

Bayesian Modeling and Inference for Nonignorably
Missing Longitudinal Binary Response Data with
Applications to HIV Prevention Trials

Supplementary Materials

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May 19, 2017

Appendix: Additional Tables

Table S1: Posterior Summaries under the Nonignorable Model with a $N(0, 1)$ Prior and a $N(0, 10)$ Prior when missing percentage is low.

| | N(0, 1) Prior | | | | | | N(0, 10) Prior | | | | |
|----------------|---------------|--------|-------|-------|-------|-------|----------------|-------|-------|-------|-------|
| | TRUE | EST | SE | SIME | RMSE | CP | EST | SE | SIME | RMSE | CP |
| t=0 | | | | | | | | | | | |
| β_{00}^* | -1.000 | -1.031 | 0.133 | 0.121 | 0.124 | 0.984 | -1.003 | 0.136 | 0.123 | 0.123 | 0.968 |
| β_{10}^* | 0.500 | 0.512 | 0.069 | 0.069 | 0.070 | 0.968 | 0.504 | 0.068 | 0.068 | 0.068 | 0.956 |
| β_{20}^* | 1.000 | 1.022 | 0.134 | 0.129 | 0.131 | 0.968 | 0.999 | 0.132 | 0.128 | 0.127 | 0.956 |
| β_{30}^* | 0.400 | 0.410 | 0.111 | 0.100 | 0.100 | 0.980 | 0.400 | 0.110 | 0.098 | 0.098 | 0.984 |
| γ_{00} | -2.500 | -2.573 | 0.239 | 0.236 | 0.247 | 0.964 | -2.717 | 0.441 | 0.442 | 0.492 | 0.960 |
| γ_{10} | 0.500 | 0.509 | 0.118 | 0.108 | 0.108 | 0.972 | 0.499 | 0.128 | 0.119 | 0.118 | 0.964 |
| γ_{20} | -0.500 | -0.469 | 0.235 | 0.222 | 0.224 | 0.964 | -0.490 | 0.254 | 0.246 | 0.245 | 0.956 |
| γ_{30} | -0.500 | -0.493 | 0.213 | 0.203 | 0.202 | 0.964 | -0.503 | 0.218 | 0.205 | 0.204 | 0.972 |
| γ_{60} | 0.000 | -0.084 | 0.581 | 0.468 | 0.474 | 0.992 | 0.040 | 0.972 | 0.843 | 0.843 | 0.976 |
| t=1 | | | | | | | | | | | |
| β_{01}^* | -1.000 | -1.048 | 0.155 | 0.155 | 0.162 | 0.964 | -0.987 | 0.168 | 0.176 | 0.176 | 0.912 |
| β_{11}^* | 0.500 | 0.499 | 0.073 | 0.068 | 0.068 | 0.964 | 0.498 | 0.072 | 0.068 | 0.068 | 0.972 |
| β_{21}^* | 1.000 | 1.017 | 0.141 | 0.140 | 0.140 | 0.964 | 0.977 | 0.143 | 0.142 | 0.143 | 0.928 |
| β_{31}^* | -0.200 | -0.193 | 0.111 | 0.105 | 0.105 | 0.956 | -0.196 | 0.110 | 0.103 | 0.103 | 0.952 |
| γ_{01} | -2.000 | -2.012 | 0.210 | 0.174 | 0.175 | 0.972 | -2.218 | 0.392 | 0.454 | 0.503 | 0.956 |
| γ_{11} | 0.500 | 0.521 | 0.088 | 0.091 | 0.093 | 0.932 | 0.503 | 0.095 | 0.098 | 0.098 | 0.928 |
| γ_{21} | -0.500 | -0.475 | 0.179 | 0.176 | 0.178 | 0.968 | -0.519 | 0.194 | 0.200 | 0.201 | 0.952 |
| γ_{31} | -0.250 | -0.279 | 0.159 | 0.151 | 0.154 | 0.956 | -0.259 | 0.164 | 0.155 | 0.155 | 0.964 |
| γ_{41} | 0.400 | 0.376 | 0.285 | 0.293 | 0.293 | 0.944 | 0.394 | 0.298 | 0.303 | 0.303 | 0.948 |
| γ_{51} | -0.250 | -0.169 | 0.263 | 0.234 | 0.247 | 0.948 | -0.268 | 0.305 | 0.309 | 0.308 | 0.936 |
