8	98.5	99.9	100	93.6	99.9	100	84.7	99.5	100
		0.2	0.0	99.5	100	100	97.5	100	100	91.9	99.9	100
			0.5	99.5	100	100	97.4	99.9	100	91.8	99.7	100
			0.8	99.6	100	100	97.0	100	100	92.0	99.9	100

Table S.6: Size and power of $\widehat{U}_n(M)$ for models (S4.6)–(S4.10) with $u_t \sim$ EGM 1

Model	ρ	d	$r \diagdown n$	$M=6$			$M=12$			$M=18$		
				100	200	500	100	200	500	100	200	500
(S4.6)	0.5	0.0	0.0	7.2	5.9	4.8	8.0	6.5	5.7	8.5	7.3	6.0
			0.5	7.8	6.6	5.4	8.0	6.5	5.8	8.5	7.3	6.2
			0.8	8.0	6.1	5.5	7.9	6.4	5.3	8.4	6.8	6.1
		0.2	0.0	7.8	6.7	5.7	8.1	6.8	5.8	9.0	7.3	5.9
			0.5	7.5	6.9	5.9	8.7	7.1	6.4	9.0	6.8	6.0
			0.8	7.4	6.9	5.3	7.6	7.1	5.3	8.5	7.2	5.9
	-0.5	0.0	0.0	8.0	6.9	5.8	8.2	5.8	5.9	8.6	6.9	6.0
			0.5	7.4	6.4	5.3	7.6	6.8	5.7	8.7	7.1	6.1
			0.8	7.8	6.5	5.2	7.5	5.5	5.1	8.7	6.3	5.0
		0.2	0.0	8.1	6.7	6.1	8.5	6.1	6.0	9.5	6.7	5.8
			0.5	7.9	7.2	6.0	8.5	7.2	6.0	8.8	7.8	5.9
			0.8	8.9	7.8	5.9	8.4	7.9	5.3	8.9	7.7	5.6
(S4.7)	0.5	0.0	0.0	16.8	28.7	47.7	16.7	32.5	61.6	15.6	31.4	64.0
			0.5	17.2	27.1	48.5	16.3	31.4	61.8	15.7	30.4	63.9
			0.8	16.8	29.1	49.4	17.0	33.3	61.7	15.8	32.3	64.4
		0.2	0.0	16.6	31.4	59.3	16.0	33.0	65.6	15.0	30.9	65.1
			0.5	16.0	31.9	58.2	15.7	32.9	65.1	14.9	30.8	64.0
			0.8	16.1	31.1	58.4	16.0	32.1	63.9	14.7	30.5	63.1
	-0.5	0.0	0.0	14.7	28.6	58.1	15.5	32.7	67.0	14.2	31.2	69.3
			0.5	16.0	27.4	57.8	16.2	31.5	67.2	15.0	31.9	69.1
			0.8	14.6	27.9	57.5	16.0	31.0	68.3	14.9	30.4	69.9
		0.2	0.0	14.4	31.8	69.9	14.7	33.3	72.4	13.8	31.7	70.5
			0.5	15.1	31.6	71.1	16.1	32.9	72.6	14.1	30.8	70.7
			0.8	14.6	30.7	70.9	15.2	33.2	74.0	13.7	30.9	71.8
(S4.8)	0.5	0.0	0.0	18.1	25.0	35.5	13.8	23.6	37.3	10.3	21.0	38.3
			0.5	17.1	25.0	35.6	14.0	24.0	39.0	11.0	21.7	38.1
			0.8	17.4	24.8	35.1	14.0	24.3	38.3	10.5	22.1	38.1
		0.2	0.0	14.4	20.8	26.3	11.3	19.4	27.9	8.2	17.3	26.9
			0.5	13.8	20.6	25.6	10.6	19.4	27.2	7.1	16.2	26.4
			0.8	14.8	20.5	26.5	10.7	19.3	28.4	7.8	16.5	28.7
	-0.5	0.0	0.0	23.1	33.7	60.2	19.8	32.4	59.5	16.1	29.6	58.1
			0.5	22.2	32.7	59.1	19.3	32.4	57.9	15.2	29.8	56.2
			0.8	23.9	33.3	59.8	20.5	31.9	59.1	16.4	29.1	57.5
		0.2	0.0	24.3	29.8	44.5	20.2	28.8	43.0	16.1	26.3	41.9
			0.5	23.9	31.1	44.5	20.4	28.9	43.2	16.2	26.3	41.8
			0.8	24.9	29.2	44.9	21.1	28.5	43.4	16.8	26.3	42.5
