

# INTEGRATED APPLICATION OF THE CUMULATIVE SCORE CONTROL CHART AND ENGINEERING PROCESS CONTROL

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*Abstract:* Recent years have witnessed heightened interest in the integration of statistical process control (SPC) and engineering process control (EPC). The present study considers the utilization of the cumulative score (Cuscore) technique as an interface between SPC and EPC in cases where a closed loop control is used for process adjustment. Specifically, this study presents a scheme for the integrated use of the Cuscore control chart with the minimum mean squared error (MMSE) control. This study demonstrates the viability and potential advantages of this scheme for combining SPC and EPC.

*Key words and phrases:* Cumulative score control chart, EPC, integration, minimum mean squared error.

## 1. Introduction

Industrial quality engineers and control engineers have embraced divergent strategies to reduce the variability of manufacturing processes and to maintain targeted quality characteristics. Quality engineers have employed statistical process control (SPC) techniques (e.g., Shewhart control charts) to monitor the underlying process, whereas control engineers have utilized engineering process control (EPC) techniques (e.g., Proportional-Integral-Differential control scheme) to regulate the process. Though the objective of both SPC and EPC is to reduce the variability of the process, the techniques differ substantially in approach.

Under Statistical Process Control (SPC), the results of an industrial process—anything from automobile assembly to pesticide production—may be represented as an ongoing series of data points plotted on any number of statistical control charts. A sound process should ideally generate independently and identically distributed (IID) random variables. However, fluctuations in this pattern may arise even when a state of statistical control is achieved. Such disturbances are reflected as irregular sequences of observations (runs that are above or below a certain level) recorded on statistical control charts. When process outputs deviate substantially from the target—when the most recently plotted point falls beyond specified bounds on the statistical control chart—an “alarm” is triggered. In such

situations, quality control engineers seek to identify and eliminate the causes. Thus Hess (1989) suggests that SPC be regarded as an “open loop advisor”.

However, these statistical control charts presuppose the statistical independence of each output observation. This assumption of independence demands a strict level of statistical control rarely achieved in practice. In reality, various factors—ranging from the continued presence of inertial elements to the results of frequent sampling—result in the serial correlation of process observations. This serial correlation skews the data reported on SPC charts and can increase the frequency of “false alarms”. Consequently, the in-control average run length (ARL) of the control charts would be much shorter than anticipated. The assumption that observations are independent is a significant handicap for SPC techniques.

In fact, the presence of autocorrelation (remains in observations) is sometimes an indication that an EPC scheme is needed. A well tuned EPC should be able to produce uncorrelated data as the output deviation from target. EPC does not seek to isolate and eliminate the causes of departures from the target; EPC is designed, instead, to compensate for disturbances in the process by continuously adjusting the process. Though EPC can mitigate the effects of disturbances, EPC can rarely succeed in fully compensating for significant disruptions of the industrial process. Consequently, the process mean (or variability) may drift considerably off target.

The present study aims to develop a technique which effectively fuses SPC and EPC so that the system is able to detect the presence of the transient disturbances in a process. A closed loop control will be discussed in this study. Although the Shewhart and cumulative sum (Cusum) control charts are well developed and are widely used in industry, they have some limitations. For example, Montgomery (1996, p.314 and p.325) reported that Shewhart charts are not sensitive to minor shifts in the process and that Cusum charts respond slowly to large process shifts. Instead, Shao (1993) has found that the Cuscore control chart is effective not only in detecting process shifts, but also in identifying transient disturbances. The present study further demonstrates and evaluates the effective integration of SPC and EPC through use of the Cuscore control chart.

