Statistica Sinica **29** (2019), 1419-1438
doi:https://doi.org/10.5705/ss.202016.0084

OPTIMAL PAIRED CHOICE BLOCK DESIGNS

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Abstract: Choice experiments help manufacturers, service providers, policymakers, and other researchers to make business decisions. Traditionally, in a discrete-choice experiment, each respondent is shown the same collection of choice pairs (i.e., the choice design). In addition, as the number of attributes and/or the number of levels under each attribute increases, the number of choice pairs in an optimal paired choice design increases rapidly. Moreover, in the literature on utility-neutral setups, random subsets of theoretically obtained optimal designs are often allocated to respondents. This raises the question of whether we can do better than simply using a random allocation of subsets. We answer this question using a linear paired-comparison model (or, equivalently, a multinomial logit model), where we first incorporate the fixed respondent effects (also referred to as the block effects), and then obtain optimal designs for the parameters of interest. Our approach is simple and theoretically tractable, unlike other approaches that are algorithmic in nature. We present several constructions of optimal block designs that can be used to estimate main effects or main plus two-factor interaction effects. Our results show when and how an optimal design for the model without blocks can be split into blocks such that the optimality properties are retained under the block model.

Key words and phrases: Choice experiment, hadamard matrix, linear paired comparison model, multinomial logit model, orthogonal array, utility-neutral setup.

1. Introduction

Choice experiments mimic situations in which individuals have to choose between a number of competing options. The goal of such experiments is to quantify the influence of the attributes that characterize the choice options. In a choice experiment, respondents are shown multiple choice sets of options, from which they must choose a preferred option. Considering choice sets of size two and r respondents, a paired choice experiment usually shows the same set of N choice pairs to each of the r respondents. Then respondents are asked to indicate their preference between the two options for each of the N choice pairs. Each option in a choice pair is described by a set of k attributes, where, for

$i = 1, \ldots, k$, the ith attribute has v_i levels, for $v_i \geq 2$. We represent the v_i levels by $0, \ldots, v_i - 1$. In a choice experiment, a paired choice design d is an allocation of choice pairs among r respondents such that each respondent observes N choice pairs. Such paired choice designs are often analyzed using a multinomial logit model.

One objective of a choice experiment is to optimally or efficiently estimate the parameters of interest, which are either the main effects or the main plus two-factor interaction effects of the k attributes. D-optimal designs have been employed in studies under utility-neutral setups or using the locally D-optimal/ Bayesian approach. D-optimal designs have also been obtained theoretically under the utility-neutral setup; for example, see Graßhoff et al. (2003), Graßhoff et al. (2004), Street and Burgess (2007), Street and Burgess (2012), Demirkale, Donovan and Street (2013), Bush (2014), Großmann and Schwabe (2015), and Singh, Chai and Das (2015). In contrast, using locally optimal and Bayesian approaches, D-optimal designs have been obtained using computer algorithms (e.g., Huber and Zwerina (1996), Sándor and Wedel (2001), Sándor and Wedel (2002), Sándor and Wedel (2005), Kessels, Goos and Vandebroek (2006), Kessels, Goos and Vandebroek (2008), Kessels et al. (2008), Kessels et al. (2009), Yu, Goos and Vandebroek (2009)). In this study, we follow the utility-neutral approach.

Traditionally, in a choice experiment, respondents are shown the same collection of N choice pairs under the assumption that the respondents are alike. However, this assumption is not always practical because respondents who are sampled randomly from a population are more likely to be heterogeneous. For example, Kessels, Goos and Vandebroek (2008) note that this heterogeneity leads to variations in the responses from different respondents.

In a paired choice experiment, there is always a constraint on the maximum number of choice pairs that can be shown to each respondent so as to maintain the overall response quality. A major concern with traditional optimal paired choice designs is that the number of choice pairs in the design increases rapidly with a moderate increase in k and/or v_i.

Attempts have been made to address the issue of heterogeneity among respondents using different models and approaches. Sándor and Wedel (2002) construct designs using a computer-intensive algorithmic approach under the so-called mixed logit model. In their approach, the same set of N choice pairs are shown to each respondent. Subsequently, Sándor and Wedel (2005) demonstrated that using different choice designs and randomly allocating respondents to these designs yields substantially higher efficiency than the designs obtained in Sándor and Wedel (2002). Later, Kessels, Goos and Vandebroek (2008), addressing the heterogeneity in conjoint experiments, introduced a random respondent effects model for estimating the main effects and used algorithmic methods to construct D-optimal designs. The conjoint designs under their setup identify as many sets of options as there are respondents. Therefore, their approach, though similar, is not applicable to our setup.

In practice, there is often a pool of choice sets, a random subset of which is allocated to each respondent Street and Burgess (2007). This process is continued until all choice sets are used once. Thereafter, the process is started again. To address this *ad hoc* approach to the allocation of choice sets, we use an additional fixed-effect term in the model to systematically split the pool of choice sets. In experimental design theory, the concept of blocking is used extensively as a tool to eliminate systematic heterogeneity in the experimental material. Following the same approach, we consider the respondents as blocks. Thus, in contrast to the computer-intensive algorithmic approaches of Sándor and Wedel (2005) and Kessels, Goos and Vandebroek (2008), we treat respondent heterogeneity as a nuisance factor by including respondent-level block-effect terms in the model. Then we design experiments that optimally estimate the parameters of interest after eliminating the respondent (block) effects. Adopting this approach enables an experimenter to obtain optimal designs, with reasonable number of choice pairs $s(< N)$ shown to each of the r respondents. Later, in Section 2, we discuss the kind of heterogeneity addressed by both our approach and seemingly similar approaches.

In what follows, a design with b blocks, each of size s, is generated, where each block is associated with a respondent. Usually, t copies of a proposed design are used for larger numbers of respondents $r = tb$, because replicating the design does not affect its optimality. Therefore, we restrict ourselves to optimal paired choice block designs with b blocks, each of size s, with $N = bs$.

