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ON-LINE PROCEDURE FOR TERMINATING AN ACCELERATED DEGRADATION TEST

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Abstract: Accelerated degradation testing (ADT) is a useful technique to extrapolate the lifetime of highly reliable products under normal use conditions if there exists a quality characteristic of the product whose degradation over time can be related to reliability. One practical problem arising from designing a degradation experiment is "how long should an accelerated degradation experiment last for collecting enough data to allow one to make inference about the product lifetime under the normal use condition?" In this paper, we propose an intuitively appealing procedure to determine an appropriate termination time for an ADT. Finally, we use some light-emitting diode (LED) data to demonstrate the proposed procedure.

Key words and phrases: Accelerated degradation test (ADT), degradation path, highly reliable product, termination time.

1. Introduction

Traditionally, reliability assessment of new products has been based on accelerated life tests (ALTs) that record failure and censoring times of products subjected to elevated stress. However, this approach may offer little help for highly reliable products which are not likely to fail during an experiment of reasonable length. An alternative approach is to assess the reliability from the changes in performance (degradation) observed during the experiment, if there exists a quality characteristic of the product whose degradation over time can be related to reliability.

Usually, in order to facilitate observing the degradation phenomenon or shorten the degradation experiment under a normal use condition, it is practical to collect the "degradation data" at higher levels of stress and, then, carry out extrapolation in stress to estimate the reliability under normal use conditions. Such an experiment is called an accelerated degradation test (ADT). Nelson (1990), chapter 11 and Meeker and Escobar (1993) survey the scant literature on the subject. Carey and Koenig (1991) describe a data-analysis strategy and a model-fitting method to extract reliability information from observations on the degradation of integrated logic devices that are components in a new generation of submarine cables.

In order to conduct an ADT efficiently, there are several factors (for example, number of stresses, the stress levels, the sample size for each stress level and the termination time, etc.) that need to be considered carefully. Boulanger and Escobar (1994) address the problem of determining both the selection of stress levels and sample size for each stress level under a "pre-determined" termination (life-testing) time. The results are interesting. However, the termination time not only affects the cost of performing an experiment, but also affects the precision of estimating a product's mean lifetime (MTTF). We use an example (in Section 2) to explain why it is more appropriate not to fix the termination time in advance. Thus, determining an appropriate termination time for an ADT is a real challenge for reliability engineers.

Tseng and Yu (1997) propose a simple rule to determine the termination time for a non-accelerated degradation model. However, for highly-reliable products, the result can be applied only to estimate the product's MTTF (under the normal use condition) when the acceleration factor (AF) is known. When the AF is unknown, we need to conduct an efficient ADT to estimate the product's MTTF. In this paper, by combining the approach of Tseng and Yu (1997) with an ALT model, we propose a procedure to achieve the above goal. Finally, we also use some LED (light emitting diode) data to demonstrate this procedure.

The rest of the paper is organized as follows: Section 2 gives an explanation why the termination time is so important. Section 3 proposes a stopping rule to determine an appropriate termination time for an ADT. Section 4 applies the proposed procedure to a numerical example. Section 5 conducts a simulation study of the proposed stopping rule. Finally, Section 6 addresses some concluding remarks.

2. Why the Termination Time is Important?

Suppose that an ADT of a product is conducted at m higher stress levels:

$$S_u \leq S_1 \leq S_2 \leq \cdots \leq S_m, \tag{1}$$

where S_u denotes the normal use condition. For the ith stress level S_i, there are n_i devices (items) which are randomly selected for performing a degradation test. Let $G(t, \mathbf{\Theta}_{ij})$ denote the quality characteristic of the jth item under the stress level S_i, which degrades over time t and $\mathbf{\Theta}_{ij}$ is a vector of parameters. Assume that D is a critical value for the degradation path. Then the failure time τ_{ij} is defined as the time when the degradation path crosses the critical degradation level D. Thus, if $\mathbf{\Theta}_{ij}$ is known, the lifetime of the jth item under S_i can be expressed by

$$\tau_{ij} = \tau(D; \mathbf{\Theta_{ij}}). \tag{2}$$

For example, if $G(t;\mathbf{\Theta_{ij}}) = e^{-\alpha_{ij}t^{\beta_{ij}}}$, then

$$\tau_{ij} = \Big(\frac{-\ln D}{\alpha_{ij}}\Big)^{\frac{1}{\beta_{ij}}}. \tag{3}$$

Applying an accelerated life test (ALT) model, the lifetime distribution under a normal use condition (say S_u) can then be easily obtained.