## 2. Relevant Works

Widespread recognition of the enormous potential for enhanced quality and efficiency has fueled recent interest in the integration of SPC and EPC. Box and Kramer (1992) provided an excellent examination of the interface between statistical process monitoring and engineering feedback control and a thorough comparison of statistical process control and automatic process control (APC). Vandel Wiel, Tucker, Faltin and Doganaksoy (1992) proposed the algorithmic

statistical process control (ASPC) as a method of reducing predictable variation; ASPC employs both feedback and feedforward control, and then monitors the system to detect and remove the assignable causes of disturbances. Hunter (1986) and Montgomery and Mastrangelo (1991) reported that the exponentially weighted moving average (EWMA) approach is equivalent to the Proportional-Integral-Differential (PID) control technique. However, MacGregor (1991) contended that the EWMA approach and PID control differ substantially. MacGregor (1992) noted that because the level of the process output variable would be a constant under the assumption of most control charts, the process should be left alone unless a disturbance arising from an assignable cause is detected. Tucker (1992) also argued that control rules will compensate for assignable variation if assignable cause variation could be predictable; when assignable cause variation is unpredictable, a search for assignable causes must be made. Vander Wiel and Vardeman (1992) indicated that Cuscore charts can be developed to quickly signal the process level shifts, additive or innovational outliers, changes in model parameters, and other fluctuations in the process. Wardrop and Garcia (1992) highlighted the difficulty of designing an appropriate model for process disturbances, which may assume a variety of forms (step change or linear change disturbances), stem from a variety of causes, and enter the process at random times. Their survey of various cases suggested that these disturbances cannot be removed in many circumstances. Alwan and Roberts (1989), Montgomery and Friedman (1989), and Montgomery and Mastrangelo (1991) have all recommended that, whenever observations are autocorrelated, an appropriate time series model should be fitted to these observations and control charts then applied to the residuals of the model.

Montgomery, Keats, Runger, and Messina (1994) examined the benefits of combining SPC and EPC techniques. Their simulations demonstrated the superiority of integrated use of SPC and EPC to the use of EPC alone. But while their simulations employed the Shewhart, Cusum and EWMA control charts, the present study focuses upon a recently developed control chart—the Cuscore control chart.

### 3. Cuscore Control Chart

Box and Ramirez (1992) provided a thorough introduction to the Cuscore control chart. Consider a model which can be written as

$$a_t = f(y_t, x_t, m), \quad t = 1, 2, \dots, n, \quad (1)$$

where  $y_t$  is the output observation,  $x_t$  is the independent variable,  $m$  is an unknown parameter, and  $f()$  is some function. If  $m$  is the true value of the unknown

parameter, then the resulting  $a_t$ 's would follow a white noise sequence. Apart from a constant, the log likelihood for  $m = m_0$  is

$$l(m) = -\frac{1}{2\sigma^2} \sum_{t=1}^n a_{t_0}^2,$$

where the  $a_{t_0}^2$ 's are obtained by setting  $m = m_0$  in Equation (1). Let  $-\frac{\partial a_t}{\partial m} \Big|_{m=m_0} = g_{t_0}$ ; then the following relationship holds:

$$\frac{\partial l(m)}{\partial m} = \frac{1}{\sigma^2} \sum_{t=1}^n a_{t_0} g_{t_0}.$$

The Cuscore statistic with the parameter value  $m = m_0$  is defined as:

$$Q_0 = \sum_{t=1}^n a_{t_0} g_{t_0}. \quad (2)$$

Box and Ramirez (1992) demonstrated that the following relationship holds if the model is linear in parameter  $m$  and is approximate otherwise:

$$a_{t_0} = k(m - m_0)g_{t_0} + a_t. \quad (3)$$

Equation (3) implies that an increase in discrepancy vector  $g_{t_0}$  would be added to  $a_t$  as long as the parameter  $m$  does not equal  $m_0$ . This results in a sequential correlation of  $a_{t_0}$  with  $g_{t_0}$ . Consequently, the Cuscore statistic defined in Equation (2) would be continuously searching for the presence of the specific discrepancy vector. In addition, the least square estimator of  $(m - m_0)$  as derived from Equation (3) would be  $\hat{m} - m_0 = \sum a_{t_0} g_{t_0} / \sum g_{t_0}^2$ . This results in  $Q = (\hat{m} - m_0) \sum g_{t_0}^2$ . Thus, when plotted against  $n$ , the Cuscore statistic should be expected to provide a sensitive check for fluctuations in parameter  $m$ . As with the Cusum control chart, changes in the slope of the Cuscore statistics may be used to detect the assignable causes of process shifts and disturbances.