In this context, the traditional paired choice designs reduce to $b = 1, s = N$, and $r = t$, where s is necessarily at least the number of model parameters. However, for $b > 1$, the block size s can be smaller than the number of model parameters, but the paired choice design with b blocks can still estimate all of the parameters. In order to estimate these parameters, we provide optimal designs with block sizes that are flexible and practical under our setup.

In Section 2, we treat respondent heterogeneity as a nuisance factor and incorporate the fixed respondent (block) effects in the model. Then, we obtain an information matrix for estimating the parameters of interest after eliminating the respondent (block) effects. In Section 3, under the main-effects block model, we provide optimal paired choice block designs for estimating the main effects for symmetric and asymmetric attributes. We also provide a simple solution to the problem of identifying generators when constructing an optimal paired choice design. In Section 4, under a broader main-effects block model, we provide optimal paired choice block designs for symmetric and asymmetric attributes. The broader main-effects model includes the main effects and the two-factor interaction effects, where we are interested only in estimating the main effects. Finally, in Section 5, we provide optimal paired choice block designs for estimating the main plus two-factor interaction effects. Finally, we conclude the paper in Section 6.

2. Preliminaries and the Model Incorporating Respondent Effects

Most studies on optimal choice designs are based on the multinomial logit models of Huber and Zwerina (1996) or Street and Burgess (2007). Großmann and Schwabe (2015) observed that the two approaches are equivalent for the purpose of finding D-optimal designs. Here we use the multinomial logit model of Huber and Zwerina (1996). This model supposes that the probability of preferring option 1 over option 2 in the ith choice pair can be expressed as $\pi_{12i} = e^{u_{1i}}/(e^{u_{1i}} + e^{u_{2i}})$, where u_{1i} and u_{2i} represent the systematic parts of the utilities attached to the two options in choice pair i. Similarly, $\pi_{21i} = 1 - \pi_{12i}$ is the probability that option 2 is preferred over option 1. Thus, it follows that for the ith choice pair, the choice probabilities depend only on the utility difference $u_{1i} - u_{2i}$. For a design d with N choice pairs, because the options are described by k attributes, the utilities are modeled using the linear predictor $u_j = P_{pj}\theta$, where θ is a $p \times 1$ vector representing the parameters of interest, P_{pj} is an $N \times p$ effects-coded matrix for the jth option, and $u_j = (u_{ji})$ is an $N \times 1$ utility vector for the jth option, for $j = 1, 2$. Then, the utility difference $u_1 - u_2 = (P_{p1} - P_{p2})\theta = P_p\theta$ is a linear function of the parameter vector θ. For the purpose of deriving optimal designs, it is often assumed that $\theta = 0$. This indifference, or the utility-neutral assumption, means that the two options in a choice set are equally attractive, which simplifies the information matrix and the design problem considerably. Under the utility-neutral multinomial logit model, the Fisher information matrix is $(1/4)P_p'P_p$ (see Großmann and Schwabe (2015)).

Graßhoff et al. (2003) and Graßhoff et al. (2004) both studied linear paired-comparison designs, which they analyzed using a linear paired-comparison model. Here, the observed utility difference Z between two choices again depends on the difference matrix $P_p = P_{p1} - P_{p2}$. More precisely, a response is described by the model $Z = u_1 - u_2 + \epsilon = (P_{p1} - P_{p2})\theta + \epsilon = P_p\theta + \epsilon$, where ϵ is a random error vector. The matrix $C = P_p' P_p$ is the information matrix in the model. Because C is proportional to the information matrix under the utility-neutral multinomial logit model, it follows that the designs that are optimal under the linear paired-comparison model are also optimal under the multinomial logit model, and vice versa.

We focus on D-optimality because, as noted in Großmann and Schwabe (2015), most of the optimality results for choice designs and linear paired-comparison designs are available for the D-criterion. A D-optimal design yields the highest determinant for the information matrix of all competing designs.

For paired choice experiments, the multinomial logit model and the linear paired-comparison model are based on the utility difference $u_1 - u_2$. By incorporating respondent effects, the relevant utility difference under the block model, with blocks being the respondents, becomes

$$u_1 - u_2 = (P_{p1} - P_{p2})\theta + W\beta = P_p\theta + W\beta, \tag{2.1}$$

where $\beta = (\beta_1, \ldots, \beta_b)'$ represents a $b \times 1$ vector of block effects, and $W = (w_{ij})$ is an $N \times b$ incidence matrix, with $w_{ij} = 1$ if the ith choice pair belongs to the jth block, and 0 otherwise. Without loss of generality, we take $W = I_b \otimes 1_s$, where I_a and 1_a denote the identity matrix of order a and the $a \times 1$ vector of ones, respectively. Here, $\otimes$ denotes the Kronecker product. Note that (2.1) corresponds to a paired choice block design with b blocks, each of size s, where the b blocks are repeated t times to accommodate $r = tb$ respondents. Each of the r respondents is associated with a single block of the design.

Unlike Sándor and Wedel (2005) and Kessels, Goos and Vandebroek (2008), where an assumed distribution on the model parameters takes care of the respondent effects, our approach, following standard block-design theory, considers β_j as a fixed-effects term. Most of the literature on theoretically obtained D-optimal designs for choice experiments employs multinomial logit models without any respondent effects. In contrast, our fixed-effects block model attempts to obtain an optimal block design theoretically under the utility-neutral setup.

In a multinomial logit model or a linear paired-comparison model, including respondent effects β can be regarded as adding b two-level attributes to the set of p predictor variables. Then, the corresponding difference matrix for the pairs, in b blocks, has an additional component, which can be written as (P_p, W). Thus, under the utility-neutral multinomial logit block model, it follows that the information matrix for estimating θ and β is

$$M = \frac{1}{4}\begin{bmatrix} C & P_p'W \\ W'P_p & W'W \end{bmatrix}, \tag{2.2}$$

where $C = P_p'P_p$, as defined earlier. Moreover, up to a constant factor of 1/4, M coincides with the information matrix in the linear paired-comparison block model. Thus, optimal designs under the latter model are also optimal under the utlity-neutral multinomial logit block model. The information matrix for estimating θ under the linear paired-comparison block model, after eliminating the block effects, is

$$\tilde{C} = C - P_p'W(W'W)^{-1}W'P_p = C - \frac{1}{s}P_p'WW'P_p. \tag{2.3}$$

This follows from the standard linear model theory, where a parameter vector is partitioned into a parameter vector of interest and the nuisance parameters (see, e.g., Page 68 of Haines (2015)).