In practical situations, however, $\mathbf{\Theta}_{ij}$ is unknown. In addition, due to the measurement errors, the observed degradation path at time t, $LP_{ij}(t)$, can only be expressed as follows:

$$LP_{ij}(t) = G(t;\mathbf{\Theta_{ij}}) + \epsilon_{ij}(t), \tag{4}$$

where $\epsilon_{ij}(t)$ is the measurement error term which is assumed to follow a distribution with mean 0 and variance σ_ϵ^2.

To obtain a precise estimate of a product's MTTF, the ascertainment of the termination time is an important issue to the experimenter. We use the following example for illustration.

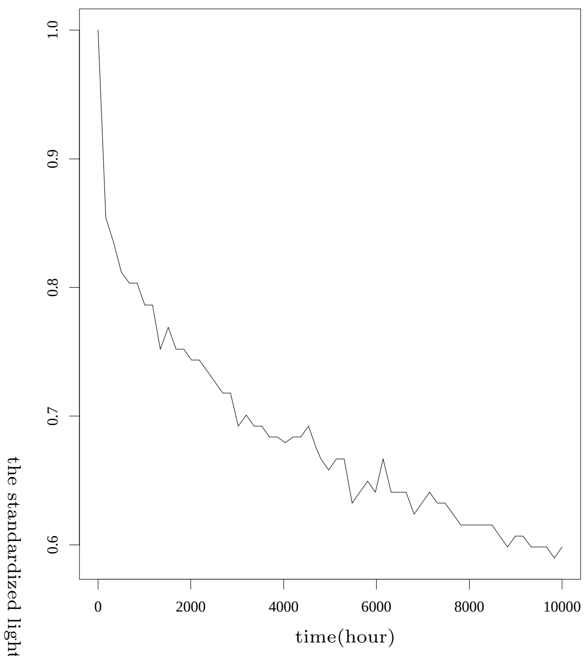

Figure 1. A typical degradation path of an LED product

Example 1. Figure 1 shows a typical degradation path of an LED product. From the plot, it is seen that $G(t,\mathbf{\Theta}) = e^{-\alpha t^\beta}$ is an appropriate model for the degradation path. Now, if the experiment is terminated at 3000 hours, then the MLEs for α and β are $\hat{\alpha}$=0.01217156 and $\hat{\beta}$=0.3972809. However, if the experiment is terminated at 8000 hours, then $\hat{\alpha}$=0.008542078, and $\hat{\beta}$=0.4448581. Assume that D=0.50. Then the corresponding estimated lifetimes are 26226 and 19581 hours, respectively. It is clear that the termination time has a significant impact on the precision of estimating a product's lifetime.

For the ADT case, we now provide a three-dimensional plot for illustration. In Figure 2, suppose that the experiment is conducted up to the time t_l. Then, based on the observed data $\{(t_k, LP_{ij}(t_k))\}_{k=1}^{l}$, the least squares estimator (LSE) of $\mathbf{\Theta_{ij}}$ and the corresponding jth product's lifetime (under S_i) can be obtained. Then, by using a statistical life-stress ALT model, we can extrapolate to obtain the MTTF under the normal use condition S_u. Let $\hat{\text{MTTF}}(l)$ denote the estimated MTTF when the ADT is conducted up to the time t_l. From the plots of $\{\hat{\text{MTTF}}(l)\}_{l\geq 1}$, it is seen that the curve (path) will oscillate drastically at the beginning; however, as the termination time t_l increases, more data are collected and the path of $\hat{\text{MTTF}}(l)$ approaches an asymptote.

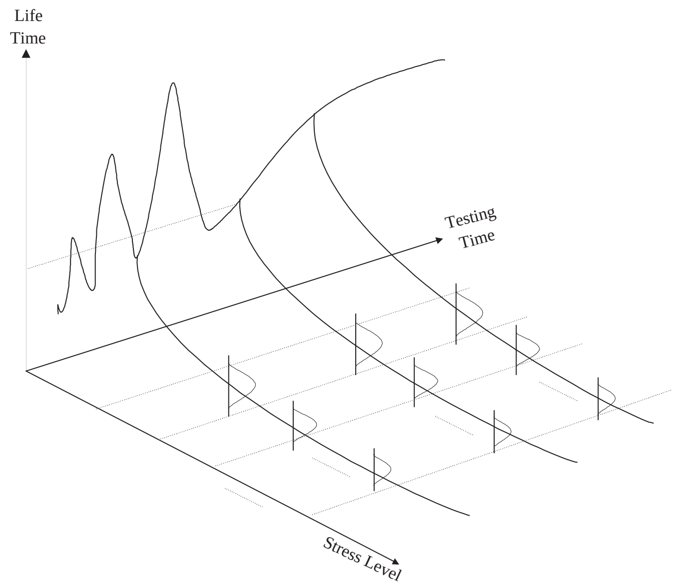

Figure 2. A typical trend of the estimators of MTTF under normal use condition for an ADT.