Box and Ramirez (1992) presented the centred Cuscore (CC), the Cuscore evaluated at  $m = \bar{m} = (m_0 + m_1)/2$ . The CC is defined as:

$$CC = \sum_{t=1}^n \bar{a}_t \bar{g}_t,$$

where  $\bar{a}_t = a_t(\bar{m})$  and  $\bar{g}_t = g_t(\bar{m})$ . They noted that the construction of control limits with a Cuscore control chart would be equivalent to a series of Wald sequential tests (Wald (1947)) with boundaries  $(0, H)$ . That is, if  $\beta$  (type II error) is negligible and can be ignored, then  $H = \sigma^2 \ln(1/\alpha)/\delta$ , where  $\delta = m_1 - m_0$ .

Therefore, to detect changes in parameter  $m$ , control limits may be determined by plotting centred Cuscore statistics (which are evaluated at  $(m_0 + m_1)/2$ ) against  $n$ . That is,

$$\begin{aligned} \text{UCL (upper control limit)} &= \frac{\sigma^2 \ln(1/\alpha)}{\delta^+} \text{ when } m_1 > m_0 \\ \text{LCL (lower control limit)} &= \frac{\sigma^2 \ln(1/\alpha)}{\delta^-} \text{ when } m_1 < m_0, \end{aligned}$$

where  $\delta^+ = m_1 - m_0$  (when  $m_1 > m_0$ ) and  $\delta^- = m_1 - m_0$  (when  $m_1 < m_0$ ).

#### 4. Cuscore Control Chart for a Closed Loop Control

##### 4.1. Transient disturbances

Manufacturing process control seeks to minimize the deviation of the process output from the target quality characteristic and to minimize the process variability as a means to enhance and maintain the quality of the finished product. However, industrial processes are frequently disturbed by unforeseen incidents and by deliberate adjustments to the process. Changes in product design (short production runs), vacillations in material properties (such as chemical impurities), or unanticipated events (power or equipment failure) may all upset the production process and demand correction. These disruptions may all be considered transient disturbances.

Transient disturbances typically fall into one of two categories: (1) step change disturbances and (2) linear change disturbances (or ramps). Step change disturbances may result from adjustments in the process target; linear change disturbances (LCD) may result from wearing out of a tool.

MacGregor, Harris, and Wright (1984) and Shao, Haddock, Runger, and Wallace (1993) have examined transient models which represent step change disturbances as (1)  $(1 - B)D_t = \beta_t$  and linear change disturbances as (2)  $(1 - B)(1 - B)D_t = \beta_t$ , where

- $B$ : Backward shift operator (e.g.,  $B^2 D_t = D_{t-2}$ .)
- $D_t$ : The transient disturbances at time  $t$ , and we assume that  $D_t$  would follow either step or linear changes disturbances.
- $\beta_t$ : A random variable, and  $\beta_t$  would be zero most of the time except when the transient disturbances have occurred.