| γ_{61} | 0.500 | 0.266 | 0.625 | 0.514 | 0.564 | 0.968 | 0.599 | 0.960 | 1.014 | 1.017 | 0.944 |
| t=2 | | | | | | | | | | | |
| β_{02}^* | -1.000 | -1.067 | 0.149 | 0.148 | 0.162 | 0.964 | -1.009 | 0.154 | 0.156 | 0.156 | 0.960 |
| β_{12}^* | 0.500 | 0.495 | 0.072 | 0.067 | 0.068 | 0.968 | 0.497 | 0.071 | 0.067 | 0.067 | 0.972 |
| β_{22}^* | 1.000 | 1.041 | 0.145 | 0.137 | 0.143 | 0.964 | 1.000 | 0.145 | 0.138 | 0.138 | 0.964 |
| β_{32}^* | -0.400 | -0.405 | 0.116 | 0.111 | 0.111 | 0.948 | -0.394 | 0.114 | 0.109 | 0.109 | 0.952 |
| γ_{02} | -2.800 | -2.783 | 0.238 | 0.243 | 0.243 | 0.948 | -2.970 | 0.338 | 0.414 | 0.446 | 0.932 |
| γ_{12} | 0.500 | 0.515 | 0.087 | 0.093 | 0.094 | 0.940 | 0.500 | 0.090 | 0.098 | 0.098 | 0.940 |
| γ_{22} | -0.500 | -0.475 | 0.179 | 0.161 | 0.163 | 0.956 | -0.529 | 0.190 | 0.180 | 0.182 | 0.964 |
| γ_{32} | 0.250 | 0.244 | 0.161 | 0.170 | 0.170 | 0.932 | 0.269 | 0.165 | 0.178 | 0.179 | 0.928 |
| γ_{42} | 1.700 | 1.704 | 0.163 | 0.168 | 0.168 | 0.944 | 1.764 | 0.181 | 0.197 | 0.207 | 0.944 |
| γ_{52} | -0.600 | -0.525 | 0.257 | 0.275 | 0.284 | 0.916 | -0.624 | 0.274 | 0.308 | 0.308 | 0.896 |
| γ_{62} | 1.300 | 1.070 | 0.492 | 0.473 | 0.525 | 0.928 | 1.417 | 0.638 | 0.740 | 0.748 | 0.940 |
| t=3 | | | | | | | | | | | |
| β_{03}^* | -1.000 | -1.029 | 0.140 | 0.136 | 0.138 | 0.944 | -1.001 | 0.143 | 0.140 | 0.140 | 0.944 |
| β_{13}^* | 0.500 | 0.506 | 0.077 | 0.080 | 0.080 | 0.940 | 0.501 | 0.076 | 0.080 | 0.080 | 0.944 |
| β_{23}^* | 1.000 | 1.027 | 0.142 | 0.134 | 0.137 | 0.956 | 1.003 | 0.140 | 0.131 | 0.131 | 0.960 |
| β_{33}^* | -0.600 | -0.614 | 0.124 | 0.122 | 0.122 | 0.944 | -0.602 | 0.122 | 0.120 | 0.120 | 0.948 |
| γ_{03} | -2.800 | -2.872 | 0.178 | 0.187 | 0.200 | 0.924 | -2.896 | 0.192 | 0.203 | 0.224 | 0.916 |
| γ_{13} | 0.500 | 0.501 | 0.089 | 0.093 | 0.092 | 0.940 | 0.499 | 0.092 | 0.098 | 0.098 | 0.936 |
| γ_{23} | -0.500 | -0.494 | 0.171 | 0.165 | 0.165 | 0.956 | -0.500 | 0.176 | 0.172 | 0.171 | 0.972 |
| γ_{33} | 0.500 | 0.515 | 0.162 | 0.168 | 0.169 | 0.936 | 0.519 | 0.165 | 0.174 | 0.174 | 0.936 |
| γ_{43} | 1.700 | 1.752 | 0.115 | 0.118 | 0.129 | 0.944 | 1.750 | 0.119 | 0.123 | 0.132 | 0.940 |
| γ_{53} | 0.600 | 0.591 | 0.247 | 0.227 | 0.226 | 0.964 | 0.578 | 0.264 | 0.263 | 0.264 | 0.940 |
| γ_{63} | -0.500 | -0.512 | 0.478 | 0.433 | 0.433 | 0.956 | -0.491 | 0.575 | 0.619 | 0.618 | 0.908 |
| ρ | 0.800 | 0.793 | 0.037 | 0.036 | 0.037 | 0.956 | 0.795 | 0.038 | 0.036 | 0.036 | 0.952 |
| α | 0.667 | 0.670 | 0.045 | 0.043 | 0.043 | 0.968 | 0.660 | 0.046 | 0.044 | 0.044 | 0.956 |