From Figure 2, it is obvious that the experiment can be terminated only if the sequence $\hat{\text{MTTF}}(l)$ is convergent. However, one usually needs to conduct a very long life-testing time to achieve a convergent value. This is impractical for experimenters. In the following section, we propose an intuitive procedure to determine an appropriate termination time for an ADT.

3. Determining the Termination Time for an ADT

The procedure for determining an appropriate termination time for an ADT consists of three major steps labelled (A) to (C) as follows:

(A) **Use the degradation paths to estimate the lifetimes of devices under each testing stress.**

Suppose that an ADT is conducted up to the time t_l. Based on the degradation data $\{(t_k, LP_{ij}(t_k))\}_{k=1}^{l}$, the least squares estimator (LSE) $\hat{\mathbf{\Theta}}_{\mathbf{ij}}(l)$ of $\mathbf{\Theta}_{\mathbf{ij}}$ can be obtained by minimizing

$$SSE(\mathbf{\Theta}_{\mathbf{ij}}) = \sum_{k=1}^{l}\{LP_{ij}(t_k) - G(t_k;\mathbf{\Theta}_{\mathbf{ij}})\}^2 \tag{5}$$

and the corresponding lifetime τ_{ij} can be estimated by

$$\hat{\tau}_{ij}(l) = \tau\left[D;\hat{\Theta}_{ij}(l)\right]. \tag{6}$$

(B) **Find a suitable life-stress model and use an ML procedure to estimate the MTTF of the device under S_u.**

Applying an ALT model to extrapolate the lifetime distribution under normal use conditions requires the following steps:

1. use probability plots to assess the lifetime distribution of $\{\hat{\tau}_{ij}(l)\}_{j=1}^{n_i}$, for all $1 \le i \le m$;
2. use scatter plots of $\{\hat{\tau}_{ij}(l)\}_{j=1}^{n_i}, 1 \le i \le m$, to determine a suitable life-stress relationship; and
3. use an ML procedure to estimate the unknown parameters in a suitable life-stress model and then the MLE of the product's MTTF at normal use condition S_u can be obtained.

(C) **Investigate the limiting property of $\hat{\text{MTTF}}(l)$ and propose an appropriate termination time.**

Intuitively, the growth trend of $\hat{\text{MTTF}}(l)$ may oscillate drastically at the beginning. As t_l increases, the growth trend will converge. Assume that l_0 is a starting point at which $\{\hat{\text{MTTF}}(k)\}_{k=l_0}^{l}$ has a convergent pattern. A convergent pattern is indicated by one of the following three cases: (1) monotonically increasing to a target; (2) monotonically decreasing to a target; and (3) slightly oscillating around a target value. Due to the asymptotic property, there exists a sigmoidal growth curve $f_l(t)$ which fits $\{\hat{\text{MTTF}}(k)\}_{k=l_0}^{l}$ (Seber and Wild (1989), Chapter 7). To obtain a more precise estimator of MTTF, we can define an asymptotic MTTF as $f_l(\infty)(= \lim_{t\to\infty} f_l(t))$. The physical meaning of $f_l(\infty)$ is that the predicted product's MTTF will converge asymptotically to this value when the experiment is conducted up to the time t_l. Obviously, $f_l(\infty)$ provides a better estimator than $\hat{\text{MTTF}}(l)$.

To measure the relative rate of change of the asymptotic mean lifetime, we consider the following h-period moving-average:

$$\rho(l) = \frac{1}{h}\Big\{\sum_{k=l-h+1}^{l}\Big|1 - \frac{f_k(\infty)}{f_{k-1}(\infty)}\Big|\Big\}. \tag{7}$$

Obviously, when $h = 1$, $\rho(l)$ reduces to a one-period change rate of the asymptotic mean lifetime. To avoid the irregular pattern of the relative change rate, we choose $h = 3$ in this study. Thus, a rule for terminating the experiment can be stated as follows:

t_l **is an appropriate termination time if** $\rho(m) \le \varepsilon,\ \forall m \ge l,$

where ε is an allowable tolerance which is commonly specified by the experimenters. Now, we state an algorithm to summarize the above procedure.