##### 4.2. Step change transient disturbance with Cuscore control chart

According to MacGregor (1988), when the noise of the zero order system follows IMA (1,1) process, the minimum mean squared error (MMSE) controller

should behave as an integral controller. Similarly, when the noise of the first-order system follows the IMA (1,1) process, the MMSE controller should behave as a proportional-integral (PI) controller. In addition, since the integral and PI controllers are widely used in industry, this study uses the MMSE (which is equivalent to integral or PI) control mode for feedback control action in the closed loop control. Appendix A shows that when a step-change disturbance enters the process, the output deviation would be  $y_{t+i} = D\theta^{i-1} + a_{t+i}$ . Therefore, after the transient disturbance has been generated, the process may be described as:

$$y_t = D\theta^{t-1} + a_t.$$

In order to employ the Cuscore control chart, the above relationship could be reconstructed as  $y_t = mD\theta^{t-1} + a_t$ . Therefore,  $a_t = y_t - mD\theta^{t-1}$ , and  $g_{t_0} = -\frac{\partial a_t}{\partial m}|_{m=m_0} = D\theta^{t-1}$ ,  $a_{t_0} = (m - m_0)g_{t_0} + a_t = y_t$ . The Cuscore statistic, thus, would be

$$Q_0 = \sum_{t=1}^n a_{t_0}g_{t_0} = \sum_{t=1}^n y_t D(\theta^{t-1}).$$

### 4.3. Linear change transient disturbance with Cuscore control chart

In the case of the linear change transient disturbance, Appendix B shows that the output deviation would be  $y_{t+i} = b\left(\frac{\theta^i-1}{\theta-1}\right) + a_{t+i}$ . Therefore, after the transient disturbance has been generated, one can represent the process as:  $y_t = b\left(\frac{\theta^t-1}{\theta-1}\right) + a_t$ . To use the Cuscore control chart, this relationship could be reconstructed as  $y_t = mb\left(\frac{\theta^t-1}{\theta-1}\right) + a_t$ . Therefore,  $a_t = y_t - mb\left(\frac{\theta^t-1}{\theta-1}\right)$ , and

$$g_{t_0} = -\frac{\partial a_t}{\partial m}|_{m=m_0} = b\left(\frac{\theta^t-1}{\theta-1}\right), \quad a_{t_0} = (m - m_0)g_{t_0} + a_t = y_t.$$

The Cuscore statistic, thus, would be

$$Q_0 = \sum_{t=1}^n a_{t_0}g_{t_0} = \sum_{t=1}^n y_t b\left(\frac{\theta^t-1}{\theta-1}\right). \quad (4)$$

However, since the MMSE control action cannot completely compensate for the linear change transient disturbance, the centred Cuscore's ability to detect the assignable causes of transient disturbances attracts considerable interest. Consider a linear change disturbance with slope  $b = 1$  (i.e., trend per period is of magnitude unity). The value of  $m_0$  is thus equal to 0,  $m_1$  is equal to 1 and  $\bar{m} = (m_0 + m_1)/2 = 0.5$ . The CC is then obtained as the following:

$$CC_t = \sum_{t=1}^n \bar{a}_t \bar{g}_t = \sum_{t=1}^n \left[ y_t - 0.5b\left(\frac{\theta^t-1}{\theta-1}\right) \right] \left[ b\left(\frac{\theta^t-1}{\theta-1}\right) \right]. \quad (5)$$

**5. Simulation Studies**

**5.1. Use of Cuscore control chart**

The performance of the Cuscore control chart may be illustrated through simulation of a process upset by a linear change disturbance. Specifically, this study examined simulations of a zero-order process (both with and without engineering process control) in which a linear change disturbance with slope 1 is introduced after observation 201 in a series of 400 observations. The value of parameters  $q = 0.8$  and  $\theta = 0.5$  are arbitrarily selected (however, they are typical of those encountered in practice). The white noise follows a normal distribution with mean 0 and standard deviation 1.