A paired choice block design is connected if all parameters of interest are estimable, which happens if and only if $\tilde{C}$ has rank p. In what follows, the class of all connected paired choice block designs with k attributes in b blocks, each of size s, is denoted by $\mathcal{D}_{k,b,s}$. From (2.3), $C - \tilde{C}$ is a non-negative definite matrix. Thus, if in the class of unblocked designs with $N = bs$, a paired choice design d is D-optimal, then d, considered as a design in $\mathcal{D}_{k,b,s}$, is also D-optimal, provided that $\tilde{C} = C$.

Note that eliminating the respondent effects simultaneously controls the within-pair order effects (see Goos and Großmann (2011) and Bush, Street and Burgess (2012)).

3. Optimal Block Designs Under the Main-Effects Model

Under the main-effects block model, from (2.1), it follows that $u_1 - u_2 = (P_{M1} - P_{M2})\tau + W\beta = P_M\tau + W\beta$, where τ is a $\sum_{i=1}^{k}(v_i - 1) \times 1$ parameter vector of the main effects, P_{Mj} is an $N \times \sum_{i=1}^{k}(v_i - 1)$ effects-coded matrix of the main effects for the jth option, for $j = 1, 2$, and $P_M = P_{M1} - P_{M2}$. A row of P_{Mj} contains the effects-coded row vector of length $v_i - 1$ for the ith attribute. The effects coding for level l is represented by a unit vector with 1 in the $(l+1)$th position, for $l = 0, \ldots, v_i - 2$, and the coding for level $v_i - 1$ is represented by a unit vector with -1 in each of the $v_i - 1$ positions, for $i = 1, \ldots, k$. For example, for $v = 3$, effects-coded vectors for $l = 0, 1, 2$ are $(1\ 0), (0\ 1)$, and $(-1\ \ -1)$, respectively.

From (2.3), the information matrix used to estimate the main effects after eliminating the block effects is

$$\tilde{C}_M = C_M - \frac{1}{s} P_M' W W' P_M, \tag{3.1}$$

where $C_M = P_M' P_M$ is the information matrix used to estimate the main effects under the unblocked model. From (3.1), it follows that a necessary and sufficient condition for $\tilde{C}_M = C_M$ to hold is $W' P_M = 0$. Therefore, by blocking the choice pairs of an optimal paired choice design into b suitable blocks, such that $W' P_M = 0$, we can obtain an optimal paired choice block design. Here, we provide a simple condition to achieve this; the proof is provided in the Supplementary Material.

Theorem 1. *$\tilde{C}_M = C_M$ if, for each block, the levels of all attributes appear equally often in the first option and in the second option.*

This property of every level of an attribute appearing the same number of times in the first and second options of pairs is also known as *position-balance* (see Großmann and Schwabe (2015)).

An orthogonal array $OA(n, k, v_1 \times \cdots \times v_k, t)$ of strength t is an $n \times k$ array with elements in the ith column from a set of v_i distinct symbols $\{0, 1, \ldots, v_i - 1\}$ $(i = 1, \ldots, k)$, such that all possible combinations of symbols appear equally often as rows in every $n \times t$ subarray. An orthogonal array is symmetric if $v_i = v$ for all i, and the corresponding OA is denoted by $OA(n, k, v^k, t)$; else it is an asymmetric orthogonal array.

Street and Burgess (2007), Demirkale, Donovan and Street (2013), and Bush (2014) provide the $OA + G$ method for constructing optimal paired choice designs using orthogonal arrays and generators G. Let G be a collection of h generators $G_1, \ldots, G_h$, where $G_j = (g_{j_1}, g_{j_2}, \ldots, g_{j_k})$. The $OA + G$ method gives a paired choice design $(A, B_j), j = 1, \ldots, h$, where $A = (A_{li})$ is an $OA(n_1, k, v_1 \times \cdots \times v_k, t)$ and $B_j = (B_{li}^j)$, with $B_{li}^j = A_{li} + g_{ji}$ reduced mod v_i, for $l = 1, \ldots, n_1$, $i = 1 \ldots, k$, and $j = 1, \ldots, h$. This method depends on the availability of the required orthogonal array, which may not always exist. The SAS link `http://support.sas.com/techsup/technote/ts723.html`, the Sloane link `http://neilsloane.com/oadir/`, and Hedayat, Sloane and Stufken (1999) provide a comprehensive summary of orthogonal arrays and their constructions.

In the literature, the generators G are usually derived through a trial-and-error approach, and no general results on the structure of such generators appear to exist. In fact, Bush (2014) highlights the complexities involved in choosing the sets of generators. We present a simple result that systematically provides h generators, the proof of which is provided in the Supplementary Material. Let $lcm(a_1, \ldots, a_k)$ denote the least common multiple of $a_1, \ldots, a_k$.

Theorem 2. *The number of generators for the optimal paired choice design with k attributes is $h = lcm(h_1, \ldots, h_k)$, where $h_i = v_i - 1$ for v_i even, and $h_i = (v_i - 1)/2$ for v_i odd, for $i = 1, \ldots, k$. Then the generators are given by $G_j = (g_{j_1}, g_{j_2}, \ldots, g_{j_k})$, where g_{j_i} takes each of the values from the set $\{1, \ldots, h_i\}$ with frequency h/h_i, for $j = 1, \ldots, h$, $i = 1, \ldots, k$.*

Note that Theorem 2 provides generators for unblocked paired choice designs. As in Street and Burgess (2007), we use several sets of generators to create the final design; thus, the number of generators given in Theorem 2 may not be the smallest possible.