Algorithm for determining an appropriate termination time

Step 0. At the beginning, arbitrarily choose $l = 4$ as a starting point.

Step 1. Use Equations (5) and (6) to compute the estimated lifetime $\hat{\tau}_{ij}(l)$ of the jth item under the stress level S_i, $1 \le j \le n_i$, $1 \le i \le m$.

Step 2. Use scatter plots to assess the life-stress relationship and compute the MLE for MTTF.

Step 3. Plot the growth trend of $\{\hat{\text{MTTF}}(k)\}_{k=2}^{l}$. If there exists a convergent pattern go to Step 4. Otherwise, let $l = l + 1$ and go to Step 1.

Step 4. Choose a suitable starting point l_0 such that the plot of $\{\hat{\text{MTTF}}(k)\}_{k=l_0}^{l}$ has a convergent trend. Then, find a suitable function $f_l(t)$ to fit $\{\hat{\text{MTTF}}(k)\}_{k=l_0}^{l}$ and compute $f_l(\infty)$.

Step 5. Compute $\rho(l)$. If $\rho(m) \le \varepsilon$, $\forall m \ge l$, then t_l is an appropriate termination time. Otherwise, let $l = l + 1$ and go to Step 1.

In the next section, we use a numerical example to illustrate the procedure.

4. A Numerical Example

Light emitting diodes (LEDs) have become widely used in a variety of fields. The fields of application range from consumer electronics to optical fiber transmission systems. Very-high-reliability is especially required in optical fiber transmissions. Thus, designing an efficient experiment to estimate its lifetime is a challenge to the producers.

From engineering knowledge, electric current is a suitable accelerated variable for LED products (see Ralston and Mann (1979)); so, three higher stress levels, $S_1 = 10$ mA, $S_2 = 20$ mA, and $S_3 = 30$ mA, are carefully chosen to perform an ADT. The goal is to estimate the product's MTTF under normal use conditions (say, 5 mA). There are $n_1 = 16$, $n_2 = 14$, and $n_3 = 18$ items which are randomly selected for performing an ADT under 10 mA, 20 mA, and 30 mA, respectively.

A key quality characteristic of LED is its light intensity. It degrades over time. Let $LP_{ij}(t)$ denote the observed standardized light intensity of the jth LED under S_i. Figure 3 shows the degradation paths of the standardized light intensity of LEDs for these three stress levels.