Figure 1 displays the process output deviations from target in the absence of engineering process controls; after observation 201, the linear change disturbance persists unabated for the duration of the process. Figure 2 shows the output deviations from target after the integral control (i.e., Equation (A.1)) is introduced to tune the process. The integral control (which is equivalent to MMSE) partially compensates for the linear change disturbance, but it fails to return the process outputs to target. Figure 3 plots both the Cuscore (defined as Equation (4)) and the CC statistic (defined as Equation (5)) against time. The linear increase in the Cuscore statistic values following observation 201 reflect the linear change disturbance. A CC value falling beyond an upper or lower bound would signal the presence of a linear change disturbance. For example,  $H$  would be equal to 4.61 with  $\alpha(\text{type I error}) = 0.01$ . Thus, according to Figure 3, an out of control alarm due to the assignable causes may be expected at time 203.

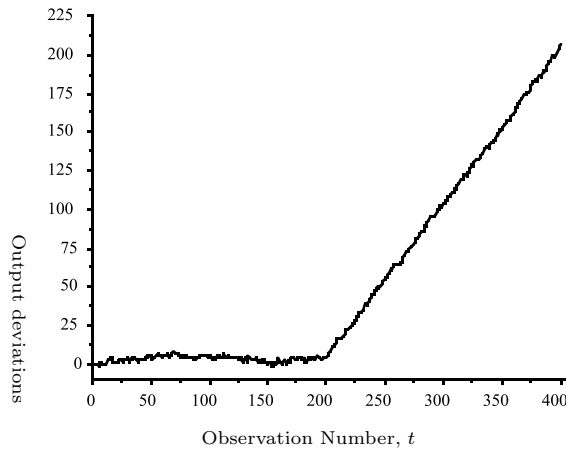


Figure 1. Output deviations from target (without use of EPC). A linear disturbance starts at  $t = 201$ .

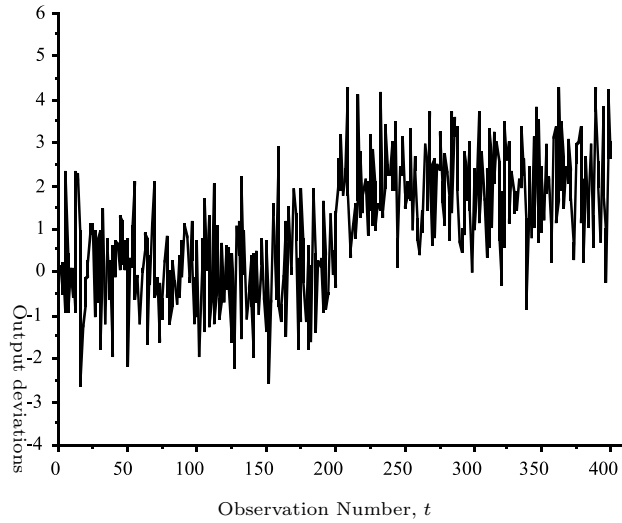


Figure 2. Output deviations from target (with use of MMSE control). A linear disturbance starts at  $t = 201$ .

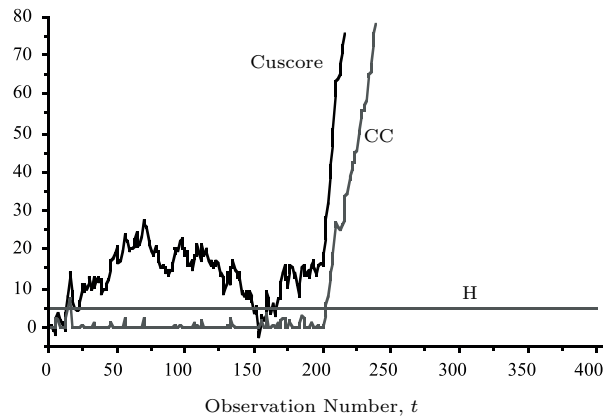


Figure 3. Plot of Cuscore and Centred Cuscore statistics. An out-of-control signal is given at  $t = 203$ .

This simulation assumed that the period needed to identify and eliminate the assignable cause of the linear change disturbance was 10 times unity from the point of initial detection by the Cuscore control chart. Thus, in this simulation, the disturbance was detected at time 203 and terminated at time 212. Figure 4 plots output deviations from target after the Cuscore control chart is applied, and it shows that the linear change disturbance is compensated by employing the integration of Cuscore control chart and MMSE control.