(S4.9)	0.5	0.0	0.0	6.3	9.9	50.5	7.3	9.9	54.9	8.1	9.6	52.9
			0.5	6.8	10.3	50.0	7.6	9.8	54.8	8.7	9.9	52.1
			0.8	6.9	10.2	50.2	7.2	9.5	54.3	8.3	9.4	52.1
		0.2	0.0	7.3	15.5	65.5	7.3	13.5	64.3	8.3	12.1	58.3
			0.5	7.2	15.8	65.4	7.9	14.1	63.9	8.6	13.1	58.7
			0.8	6.8	15.1	64.9	7.3	13.1	64.2	8.5	12.3	58.4
	-0.5	0.0	0.0	42.1	75.4	89.4	40.8	74.3	88.9	41.7	73.6	88.9
			0.5	42.5	74.8	89.9	40.9	74.2	89.8	42.2	73.5	89.4
			0.8	43.5	76.0	90.1	41.8	74.7	89.9	42.6	74.5	89.8
		0.2	0.0	54.9	77.2	88.5	54.6	76.7	88.5	54.3	76.3	88.4
			0.5	55.0	77.9	88.8	54.2	77.0	88.6	54.5	76.8	88.3
			0.8	54.5	78.3	88.2	53.6	77.5	88.1	54.0	76.9	88.1
(S4.10)	0.5	0.0	0.0	94.5	98.2	99.4	92.3	96.7	98.8	91.2	95.6	98.3
			0.5	95.0	97.5	99.3	92.8	96.4	98.6	91.0	95.5	98.1
			0.8	94.1	97.4	99.3	91.9	95.9	98.6	90.3	95.1	98.1
		0.2	0.0	94.4	96.8	98.4	91.4	95.0	96.9	90.2	93.2	96.1
			0.5	94.7	96.4	98.2	92.1	93.8	97.3	90.6	92.2	96.4
			0.8	94.3	97.2	98.7	91.2	94.9	97.4	89.6	93.2	96.5
	-0.5	0.0	0.0	94.0	97.6	99.3	91.8	96.1	98.7	89.9	95.2	98.3
			0.5	93.7	98.0	99.6	91.4	96.7	99.0	89.9	95.2	98.6
			0.8	93.5	97.5	99.2	91.2	96.1	98.7	89.2	95.0	98.2
		0.2	0.0	94.3	96.2	98.4	91.3	94.2	97.1	89.9	92.6	96.1
			0.5	94.7	96.7	98.4	91.9	94.2	96.9	90.7	92.5	95.9
			0.8	94.5	96.7	98.3	91.7	94.4	97.1	90.2	92.9	96.1

Table S.7: Size and power of $\widehat{U}_n(M)$ for models (S4.6)–(S4.10) with $u_t \sim$ EGM 2

				$M=6$			$M=12$			$M=18$		
Model	ρ	d	r \ n	100	200	500	100	200	500	100	200	500
(S4.6)	0.5	0.0	0.0	3.1	3.4	4.1	2.1	2.8	3.0	2.2	2.2	2.9
			0.5	3.9	3.9	4.0	2.8	3.2	3.7	2.2	3.2	3.4
			0.8	2.9	3.6	4.5	2.5	2.9	3.1	2.2	2.5	3.2
		0.2	0.0	3.8	3.6	4.9	2.6	3.3	3.8	2.0	2.3	4.0
			0.5	3.3	3.5	4.0	2.5	3.1	3.2	2.4	2.5	3.2
			0.8	3.4	3.5	4.1	2.8	3.3	3.1	2.0	2.3	3.2
	-0.5	0.0	0.0	4.4	4.1	4.4	2.3	3.2	3.3	1.6	2.4	3.0
			0.5	4.3	4.2	4.7	2.5	3.3	3.5	2.2	2.7	3.2
			0.8	4.2	3.7	4.2	2.1	2.7	3.3	1.6	2.1	3.1
		0.2	0.0	4.8	4.9	4.5	2.8	3.4	3.1	2.1	2.7	3.4
			0.5	4.9	5.2	5.2	2.8	3.5	3.9	2.1	3.1	3.4
			0.8	4.8	5.2	5.2	2.5	3.5	3.6	1.9	2.7	3.3
(S4.7)	0.5	0.0	0.0	11.7	10.1	9.6	37.8	27.5	20.6	58.8	42.3	31.4
			0.5	12.2	9.3	8.0	37.7	26.5	19.8	57.8	42.2	31.4
			0.8	11.4	9.2	9.9	37.8	27.0	21.4	57.9	41.2	33.3
		0.2	0.0	16.7	13.9	14.2	51.3	40.5	33.1	69.4	58.0	46.2
			0.5	16.2	14.8	14.5	51.3	40.5	32.1	69.4	56.6	46.0