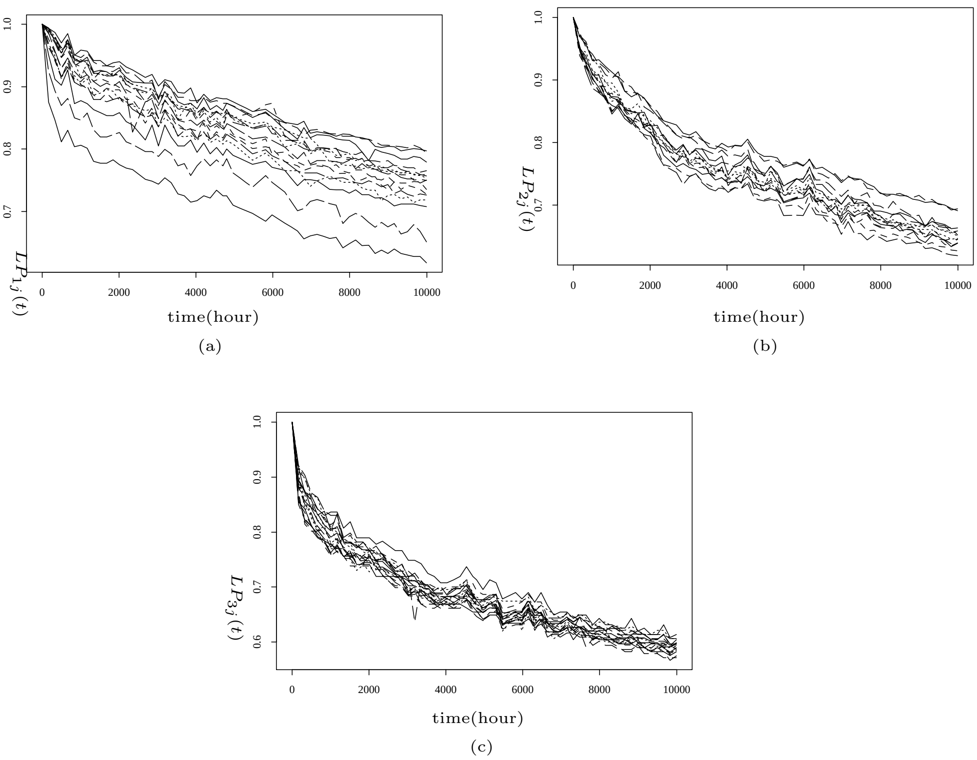

Figure 3. (a), (b), and (c) are the sample degradation paths under 10 mA, 20 mA, and 30 mA, respectively.

The experiment was conducted up to 9998 hours for each stress. A practical decision that the experimenter faces is: "Is 9998 hours long enough to provide a precise estimation for the product's MTTF?" If the testing time is long enough, what is the most appropriate termination time? Next, we apply the proposed method to address this problem.

(A) **Estimate the lifetimes of devices under each testing stress**

Figure 4 is a plot of $\log(-\log\ LP_{ij}(t))$ vs $\log\ t$. From the linear patterns, it is seen that $G(t;\mathbf{\Theta_{ij}}) = G(t;\alpha_{ij},\beta_{ij}) = e^{-\alpha_{ij}t^{\beta_{ij}}}$ is an appropriate model to describe the LED data.

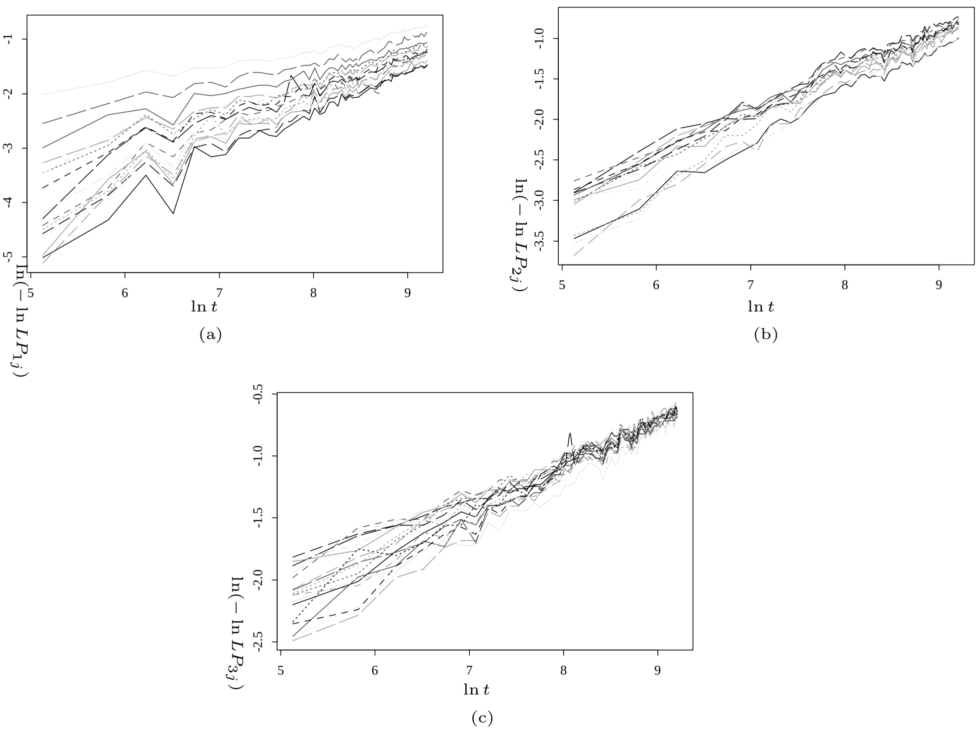

Figure 4. (a), (b) and (c) are the plots of $\ln(-\ln LP_{ij})$ vs $\ln t$ for 10 mA, 20 mA, and 30 mA, respectively.