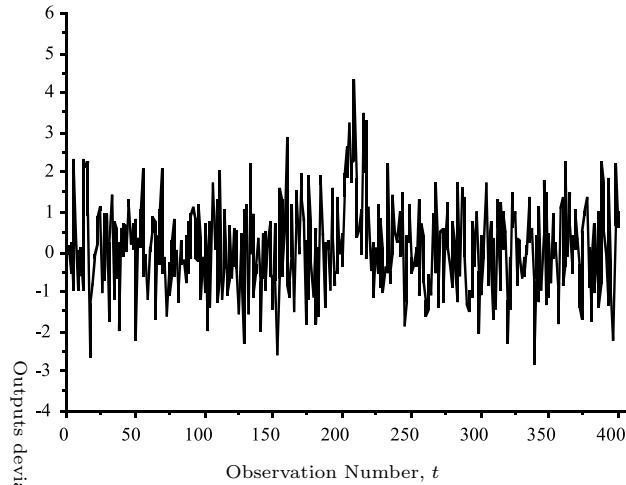


Figure 4. Output deviations from target (with use of MMSE/Cuscore). A linear disturbance starts at  $t = 201$ .

### 5.2. Performance of Cuscore control chart in comparison

The first simulation demonstrated the effectiveness of the Cuscore control chart as a tool for integrating SPC and EPC. As a supplementary consideration, a second comprehensive simulation was conducted to compare the performance of the Cuscore control chart with the performance of other SPC techniques in integrated EPC/SPC schemes.

As in the previous simulation, this simulation examines zero-order processes (both with and without engineering process control), with parameter values of  $q = 0.8$  and  $\theta = 0.5$ , in which a linear change disturbance is introduced after observation 201 in a series of 400 observations. The performance measure (PM) applied is the average squared deviation from the target (T). That is,  $PM = 1/n \sum_{t=1}^n (y_t - T)^2$ .

In Figure 1, with no EPC (i.e., integral control) action,  $PM = 7103.292$ ; in Figure 2 with applying integral control action,  $PM = 3.0805$ . This simulation also assumes that the linear change disturbance results from the assignable cause, and that the magnitudes of the trend are 0.1, 0.5, 1.0, 1.5, and 2.0 units per period. As in the previous simulation, the assignable cause occurs at time 201 and the period needed to identify and eliminate the assignable cause is 10 times unity once detected.

Three different SPC control charts for the output deviation from target were examined: a Shewhart chart for individuals with  $3\sigma$  control limits, a Cusum chart with  $k = 0.5$  and  $h = 5$ , and a Cuscore chart with  $H = 4.61$ . The values of  $k = 0.5$  and  $h = 5$  for the Cusum chart were selected because they

are effective across a broad range of process shifts (Montgomery, Keats, Runger, and Messina (1994)). Performance measures are computed across 400 periods for 1000 simulation runs; likewise, the integral control action is employed for all 400 periods of 1000 simulation runs.

Table 1 shows the simulation results for 5 different conditions with 5 different trend of magnitudes per period. The results reported are average values of the PM and the associated standard error (in parentheses) based on 1000 simulation runs. The first condition (second column of Table 1) gives the average value of the PM prior to the occurrence of the linear change transient disturbance (i.e., for period 1-200). The second condition (third column of Table 1) gives the average value of the PM for period 201-400 with integral control action (without applying any SPC charts). The remaining columns report the average value of the PM for period 201-400 with integral control together with use of the Cuscore (column 4), Shewhart (column 5) and Cusum (column 6) charts.

Table 1. Averages of the performance measures for EPC/SPC schemes based on 1000 simulations. Standard deviations of the performance measures are given in parenthesis.