Example 1. Suppose there are three attributes with $v_1 = 2, v_2 = 3$, and $v_3 = 4$. Then, we have $h_1 = 1$, $g_{j_1} = 1$; $h_2 = 1$, $g_{j_2} = 1$; and $h_3 = 3$, $g_{j_3} = 1, 2, 3$. Thus, $h = lcm(1, 1, 3) = 3$. This leads to the generators $G_1 = (111), G_2 = (112)$, and $G_3 = (113)$. Thus, for a given $OA(24, 3, 2 \times 3 \times 4, 2)$, the corresponding optimal paired choice design with parameters k, $v_1 = 2, v_2 = 3, v_3 = 4$, $b = 1$, and $N = s = hn_1 = 3 \times 24 = 72$ is obtained using the $OA + G$ method of construction with three generators. The corresponding design is given in the Supplementary Material.

Example 2. Suppose there are two attributes, with $v_1 = 4$ and $v_2 = 5$. Then, we have $h_1 = 3$, $g_{j_1} = 1, 2, 3$ and $h_2 = 2$, $g_{j_2} = 1, 2$. Thus, $h = lcm(3, 2) = 6$. This leads to the six generators $G_1 = (11), G_2 = (12)$, $G_3 = (21)$, $G_4 = (22)$, $G_5 = (31)$, and $G_6 = (32)$, which yield an optimal paired choice design when used in conjunction with $OA(20, 2, 4 \times 5, 2)$.

In general, for a given $OA(n_1, k, v_1 \times \cdots \times v_k, 2)$, the corresponding optimal paired choice design d_1 with parameters k, $v_1, \ldots, v_k$, $b = 1$, and $N = s = hn_1$, is obtained using the $OA + G$ method of construction with generators $G_j, j = 1, \ldots, h$. When $N = s$ is large, we find that practitioners advocate allocating choice pairs into more than one block either randomly or using a spare attribute (see Street and Burgess (2007), Bliemer and Rose (2011)). Based on Theorem 1, it follows that under our block model, we can retain the optimality of the design obtained through the $OA + G$ method if blocking is done using a column corresponding to an attribute. Any other blocking approach may jeopardize the characteristics of the design. We now provide four theorems and their constructions, detailed proofs of which are provided in the Supplementary Material.

Theorem 3. *For $\delta \geq 1$ and an $OA(n_1, k+1, v_1 \times \cdots \times v_k \times \delta, 2)$, there exists an optimal paired choice block design $d_2 \in \mathcal{D}_{k,b,s}$, with parameters k, $v_1, \ldots, v_k$, $b = h\delta$, and $s = n_1/\delta$, where $h = lcm(h_1, \ldots, h_k)$.*

Construction. *For a given $OA(n_1, k+1, v_1 \times \cdots \times v_k \times \delta, 2)$, corresponding to the k attributes at levels $v_i, i = 1, \ldots, k$, let d_1 be the design constructed using the $OA+G$ method and $h = lcm(h_1, \ldots, h_k)$ generators from Theorem 2. Then, d_1, with parameters k, v_i, for $i = 1, \ldots, k$, $b = 1$, and $s = hn_1$ is an optimal paired choice design. From d_1, the choice pairs obtained from each of the h generators constitute a block of size n_1. Finally, we use the δ symbols of the $(k+1)$th column of the orthogonal array for further blocking. This yields a paired choice block design d_2, with parameters k, $v_1, \ldots, v_k$, $b = h\delta$, and $s = n_1/\delta$.*

Example 3. From an $OA(24, 15, 2^{13} \times 3 \times 4, 2)$, in order to estimate the main effects of $k = 14$ attributes, of which 13 attributes are at two levels and one attribute is at three levels, an optimal paired choice block design can be constructed for $\delta = 4, h = 1, k = 14, b = 4$, and $s = 6$. As an illustration, we give a $2^4 \times 3$ paired choice block design d_2, with parameters $k = 5, b = 4$, and $s = 6$.

	B_1	B_2	B_3	B_4
	(00000,11111)	(01102,10010)	(10112,01000)	(10001,01112)
	(11010,00101)	(11110,00001)	(00111,11002)	(00012,11100)
$d_2 =$	(01101,10012)	(11011,00102)	(01002,10110)	(10100,01011)
	(11002,00110)	(00100,11011)	(11101,00012)	(01011,10102)
	(10111,01002)	(10012,01100)	(01010,10101)	(01110,10001)
	(00112,11000)	(00001,11112)	(10000,01111)	(11102,00010)

Note that when the attributes have mixed levels greater than three, the $OA+G$ method leads to choice designs with a large number of choice pairs. However, blocking still helps to reduce the number of choice pairs shown to a respondent from $N = s = 96$ to $s = 24$. For example, an $OA(32, 11, 2^3 \times 4^7 \times 8, 2)$ can be used to construct a paired choice block design with three two-level attributes and seven four-level attributes in $N = 96$ choice pairs with $b = 24$ and $s = 4$.

For many parameter sets corresponding to k attributes, each at v levels, Graßhoff et al. (2004) and Demirkale, Donovan and Street (2013) provide optimal paired choice designs with fewer choice pairs than those constructed using the $OA + G$ method. We now show how an optimal paired choice block design can be constructed, starting with its design.

Theorem 4. *For a Hadamard matrix H_m, an optimal paired choice design d_3 exists, with parameters $k, v, b = 1$, and $s = mv(v-1)/2$, for $k \leq m$. Furthermore, for v odd, a paired choice block design d_4 exists, with parameters $k, v, b = m(v-1)/2$, and $s = v$, that is optimal in $\mathcal{D}_{k,b,s}$.*

Construction. *For a given H_m, an optimal paired choice design d_3 is obtained through Theorem* 3 *of* Graßhoff et al. (2004)*, with parameters $k, v, b = 1$, and $s = mv(v-1)/2$. Moreover, for v odd, the choice pairs corresponding to each of the rows of $\{H_m, -H_m\}$ form a block; thus, the design is an optimal paired choice block design. Now, using a result from* Dey (2009)*, $v(v-1)/2$ combinations involving v levels, taken two at a time, can be grouped into $(v-1)/2$ replicas, each comprising v elements. Therefore, the blocks generated by each row of H_m can be broken further into $(v-1)/2$ blocks, each of size v, which gives the optimal paired choice block design d_4.*

Example 4. Consider $v = 3$, with combinations $(0,1), (1,2)$, and $(2,0)$, and the Hadamard matrix H_4. An optimal paired choice design d_3 exists, with parameters $k = 4, v = 3, b = 1$, and $s = 12$. Furthermore, because v is odd, an optimal paired choice block design d_4 is constructed with parameters $k = 4, v = 3, b = 4$, and $s = 3$ by considering the choice pairs generated by each row of $\{H_4, -H_4\}$ as a block.