Based on the observations $\{(t_k, LP_{ij}(t_k))\}_{k=1}^{l}$ and Equation (5), the LSEs $(\hat{\alpha}_{ij}(l),\ \hat{\beta}_{ij}(l))$ of (α_{ij},β_{ij}) can be computed. Then the lifetimes $\{\hat{\tau}_{ij}(l)\}_{j=1}^{n_i}$ can also be obtained by the following equation:

$$\hat{\tau}_{ij}(l) = \left[\frac{-\ln D}{\hat{\alpha}_{ij}(l)}\right]^{\frac{1}{\hat{\beta}_{ij}(l)}}. \tag{8}$$

(B) **Find a suitable life-stress relation and use an ML procedure to estimate product's MTTF**

Figure 5 shows two typical lognormal probability plots of $\{\hat{\tau}_{1j}(l)\}_{j=1}^{16}$, $\{\hat{\tau}_{2j}(l)\}_{j=1}^{14}$, and $\{\hat{\tau}_{3j}(l)\}_{j=1}^{18}$ for $l = 46$ (7984 hours) and $l = 58$ (9998 hours). It is seen that the lognormal distribution is an appropriate model to fit the lifetime data. Besides, the patterns of three approximately parallel lines in these probability plots imply that the scale parameters are equal.

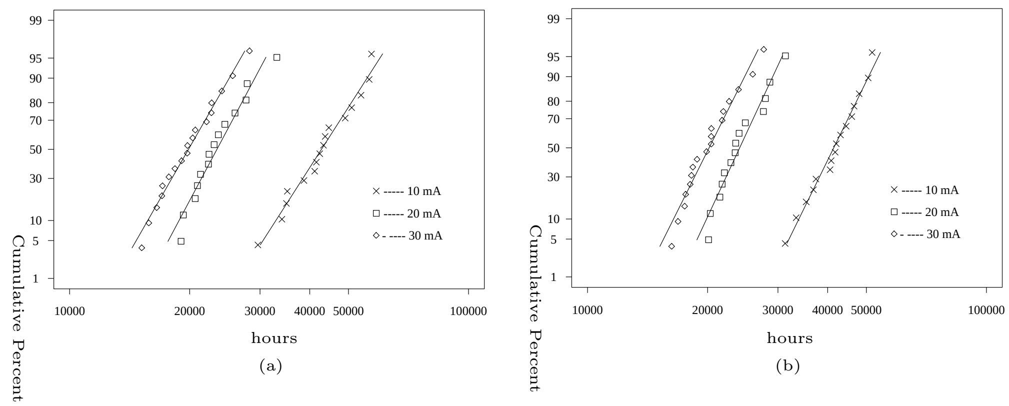

Figure 5. (a) and (b) are the lognormal probability plots of $\{\hat{\tau}_{ij}(l)\}_{j=1}^{n_i}$, $i = 1, 2, 3$, for $l = 46$ and $l = 58$, respectively.

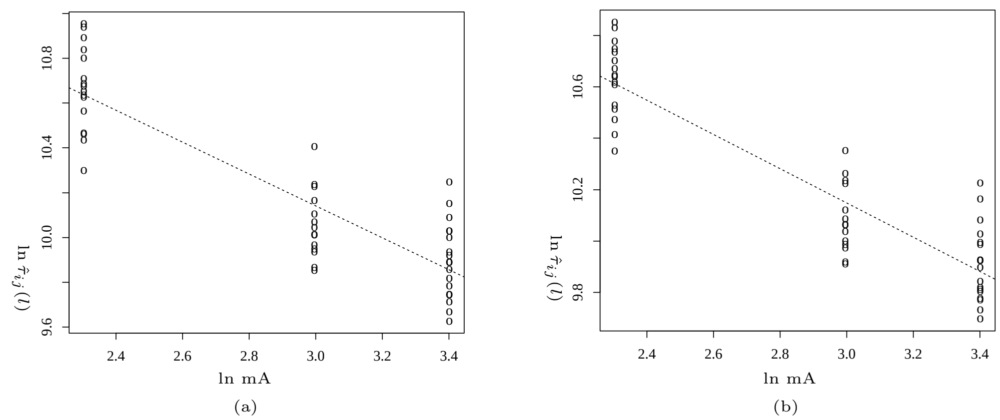

Figure 6. (a) and (b) are the scatter plots of $\ln \hat{\tau}_{ij}(l)$ vs ln mA for $l = 46$ and $l = 58$, respectively.

Furthermore, from the log-log scale scatter plots shown in Figure 6, it is seen that the inverse-power relationship is an appropriate model to describe the life and current relation. Hence, the lognormal-inverse power is a suitable life-stress model. Let $\hat{\mu}_l$ and $\hat{\sigma}_l$ denote the MLEs of the location and scale parameters of log lifetime under the normal use condition 5 mA. The $\hat{\mu}_l$, $\hat{\sigma}_l$, and $\widehat{\text{MTTF}}(l)$ for $4 \leq l \leq 58$ are listed in Table 1. Figure 7 shows the growth trends of $\{\widehat{\text{MTTF}}(l)\}_{l=4}^{58}$.