LCD with Slope Per Period	Prior to LCD	EPC Only	EPC/ Cuscore	EPC/ Shewhart	EPC/ Cusum
0.1	1.04715 (0.06348)	1.06584 (0.07259)	1.31585 (0.46651)	1.06677 (0.07987)	1.51963 (0.40027)
0.5	1.04715 (0.06348)	1.91691 (0.40273)	1.55491 (0.37007)	1.74468 (0.42350)	1.53653 (0.34791)
1.0	1.04715 (0.06348)	4.76576 (0.20206)	1.70931 (0.39225)	1.79161 (0.35505)	1.71972 (0.38624)
1.5	1.04715 (0.06348)	9.59794 (0.08809)	1.95228 (0.40750)	1.99800 (0.41713)	1.97721 (0.40741)
2.0	1.04715 (0.06348)	16.41345 (0.35560)	2.29058 (0.42474)	2.28466 (0.40908)	2.32172 (0.41667)

To ensure a consistent comparison for all conditions, this study used the same seed for random number generators. Thus, column one indicates that the PM prior to the linear change disturbance is the same for all five trend magnitude sizes evaluated. The results in Table 1 suggest that the combined EPC/SPC scheme has a smaller PM than the application of EPC alone, especially with larger trends of magnitude. The significant exception occurs with a trend magnitude of 0.1. As established earlier, EPC succeeds in partially compensating for linear change disturbances and provides virtually complete compensation for negligible disturbances. Thus, despite a disturbance of trend magnitude 0.1, the process

operating with EPC alone acts as if there is no linear change disturbance for the period of 201-400.

Nevertheless, there is a firm indication that the PM for the integrated EPC-SPC schemes is smaller than the PM for the EPC mode in the remaining conditions. Furthermore, when the condition of a disturbance with trend magnitude of 0.1 is discarded, the PM for the EPC/Cuscore scheme seems to appear smaller than the PM for the EPC/Shewhart and EPC/Cusum schemes. Although the values of averaged PM in Table 1 do not show much superiority of the Cuscore to other charts, this might be due to the PM which takes the average squared deviation from the target for the 400 observations and makes the PM's for different control charts quite close. Table 2 shows the simulation results for average detecting time of 4 different trend magnitudes per period by applying Cuscore, Shewhart, and Cusum charts. This sensitive measure apparently indicates that Cuscore control chart has the shortest average time for detecting the linear change disturbances.

Table 2. Average time for detecting the linear change disturbances based on 1000 simulations.

LCD with Slope Per Period	EPC/ Cuscore	EPC/ Shewhart	EPC/ Cusum
0.5	10.24	35.99	10.98
1.0	3.59	7.44	4.05
1.5	1.95	2.89	2.44
2.0	1.34	1.34	1.72

### 6. Conclusions

This study addresses the integration of EPC and SPC techniques in quality control with specific emphasis upon application of the Cuscore control chart. The integrated use of the Cuscore control chart and EPC tactics in a closed loop process is discussed and demonstrated through simulation.

This study's principal purpose was to examine the feasibility of using the Cuscore control chart as an interface between EPC and SPC techniques. Because the Cuscore control chart is simple to use and sensitive to the appearance of transient disturbances in an industrial process, its use may be integrated with the application of EPC techniques to both monitor an industrial process and to detect and eliminate the assignable causes of transient disturbances. This present effort has demonstrated the effective incorporation of Cuscore and EPC techniques. Additionally, the simulation results appear to confirm the superiority of integrated SPC and EPC schemes over control schemes employing EPC

alone in many circumstances and, furthermore, to suggest the superiority of Cus-core/EPC scheme to other SPC/EPC integrations in certain circumstances.