$d_4 =$

B_1	B_2	B_3	B_4
(0000,1111)	(0101,1010)	(0011,1100)	(0110,1001)
(1111,2222)	(1212,2121)	(1122,2211)	(1221,2112)
(2222,0000)	(2020,0202)	(2200,0022)	(2002,0220)

Theorem 5. *For an $OA(n_2, k+1, v^k \times v_{k+1}, 2)$, with $v_{k+1} = n_2/v$, an optimal paired choice design d_5 exists, with parameters $k, v, b = 1, s = n_2(v-1)/2$. Furthermore, for v odd, a paired choice block design d_6 exists, with parameters $k, v, b = n_2(v-1)/2v$, and $s = v$, that is optimal in $\mathcal{D}_{k,b,s}$.*

Construction. *For a given $OA(n_2, k+1, v^k \times v_{k+1}, 2)$, with $v_{k+1} = n_2/v$, an optimal paired choice design d_5 is obtained from Construction* 3.2 *of* Demirkale, Donovan and Street (2013)*, with parameters $k, v, b = 1$, and $s = v_{k+1}\binom{v}{2}$. Moreover, for v odd, the choice pairs corresponding to each of the parallel sets of the* *orthogonal array form a block; thus, the design is an optimal paired choice block design. Now, following* Dey (2009)*, the blocks generated by each parallel set can be broken further into* $(v-1)/2$ *blocks, each of size* v*, which gives the optimal paired choice block design* d_6*.*

Theorem 6. *For* $\delta \geq 1$ *and an* $OA(n_3, k+1, m_1 \times \cdots \times m_k \times \delta, 2)$*, with* $m_i = v_i(v_i-1)/2$ *for some odd* v_i*, an optimal paired choice block design* d_8 *exists, with parameters* $k, v_i, \ldots, v_k, b = \delta$*, and* $s = n_3/\delta$*.*

Construction. *For a given* $OA(n_3, k+1, m_1 \times \cdots \times m_k \times \delta, 2)$*, with* $m_i = v_i(v_i-1)/2$ *for some odd* v_i*, an optimal paired choice design* d_7 *is obtained from Theorem* 4 *of* Graßhoff et al. (2004)*, with parameters* $k, v_i, \ldots, v_k, b = 1$*, and* $s = n_3$*. Then, similarly to the construction of Theorem* 3*, we use the* δ (≥ 1) *symbols of the* $(k+1)$*th column of the orthogonal array for blocking. This yields an optimal paired choice block design* d_8*, with parameters* $k, v_i, \ldots, v_k, b = \delta$*, and* $s = n_3/\delta$*. Note that this method of blocking is applicable for odd* v_i *only.*

Table 1 highlights the flexibility in the number of blocks while blocking the traditional optimal symmetric paired choice designs listed in Table 2 of Demirkale, Donovan and Street (2013). We list the values of s and b corresponding to the optimal designs obtained using Theorem 3 and Theorem 4. Note that in the parameter range of Table 1, Theorem 5 and Theorem 6 do not provide additional designs that are not obtainable from Theorem 3 and Theorem 4. Some of the traditional optimal paired choice designs, marked $*$, are not optimal under the block setup for blocks of size $s = N$ and $b = 1$, because the design matrices are not orthogonal to the vector of ones. However, because $b > 1$, optimal designs with blocks of size $s = N/b$ are feasible using Theorem 3.

Note that, from a given optimal paired choice design in $\mathcal{D}_{k,b,s}$, we can randomly group the b blocks into b/x blocks, each of size xs, to obtain optimal paired choice designs in $\mathcal{D}_{k,b/x,sx}$. In Table 1, the designs with $x = 1$ are first obtained using the Theorems listed in the corresponding column headers, whereas the designs with $x > 1$ are obtained thereafter using random grouping. We can obtain a table similar to Table 1 for optimal asymmetric paired choice designs based on a list of more than 600 orthogonal arrays with $n \leq 100$.

4. Optimal Block Designs Under the Broader Main-Effects Model

In this section, we estimate the main effects under the broader main-effects model for an asymmetric paired choice design, where the ith attribute is at v_i

Table 1. Optimal designs in $\mathcal{D}_{k,b,s}$.