Table 1. The estimates $\hat{\mu}_l$, $\hat{\sigma}_l$, $\hat{\text{MTTF}}(l)$, $\hat{f}_l(\infty)$, $\rho(l)$, and $\rho^*(l)$

l	time t_l (hours)	$\hat{\mu}_l$	$\hat{\sigma}_l$	$\hat{\text{MTTF}}(l)$	$\hat{f}_l(\infty)$	$\rho(l)$	$\rho^*(l)$
4	672	10.88282	0.8392298	75733.70			
5	840	10.33716	0.7177683	39925.00			
6	1008	10.38654	0.6822777	40916.77			
7	1176	10.65507	0.6798830	53433.79			
8	1344	10.56699	0.6163895	46955.72			
9	1512	10.56137	0.5733802	45513.13			
10	1680	10.69152	0.5502431	51170.25			
11	1848	10.84014	0.5383829	58987.15			
12	2016	11.01169	0.5235960	69478.49			
13	2184	11.05117	0.5113794	71820.66			
14	2352	11.05379	0.5103521	71971.57			
15	2688	11.03400	0.4919222	69912.45			
16	2856	11.10751	0.4802682	74820.06			
17	3024	11.07192	0.4611743	71557.86			
18	3192	11.19982	0.4578258	81196.78			
19	3360	11.25725	0.4514328	85746.92			
20	3528	11.22251	0.4326228	82132.92			
21	3696	11.21631	0.4183135	81130.02			
22	3864	11.22478	0.3978159	81138.83			
23	4032	11.20680	0.3882441	79393.19			
24	4200	11.12957	0.3168698	71666.48			
25	4368	11.12286	0.3115891	71069.12			
26	4536	11.14599	0.3140012	72786.83			
27	4704	11.15853	0.3115086	73648.01			
28	4800	11.20639	0.3103663	77231.05			
29	4968	11.22875	0.3038130	78818.73			
30	5136	11.23306	0.2966744	78989.90			
31	5304	11.22737	0.2898643	78384.71			
32	5472	11.23355	0.2795926	78640.07			
33	5808	11.25729	0.2725032	80372.36			
34	5976	11.27010	0.2668184	81283.49			
35	6144	11.25848	0.2621115	80244.58			
36	6312	11.24769	0.2551912	79241.55			
37	6480	11.22495	0.2480252	77320.14			
38	6640	11.20611	0.2389135	75709.55			
39	6808	11.19458	0.2295610	74677.60	71155.27		0.04950206
40	6976	11.17881	0.2243968	73422.80	68910.15		0.06548613
41	7144	11.16832	0.2198768	72583.83	68641.46		0.05743435
42	7312	11.16162	0.2149444	72021.60	69128.86	0.01418406	0.04184576
43	7480	11.15189	0.2106033	71258.87	68710.26	0.00568505	0.03709216
44	7648	11.14633	0.2063289	70800.15	68604.46	0.00489860	0.03200519
45	7816	11.13770	0.2000467	70102.55	68076.63	0.00509632	0.02975953
46	7984	11.13094	0.1957814	69571.74	67586.29	0.00547879	0.02937652
47	8152	11.12573	0.1919512	69158.57	67244.02	0.00665359	0.02847181
48	8320	11.12120	0.1881049	68795.63	67003.35	0.00528200	0.02674917
49	8488	11.11544	0.1844696	68354.17	66705.45	0.00436309	0.02471652
50	8656	11.10968	0.1806594	67914.49	66360.53	0.00439863	0.02341703
51	8824	11.10162	0.1775599	67331.79	65843.33	0.00580354	0.02260610
52	8992	11.09496	0.1740044	66842.81	65274.44	0.00720154	0.02402735
53	9160	11.09089	0.1708101	66534.79	64819.96	0.00779881	0.02645538
54	9328	11.08587	0.1673328	66162.92	64389.59	0.00741404	0.02754064
55	9494	11.08197	0.1647577	65877.33	64038.80	0.00635000	0.02870968
56	9662	11.07871	0.1630199	65643.82	63779.25	0.00538013	0.02923489
57	9830	11.07811	0.1604922	65577.91	63689.31	0.00363703	0.02965336
58	9998	11.07372	0.1577884	65262.30	63548.85	0.00255619	0.02696280