Table S2: Posterior Summaries under the Nonignorable Model with a $N(0, 1)$ Prior and a $N(0, 10)$ Prior when missing percentage is high.

| | N(0, 1) Prior | | | | | | N(0, 10) Prior | | | | |
|----------------|---------------|--------|-------|-------|-------|-------|----------------|-------|-------|-------|-------|
| | TRUE | EST | SE | SIME | RMSE | CP | EST | SE | SIME | RMSE | CP |
| t=0 | | | | | | | | | | | |
| β_{00}^* | -1.000 | -1.036 | 0.143 | 0.128 | 0.132 | 0.992 | -1.002 | 0.146 | 0.131 | 0.131 | 0.972 |
| β_{10}^* | 0.500 | 0.512 | 0.074 | 0.072 | 0.073 | 0.968 | 0.503 | 0.072 | 0.070 | 0.070 | 0.952 |
| β_{20}^* | 1.000 | 1.026 | 0.144 | 0.136 | 0.138 | 0.960 | 0.997 | 0.141 | 0.134 | 0.134 | 0.968 |
| β_{30}^* | 0.400 | 0.412 | 0.114 | 0.102 | 0.103 | 0.984 | 0.400 | 0.112 | 0.100 | 0.099 | 0.984 |
| γ_{00} | -2.500 | -2.568 | 0.240 | 0.234 | 0.243 | 0.960 | -2.730 | 0.450 | 0.468 | 0.520 | 0.944 |
| γ_{10} | 0.500 | 0.511 | 0.118 | 0.107 | 0.107 | 0.984 | 0.499 | 0.129 | 0.118 | 0.118 | 0.976 |
| γ_{20} | -0.500 | -0.465 | 0.236 | 0.222 | 0.224 | 0.968 | -0.492 | 0.257 | 0.244 | 0.244 | 0.960 |
| γ_{30} | -0.500 | -0.492 | 0.214 | 0.202 | 0.202 | 0.964 | -0.502 | 0.218 | 0.205 | 0.205 | 0.960 |
| γ_{60} | 0.000 | -0.106 | 0.589 | 0.467 | 0.478 | 0.984 | 0.049 | 0.991 | 0.876 | 0.876 | 0.968 |
| t=1 | | | | | | | | | | | |
| β_{01}^* | -1.000 | -1.057 | 0.167 | 0.165 | 0.175 | 0.972 | -0.977 | 0.183 | 0.185 | 0.186 | 0.928 |
| β_{11}^* | 0.500 | 0.498 | 0.077 | 0.074 | 0.074 | 0.972 | 0.499 | 0.077 | 0.074 | 0.074 | 0.964 |
| β_{21}^* | 1.000 | 1.022 | 0.152 | 0.150 | 0.151 | 0.948 | 0.970 | 0.154 | 0.149 | 0.152 | 0.924 |
| β_{31}^* | -0.200 | -0.193 | 0.112 | 0.106 | 0.106 | 0.956 | -0.197 | 0.111 | 0.104 | 0.104 | 0.948 |
| γ_{01} | -2.000 | -2.008 | 0.214 | 0.183 | 0.182 | 0.956 | -2.286 | 0.467 | 0.499 | 0.574 | 0.964 |
| γ_{11} | 0.500 | 0.523 | 0.089 | 0.092 | 0.094 | 0.924 | 0.500 | 0.097 | 0.101 | 0.101 | 0.924 |
| γ_{21} | -0.500 | -0.470 | 0.180 | 0.176 | 0.178 | 0.968 | -0.529 | 0.199 | 0.201 | 0.202 | 0.936 |
| γ_{31} | -0.250 | -0.282 | 0.159 | 0.150 | 0.153 | 0.952 | -0.256 | 0.166 | 0.156 | 0.156 | 0.964 |
| γ_{41} | 0.400 | 0.375 | 0.286 | 0.293 | 0.294 | 0.944 | 0.396 | 0.301 | 0.305 | 0.304 | 0.952 |
| γ_{51} | -0.250 | -0.157 | 0.267 | 0.240 | 0.257 | 0.952 | -0.285 | 0.316 | 0.320 | 0.322 | 0.932 |
| γ_{61} | 0.500 | 0.228 | 0.653 | 0.535 | 0.599 | 0.972 | 0.684 | 1.077 | 1.090 | 1.104 | 0.932 |