v	k	Traditional $(s,1)$	Theorem 3 (s,b)	Theorem 4 (s,b)
2	3	4	(4,1)	
2	4	4*	(4x,2/x), x=1,2	
2	5-6	8	(4x,2/x), x=1,2 (6x,2/x), x=1,2	
2	7	8	(8,1) (6x,2/x), x=1,2 (4x,4/x), x=1,2,4	
2	8	8*	(6x,2/x), x=1,2 (4x,4/x), x=1,2,4	
2	9-10	12	(6x,2/x), x=1,2 (4x,4/x), x=1,2,4 (10x,2/x), x=1,2	
2	11	12	(12,1) (4x,4/x), x=1,2,4 (10x,2/x), x=1,2 (6x,4/x), x=1,2,4	
2	12	12*	(4x,4/x), x=1,2,4 (10x,2/x), x=1,2 (6x,4/x), x=1,2,4	
3	3	9,12	(3x,3/x), x=1,3	(3x,4/x), x=1,2,4
3	4	9,12,18	(9,1) (3x,6/x), x=1,2,3,6	(3x,4/x), x=1,2,4
3	5,6	18,24	(3x,6/x), x=1,2,3,6	(3x,8/x), x=1,2,4,8
3	7	18,24,27	(9x,2/x), x=1,2 (3x,9/x), x=1,3,9	(3x,8/x), x=1,2,4,8
3	8	24,27	(3x,9/x), x=1,3,9	(3x,8/x), x=1,2,4,8
3	9	27,36	(3x,9/x), x=1,3,9	(3x,12/x), x=1,2,3,4,6,12
3	10-12	27,36	(9x,3/x), x=1,3 (3x,12/x), x=1,2,3,4,6,12	(3x,12/x), x=1,2,3,4,6,12
4	3-4	24*,28	(4x,12/x), x=1,2,3,4,6,12	
4	5	48	(16x,3/x), x=1,3 (4x,24/x), x=1,2,3,4,6,8,12,24	
4	6-8	48*,96	(4x,24/x), x=1,2,3,4,6,8,12,24	
4	9	72*,96	(16x,6/x), x=1,2,3,6 (4x,36/x), x=1,2,3,4,6,9,12,18,36	
4	10-12	72*,144	(4x,36/x), x=1,2,3,4,6,9,12,18,36	
5	3-4	40,50	(5x,10/x), x=1,2,5,10	(5x,8/x), x=1,2,4,8
5	5	50,80	(5x,10/x), x=1,2,5,10	(5x,16/x), x=1,2,4,8,16
5	6	50,80,100	(25x,2/x), x=1,2 (5x,20/x), x=1,2,4,5,10,20	(5x,16/x), x=1,2,4,8,16
5	7-8	80,100	(5x,20/x), x=1,2,4,5,10,20	(5x,16/x), x=1,2,4,8,16
5	9-10	100,120	(5x,20/x), x=1,2,4,5,10,20	(5x,24/x), x=1,2,3,4,6,12,24
6	3	60*,180	(12x,15/x), x=1,3,5,15 (18x,10/x), x=1,2,5,10	
6	4	60*,180*,360	(6x,60/x), x=1-6,10,12,15,20,30,60	
6	5-6	120*,180*,360	(6x,60/x), x=1-6,10,12,15,20,30,60	
7	3-4	84,147	(7x,21/x), x=1,3,7,21	(7x,12/x), x=1,2,3,4,6,12
7	5-7	147,168	(7x,21/x), x=1,3,7,21	(21x,8/x), x=1,2,4,8
7	8	147,168,294	(49x,3/x), x=1,3 (7x,42/x), x=1,2,3,6,7,14,21,42	(21x,8/x), x=1,2,4,8

levels, for $i = 1, \ldots, k$. The broader main-effects model constitutes the main effects and the two-factor interaction effects, where we are interested only in estimating the main effects. For symmetric paired choice designs, Graßhoff et al. (2003) characterized optimal paired choice designs under the broader main-effects model. More recently, for $v_i = 2$, Singh, Chai and Das (2015) obtained optimal designs under the same model.

After introducing the respondent effects, from (2.1), the relevant utility differences become

$$u_1 - u_2 = (P_{M1} - P_{M2})\tau + (P_{I1} - P_{I2})\gamma + W'\beta = P_M\tau + P_I\gamma + W'\beta, \quad (4.1)$$

where γ is a $\sum_{i=1}^{k-1}\sum_{j=i+1}^{k}(v_i - 1)(v_j - 1) \times 1$ parameter vector for the two-factor interaction effects, P_{Ij} is an $N \times \sum_{i=1}^{k-1}\sum_{j=i+1}^{k}(v_i-1)(v_j-1)$ effects-coded matrix of the two-factor interaction effects for the jth option, for $j = 1, 2$, and $P_I = P_{I1} - P_{I2}$. Let $P_{Ij} = (P_{Ij}^{1'}, \ldots, P_{Ij}^{n'})'$, where P_{Ij}^l corresponds to the lth choice pair in P_{Ij}. In addition, let $P_{Mj(i)}^l$ represent the columns of P_{Mj} corresponding to the lth choice pair and ith attribute. Then, $P_{Ij}^l = (P_{Mj(1)}^l \otimes P_{Mj(2)}^l, P_{Mj(1)}^l \otimes P_{Mj(3)}^l, \ldots, P_{Mj(k-1)}^l \otimes P_{Mj(k)}^l)$.

The information matrix used to estimate the main effects after eliminating the two-factor interaction effects and the block effects is

$$\tilde{C}_B = C_M - [P_M'P_I \; P_M'W]\begin{bmatrix} P_I'P_I & P_I'W \\ P_I'W & W'W \end{bmatrix}^{-}\begin{bmatrix} P_I'P_M \\ W'P_M \end{bmatrix}. \quad (4.2)$$

Therefore, a paired choice design that is optimal under the main-effects model is also optimal under the broader main-effects block model if $\tilde{C}_B = C_M$, that is, if $P_I'P_M = 0$ and $W'P_M = 0$. The designs in Theorem 3 satisfy $W'P_M = 0$. Furthermore, for symmetric designs with $v = 2$, it follows from Singh, Chai and Das (2015) that the designs also satisfy $P_I'P_M = 0$. Therefore, in particular, for symmetric designs with $v = 2$, the paired choice block designs of Theorem 3 are also optimal under the broader main-effects block model.

We now give the following construction for optimal paired choice block designs under the broader main-effects model.

Theorem 7. *Under the broader main-effects model, for an $OA(n_1, k, v_1 \times \cdots \times v_k, 3)$ and $h = lcm(v_1, \ldots, v_k)$, there exists a paired choice block design d_1^B, with parameters $k, v_1, \ldots, v_k, b = 1$, and $s = hn_1$, that is optimal in $\mathcal{D}_{k,b,s}$.*

Construction. *We obtain d_1^B using the $OA + G$ method of construction and h generators, as in Theorem 2. A detailed proof is provided in the Supplementary Material.*

Theorem 8. *Under the broader main-effects model, for* $\delta \geq 1$ *and an* $OA(n_1, k+1, v_1 \times \cdots \times v_k \times \delta, 3)$*, there exists a paired choice block design* d_2^B*, with parameters* $k, v_1, \ldots, v_k, b = h\delta$*, and* $s = n_1/\delta$*, that is optimal in* $\mathcal{D}_{k,b,s}$*.*

Construction. *In a similar manner to Theorem* 3*, the construction here is based on using sets of generators, from Theorem* 2*, on an orthogonal array of strength three.*

We now provide another method to obtain symmetric optimal paired choice block designs with $s = v; v \geq 3$.