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### Appendix A. Output Deviations Under Zero Order System with Integral Control When Step Change Transient Disturbance Exists

The zero order system with noise of IMA(1,1) process could be modelled as  $y_{t+1} = qx_t + d_{t+1}$ , and  $d_{t+1} = \frac{(1-\theta B)a_t}{(1-B)}$ , where  $d_{t+1}$  is the noise at time  $t + 1$  and it follows IMA(1,1) process,  $y_{t+1}$  is the output deviation from target at time  $t + 1$ , and  $x_t$  is control variable's deviation from nominal value at time  $t$ . The integral (I) control action is (MacGregor (1988))

$$x_t = -\frac{(1-\theta)}{q} \sum_{j=-\infty}^t y_j. \quad (\text{A.1})$$

Now, suppose the step change transient disturbance has started to affect the process at time  $t + 1$ , then the following consequences would happen.

(1) At time  $t + 1$ .

Since the step change transient disturbance has started to affect the process at time  $t + 1$ , therefore the process should be represented as

$$y_{t+1} = qx_t + d_{t+1} + D_{t+1}. \quad (\text{A.2})$$

Substituting Equation (A.1) into Equation (A.2), it can be shown that:

$$\begin{aligned} y_{t+1} &= q \left[ -\frac{(1-\theta)}{q} \sum_{j=-\infty}^t y_j \right] + d_{t+1} + D_{t+1} \\ &= D_{t+1} + a_{t+1} \quad \left( \text{since } \sum_{j=-\infty}^t y_j = \frac{a_t}{(1-B)} \right). \end{aligned}$$

Therefore, the control action would be

$$x_{t+1} = x_t - \frac{(1-\theta)}{q} D_{t+1} - \frac{(1-\theta)}{q} a_{t+1}. \quad (\text{A.3})$$

(2) At time  $t + 2$ .

The underlying process should be represented as

$$y_{t+2} = qx_{t+1} + d_{t+2} + D_{t+2}. \tag{A.4}$$

Substituting Equation (A.3) into Equation (A.4), it can be shown that:  $y_{t+2} = a_{a+2} + D_{t+2} - (1 - \theta)D_{t+1}$ . Therefore, the control action would be  $x_{t+1} = x_t - \frac{(1-\theta)}{q}y_{t+2}$ . Continuing doing this way, we are able to conclude that

$$y_{t+i} = D_{t+i} - (1 - \theta)D_{t+i-1} - (1 - \theta)\theta D_{t+i-2} - (1 - \theta)\theta^2 D_{t+i-3} + \dots + a_{t+i}, \tag{A.5}$$

where  $i$  stands for the time when the transient disturbance started affecting the process. In the case of step change transient disturbance, the magnitude of the level is assumed to be  $D$ . Therefore,  $D_{t+i} = D_{t+i-1} = D_{t+i-2} = \dots = D_{t+1} = D$ . Equation (A.5) can be rewritten as

$$\begin{aligned} y_{t+i} &= D[1 - (1 - \theta) - (1 - \theta)\theta - \dots - (1 - \theta)\theta^{i-2}] + a_{t+i} \\ &= D[1 - (1 - \theta)(1 + \theta + \theta^2 + \dots + \theta^{i-2})] + a_{t+i}. \end{aligned}$$

Then, it can be shown that  $y_{t+i} = D\theta^{i-1} + a_{t+i}$ .

### Appendix B. Output Deviations Under Zero Order System with Integral Control When Linear Transient Disturbance Exists

In the case of linear transient disturbance, the disturbance can be reformed as  $D_t = a + bi$ , where  $a$  is the magnitude of the level and  $b$  is the slope. Suppose  $a = 0$  without lose generality, then Equation (A.5) can be reformed as

$$y_{t+i} = b\{i - (1 - \theta)[(i - 1) + \theta(i - 2) + \theta^2(i - 3) + \dots + \theta^{i-2}]\} + a_{t+i}.$$

It can be shown that  $y_{t+i} = b\left(\frac{\theta^i - 1}{\theta - 1}\right) + a_{t+i}$ .

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