| t=2 | | | | | | | | | | | |
| β_{02}^* | -1.000 | -1.083 | 0.163 | 0.154 | 0.174 | 0.940 | -1.006 | 0.167 | 0.166 | 0.166 | 0.928 |
| β_{12}^* | 0.500 | 0.491 | 0.077 | 0.073 | 0.073 | 0.956 | 0.495 | 0.076 | 0.071 | 0.071 | 0.960 |
| β_{22}^* | 1.000 | 1.051 | 0.156 | 0.148 | 0.156 | 0.960 | 0.996 | 0.155 | 0.149 | 0.149 | 0.956 |
| β_{32}^* | -0.400 | -0.408 | 0.119 | 0.114 | 0.114 | 0.956 | -0.393 | 0.117 | 0.112 | 0.112 | 0.948 |
| γ_{02} | -2.800 | 2.760 | 0.245 | 0.246 | 0.249 | 0.940 | -3.002 | 0.367 | 0.444 | 0.487 | 0.924 |
| γ_{12} | 0.500 | 0.519 | 0.088 | 0.095 | 0.096 | 0.936 | 0.500 | 0.092 | 0.100 | 0.100 | 0.940 |
| γ_{22} | -0.500 | -0.461 | 0.182 | 0.163 | 0.167 | 0.968 | -0.530 | 0.195 | 0.185 | 0.187 | 0.964 |
| γ_{32} | 0.250 | 0.237 | 0.161 | 0.171 | 0.171 | 0.920 | 0.270 | 0.168 | 0.181 | 0.182 | 0.916 |
| γ_{42} | 1.700 | 1.699 | 0.164 | 0.167 | 0.166 | 0.944 | 1.777 | 0.186 | 0.198 | 0.212 | 0.948 |
| γ_{52} | -0.600 | -0.495 | 0.270 | 0.284 | 0.302 | 0.924 | -0.618 | 0.286 | 0.334 | 0.334 | 0.908 |
| γ_{62} | 1.300 | 0.983 | 0.546 | 0.507 | 0.597 | 0.912 | 1.426 | 0.717 | 0.841 | 0.848 | 0.892 |
| t=3 | | | | | | | | | | | |
| β_{03}^* | -1.000 | -0.999 | 0.210 | 0.220 | 0.219 | 0.936 | -0.972 | 0.221 | 0.243 | 0.244 | 0.904 |
| β_{13}^* | 0.500 | 0.516 | 0.102 | 0.098 | 0.100 | 0.976 | 0.505 | 0.101 | 0.101 | 0.101 | 0.944 |
| β_{23}^* | 1.000 | 1.019 | 0.176 | 0.168 | 0.168 | 0.956 | 0.987 | 0.174 | 0.163 | 0.164 | 0.952 |
| β_{33}^* | -0.600 | -0.609 | 0.155 | 0.159 | 0.159 | 0.944 | -0.597 | 0.152 | 0.157 | 0.157 | 0.948 |
| γ_{03} | -0.500 | -0.543 | 0.127 | 0.131 | 0.138 | 0.936 | -0.547 | 0.134 | 0.148 | 0.154 | 0.928 |
| γ_{13} | 0.500 | 0.502 | 0.062 | 0.063 | 0.062 | 0.956 | 0.503 | 0.064 | 0.067 | 0.067 | 0.952 |
| γ_{23} | -0.500 | -0.505 | 0.116 | 0.122 | 0.122 | 0.924 | -0.504 | 0.119 | 0.127 | 0.127 | 0.932 |
| γ_{33} | 0.500 | 0.512 | 0.108 | 0.113 | 0.113 | 0.936 | 0.510 | 0.110 | 0.115 | 0.115 | 0.944 |
| γ_{43} | 1.700 | 1.738 | 0.135 | 0.140 | 0.144 | 0.948 | 1.733 | 0.138 | 0.142 | 0.146 | 0.956 |
| γ_{53} | 0.600 | 0.572 | 0.180 | 0.182 | 0.184 | 0.948 | 0.580 | 0.188 | 0.206 | 0.207 | 0.932 |
| γ_{63} | -0.500 | -0.440 | 0.405 | 0.416 | 0.419 | 0.948 | -0.475 | 0.446 | 0.517 | 0.517 | 0.916 |
| ρ | 0.800 | 0.794 | 0.043 | 0.041 | 0.041 | 0.960 | 0.796 | 0.044 | 0.041 | 0.041 | 0.960 |
| α | 0.667 | 0.670 | 0.050 | 0.047 | 0.047 | 0.968 | 0.656 | 0.052 | 0.048 | 0.049 | 0.960 |