Theorem 9. *For an* $OA(n_1, k-1, v^{k-1}, 3)$*, there exists a paired choice block design* d_3^B*, with parameters* $k, v \geq 3, s = v$*, and* $b = hn_1$*, that is optimal in* $\mathcal{D}_{k,b,s}$*.*

Construction. *We adopt the following method of construction:*
(i) Following Theorem 8*, construct* d_2^B *from an* $OA(n_1, k, v^{k-1} \times 1, 3)$ *for* $k-1$ *attributes, each at* v *levels. While constructing* d_2^B*, the* h *generators, as in Theorem* 2*, are* $(k-1)$*-tuples of the form* $(1\ldots1), \ldots, (v-1\ldots v-1)$ *for* v *even (*$h = (v-1)$*), and of the form* $(1\ldots1), \ldots, ((v-1)/2\ldots(v-1)/2)$ *for* v *odd (*$h = (v-1)/2$*). Then, for each choice pair, add the* k*th attribute at level zere in option one and, similarly, generate the* k*th attribute in the second option using the same generator as that used for the other* $k-1$ *attributes.*

(ii) For each of the h *generators, generate* $v-1$ *additional copies of the design obtained in (i) by adding* 1 *(mod* v*),...,* $(v-1)$ *(mod* v*) to every attribute under both options. Note that every copy in (ii) is just a recoding of the design obtained in (i) and, hence, the resultant design with parameters* $k, v, s = hn_1v$*, and* $b = 1$ *is also optimal.*

(iii) Finally, for each of the h *generators, the* i*th block of size* v *comprises the* i*th row from each of the* v *copies created in (ii), for* $i = 1, \ldots, n_1$*.*

The hn_1 *blocks obtained in this way with* $s = v$ *form the required optimal design* d_3^B*. This design has distinct choice pairs in every block.*

5. Optimal Block Designs for Estimating the Main Plus Two-Factor Interaction Effects

The literature on optimal paired choice designs for estimating the main plus two-factor interaction effects is very limited because such designs require a large number of choice pairs to be shown to every respondent. Graßhoff et al. (2003), Street and Burgess (2004), and Großmann, Schwabe and Gilmour (2012) provide optimal and/or efficient paired choice designs under this setup for k attributes, each at two levels. In this section, we consider each of the k attributes to be at two levels. Let $q = \lceil k/2 \rceil$, where $\lceil z \rceil$ represents the smallest integer greater than or equal to z. The construction method of Street and Burgess (2007) entails starting with an orthogonal array $OA(n_1, k, 2^k, 4)$ as a set of n_1 first options, and then taking the foldover of α attributes in the second option, keeping the rest of the $k - \alpha$ attributes the same for each of the n_1 choice pairs. Here $\alpha = q$ for k odd, and $\alpha = q$ and $q + 1$ for k even. This process is repeated for $\binom{k}{\alpha}$ possible combinations of the attributes. Here, the foldover of an attribute in the second option of a choice pair means that the attribute level in the second option is different from that in the first. This paired choice design d_1^I with parameters $k, v, s, b = 1$ is optimal, where $s = n_1\binom{k}{q}$ for k odd, and $s = n_1\binom{k+1}{q+1}$ for k even.

Incorporating respondent effects, the model is as given in (4.1). However, in contrast to Section 4, here we want to estimate both the main effects and the two-factor interaction effects. The information matrix used to estimate the main plus two-factor interaction effects under the multinomial logit model incorporating respondent effects is

$$\tilde{C}_I = \begin{bmatrix} C_M & P_M'P_I \\ P_I'P_M & P_I'P_I \end{bmatrix} - \frac{1}{s}[P_M'W \; P_I'W] \begin{bmatrix} W'P_M \\ W'P_I \end{bmatrix}. \tag{5.1}$$

As earlier, in order to achieve optimal paired choice block designs, we start with an optimal paired choice design d_1^I and then enforce blocking, such that $W'P_M = 0$ and $W'P_I = 0$. We provide a simple condition to achieve this, the proof of which is provided in the Supplementary Material.

Let the pair (a_1, b_1) mean that a_1 and b_1 are the levels corresponding to an attribute for the first and second options, respectively. Similarly, let the pair (a_1a_2, b_1b_2) mean that a_1a_2 and b_1b_2 are the levels corresponding to the two attributes for the first and second options, respectively.

Theorem 10. *$W'P_M = 0$ and $W'P_I = 0$ if and only if, for every block,*

(i) the frequency of the pair $(1, 0)$ is same as the frequency of the pair $(0, 1)$ for every attribute;

(ii) the frequency of pairs from the set $\{(01, 00), (01, 11), (10, 00), (10, 11)\}$ is the same as the frequency of pairs from the set $\{(00, 01), (00, 10), (11, 01), (11, 10)\}$, for every combination of two attributes.

We now provide the method used to construct optimal paired choice block designs with $s = 4$.

Theorem 11. *For $k > 4$, there exists a paired choice block design d_2^I, with parameters $k, v = 2, s = 4$, and b, that is optimal in $\mathcal{D}_{k,b,s}$. Here, $b = 2^{k-3}\binom{k}{q}$ for k odd, and $b = 2^{k-3}\binom{k+1}{q+1}$ for k even.*

Construction. *Let F be a set of $\binom{k}{\alpha}$ attribute indices of size $\alpha = q$ obtained from the attribute labels $1, \ldots, k$, taking α labels at a time, such that $2 \le \alpha \le k-2$. For an element $f = (f_1, \ldots, f_i, \ldots, f_\alpha)$ of F, let $f' = \{1, \ldots, k\} - f = (f'_1, \ldots, f'_j, \ldots, f'_{(k-\alpha)})$ be the complement of f. Similarly to the construction of design d_1^I, we adopt steps (i)–(v) to construct an optimal paired choice block design d_2^I for k attributes:*

(i) Write the complete factorial involving 2^α combinations. Divide this set into two halves, such that the second half is a foldover of the first half.