			0.8	18.0	14.7	14.5	51.8	39.8	32.9	70.8	55.6	46.6
	-0.5	0.0	0.0	12.0	8.7	10.0	38.8	27.7	21.7	58.9	42.7	32.9
			0.5	11.6	9.4	9.8	39.6	27.5	21.2	59.4	43.1	32.6
			0.8	12.1	7.9	10.7	38.9	28.3	23.0	58.9	43.9	33.8
		0.2	0.0	16.8	13.3	16.1	52.1	40.0	34.6	70.9	57.5	47.8
			0.5	15.4	13.0	15.8	51.5	39.4	33.9	69.6	56.0	46.9
			0.8	15.8	13.7	15.4	50.5	41.3	32.8	69.4	57.3	46.4
(S4.8)	0.5	0.0	0.0	23.6	21.7	18.3	43.6	36.2	31.2	56.4	46.3	39.7
			0.5	22.9	21.1	17.8	44.5	35.5	30.7	56.2	45.9	38.3
			0.8	24.2	21.6	19.2	44.0	37.3	31.9	56.5	47.6	40.6
		0.2	0.0	28.5	25.9	27.0	48.0	41.5	38.5	59.4	50.6	46.6
			0.5	28.2	27.1	26.9	47.8	41.7	39.0	58.1	51.0	46.0
			0.8	27.5	26.9	26.5	47.2	42.4	38.9	57.4	52.2	45.3
	-0.5	0.0	0.0	34.0	27.9	26.7	54.8	43.1	38.1	66.6	52.9	46.0
			0.5	32.0	28.8	28.3	52.5	43.4	40.5	64.4	54.0	47.5
			0.8	33.3	28.5	27.6	53.4	43.5	40.3	65.3	53.8	48.2
		0.2	0.0	42.5	37.4	38.8	62.6	53.9	49.4	72.4	63.1	56.1
			0.5	40.0	37.4	37.5	60.4	52.8	48.9	70.3	61.7	55.7
			0.8	41.2	37.8	38.2	62.1	54.0	49.5	71.1	64.4	56.6
(S4.9)	0.5	0.0	0.0	5.5	11.1	41.6	4.0	8.8	35.4	5.4	7.5	30.1
			0.5	5.5	11.4	41.1	4.0	9.1	35.8	6.2	7.6	30.0
			0.8	5.9	11.8	42.7	4.4	8.7	36.9	6.0	7.4	32.0
		0.2	0.0	13.9	32.9	67.3	8.9	27.1	65.2	11.5	23.4	61.7
			0.5	14.3	31.5	66.0	8.9	26.7	63.9	10.0	23.2	61.1
			0.8	13.3	32.0	66.4	8.7	27.1	65.0	10.3	23.8	62.5
	-0.5	0.0	0.0	66.4	84.2	92.8	55.4	84.0	92.7	44.0	83.6	93.0
			0.5	68.2	84.1	92.5	56.2	83.9	92.5	44.2	83.2	92.6
			0.8	65.7	83.1	92.2	55.5	83.0	92.2	43.8	83.1	92.2
		0.2	0.0	68.2	83.1	90.8	59.7	83.1	90.9	47.8	83.3	90.9
			0.5	67.3	82.7	91.6	59.5	82.7	91.7	48.4	82.8	91.9
			0.8	68.4	83.0	91.3	60.9	83.4	91.1	49.5	83.3	91.2
(S4.10)	0.5	0.0	0.0	99.1	100	100	94.8	100	100	84.1	99.7	100
			0.5	99.2	100	100	94.3	100	100	83.5	99.7	100
			0.8	98.8	99.9	100	94.5	99.8	100	83.4	99.6	100
		0.2	0.0	99.5	100	100	97.5	99.9	100	91.3	99.8	100
			0.5	99.4	99.9	100	97.2	100	100	90.5	99.9	100
			0.8	99.4	100	100	97.3	100	100	91.3	99.9	100
	-0.5	0.0	0.0	98.7	99.9	100	93.4	99.9	100	82.7	99.6	100
			0.5	98.9	100	100	93.6	99.9	100	83.2	99.6	100
			0.8	98.7	100	100	93.2	99.9	100	82.6	99.4	100
		0.2	0.0	99.4	100	100	97.3	100	100	90.8	99.8	100
			0.5	99.4	100	100	96.9	100	100	90.2	99.8	100
			0.8	99.6	100	100	97.3	99.9	100	90.5	99.9	100