(ii) Write the complete factorial involving $2^{k-\alpha}$ combinations. Divide this set into two halves, such that the second half is a foldover of the first half.

(iii) Take one combination from the first half of (i), say a, and two combinations from the first half of (ii), say b and c. Let a', b', and c' be the foldovers of a, b, and c, respectively. Corresponding to the element f of F, create a block with choice pairs $(ab, a'b)$, $(ab', a'b')$, $(a'c, ac)$, and $(a'c', ac')$. Here, in a choice pair, option ab implies that if $a = a_1 \ldots a_i \ldots a_\alpha$ and $b = b_1 \ldots b_j \ldots b_{k-\alpha}$, then a_i corresponds to the attribute index f_i and b_j corresponds to the attribute index f'_j.

(iv) Repeat (iii) for each of the $2^{\alpha-1}$ combinations in the first half of (i) using the same b and c as in (iii). Then, repeat the entire process for two different combinations from the first half of (ii).

(v) Repeating (i)–(iv) for every element f of F corresponding to $\alpha = q$ for k odd, and $\alpha = q$ and $q+1$ for k even, an optimal paired choice block design d_2^I is obtained with parameters $k, v = 2, s = 4$, and b, where $b = 2^{k-3}\binom{k}{q}$ for k odd, and $b = 2^{k-3}\binom{k+1}{q+1}$ for k even.

Example 5. Let $k = 4, v = 2, b = 10$, and $s = 8$. For $k = 4$, α takes the values 2 and 3. Because $\alpha = 3 > 2 = k - 2$, Theorem 11 does not enable us to derive d_2^I from d_1^I. However, for $\alpha = 2$, the proposed construction method still holds, yielding 12 blocks, each of size four, as below.

B_1	B_2	B_3	B_4	B_5	B_6
(0000,1100)	(0100,1000)	(0000,1010)	(0010,1000)	(0000,1001)	(0001,1000)
(0011,1111)	(0111,1011)	(0101,1111)	(0111,1101)	(0110,1111)	(0111,1110)
(1101,0001)	(1001,0101)	(1011,0001)	(1001,0011)	(1011,0010)	(1010,0011)
(1110,0010)	(1010,0110)	(1110,0100)	(1100,0110)	(1101,0100)	(1100,0101)
B_7	B_8	B_9	B_{10}	B_{11}	B_{12}
(0000,0110)	(0010,0100)	(0000,0101)	(0001,0100)	(0000,0011)	(0001,0010)
(1001,1111)	(1011,1101)	(0110,0011)	(0111,0010)	(1100,1111)	(1101,0010)
(0111,0001)	(0101,0011)	(1011,1110)	(1010,1111)	(0111,0100)	(0110,0101)
(1110,1000)	(1100,1010)	(1101,1000)	(1100,1001)	(1011,1000)	(1010,1001)

For $\alpha = 3$, we obtain a design with four blocks, each of size eight, as below.

B_{13}	B_{14}	B_{15}	B_{16}
(0000,1110)	(0000,1011)	(0000,1101)	(0000,0111)
(0110,1000)	(0011,1000)	(0101,1000)	(0011,0100)
(1010,0100)	(1010,0001)	(1100,0001)	(0101,0010)
(1100,0010)	(1001,0010)	(1001,0100)	(0110,0001)
(0001,1111)	(0100,1111)	(0010,1111)	(1000,1111)
(0111,1001)	(0111,1100)	(0111,1010)	(1011,1100)
(1011,0101)	(1110,0101)	(1110,0011)	(1101,1010)
(1101,0011)	(1101,0110)	(1011,0110)	(1110,1001)

We form six blocks, each of size eight by combining blocks B_i and B_{i+6}, for $i = 1, \ldots, 6$, which, together with the four blocks B_i, for $i = 13, \ldots, 16$, gives the optimal design with parameters $k = 4, b = 10$, and $s = 8$.

6. Discussion

In situations in which an optimal design has more choice pairs than a respondent can complete, the N choice pairs can be split among the respondents (blocks), either randomly or using a spare attribute, if one is available (see Street and Burgess (2007)). As a result, we have instances in which respondents are considered as blocks in choice experiments, although without much theoretical rigor. Bliemer and Rose (2011) reported that, in their sample, 64% of studies used a blocking column to allocate choice sets to respondents, 13% assigned choice sets randomly to respondents, and 5% provided the full factorial to each respondent. In the remaining 18%, of the studies, the authors were not able to determine how choice sets were assigned to respondents.

With an objective of assessing the main or the interaction effects, wherever practical, the same set of N optimal choice pairs are shown to every respondent. As such, there are no theoretical results on optimal designs under the utility-neutral setup, where different respondents see smaller and different designs. In contrast, the approach adopted here allows for the construction of an optimal design with a smaller and flexible number of choice pairs that will be shown to every respondent. Even in situations where simple techniques (e.g. blocking using a spare attribute) cannot be used, we provide optimal paired choice block designs.

In contrast to the approaches of Sándor and Wedel (2005) and Kessels, Goos and Vandebroek (2008), following block-design theory, we use the fixed-effects block model to obtain optimal designs. This approach treats respondent heterogeneity as a nuisance factor by including respondent-level fixed-effect terms in the model, thus, enabling the derivation of analytical results. However, there is no guarantee that the optimal block designs obtained under this setup and the heterogeneous designs obtained by Sándor and Wedel (2005) are the same. Thus, further research is needed to compare the optimal designs obtained under the two approaches.

Furthermore, unlike their designs, which can only be used to optimally estimate the main effects, we have provided optimal paired choice block designs for the main-effects model, as well as for the broader main-effects model and the main plus two-factor interaction effects model.

Supplementary Material

The Supplementary Material available online includes all proofs.

Acknowledgments

We thank the referees for their valuable comments, which helped us to significantly improve this paper. Ashish Das's work is partially supported by the Science and Engineering Research Board, India.

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(Received February 2016; accepted November 2017)