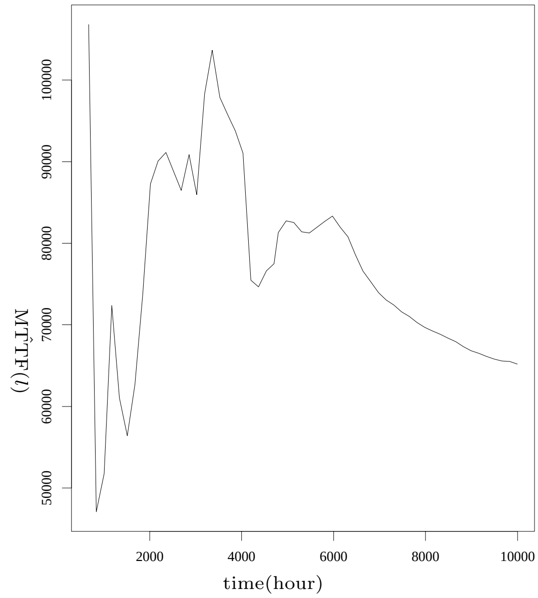

Figure 7. The trend of $\{\hat{\mathrm{MTTF}}(l)\}_{l=4}^{58}$.

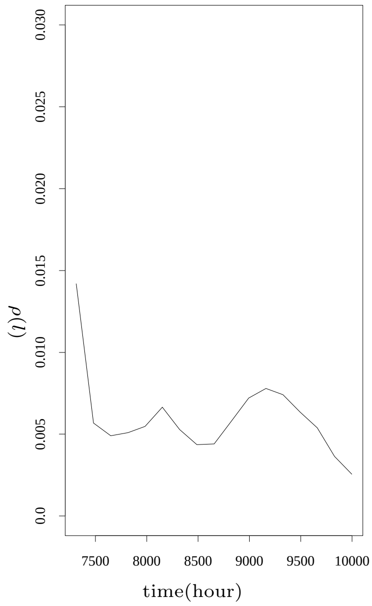

Figure 8. The trend of $\rho(l)$.

(C) **Investigate the limiting property of $\hat{\mathrm{MTTF}}(l)$ and determine an appropriate termination time.**

Observing Figure 7, it is seen that the $\hat{\mathrm{MTTF}}(l)$ curve changes drastically before $t_{34} = 5976$ hours. After t_{34}, there appears an exponentially decreasing pattern and the curve of $\hat{\mathrm{MTTF}}(k)$ levels off after $t_{42} = 7312$ hours. Hence, we use the following growth curve to describe $\{\hat{\mathrm{MTTF}}(k)\}_{k=34}^{l}$ for $l \geq 42$:

$$f_l(t) = a_l + e^{(b_l + c_l * t)}. \tag{9}$$

Obviously,

$$f_l(\infty) = \lim_{t \to \infty} f_l(t) = a_l. \tag{10}$$

Using the method of non-linear least squares, we obtain the asymptotic mean lifetime $\hat{a}_l = \hat{f}_l(\infty)$ and the value $\rho(l)$. The results are shown in Columns 6 and 7 of Table 1. Figure 8 also shows the plot of $\rho(l)$.

From Table 1, it is seen that the estimated asymptotic mean lifetime is near 63550 hours if the experiment is conducted up to 9998 hours. Besides, from Figure 8, we can obtain a reasonable estimate of MTTF within 1% error if the experiment time is conducted at least 7480 hours (which is about 11% of the product's MTTF).

5. A Simulation Study of the Proposed Rule

The proposed stopping rule is very intuitive. Due to the complexity of the model, it is not easy to provide analytical support for this rule. Instead, we conducted a simulation study to investigate the performance of this rule. Assume that the degradation path $LP_{ij}(t)$ satisfies equation (4), where $G_{ij}(t) = e^{-\alpha_{ij}t^{\beta_{ij}}}$ and $\epsilon_{ij}(t)$ follows $N(0, \sigma_\epsilon^2)$. In order to conduct a simulation study, we specify the joint distribution of $(\alpha_{ij}, \beta_{ij})$, $\forall 1 \le j \le n_i, 1 \le i \le 3$. Then, we use the termination time of 9998 hours as a benchmark to estimate these values. The LSEs $(\hat{\alpha_{ij}}, \hat{\beta_{ij}})$ of $(\alpha_{ij}, \beta_{ij})$ have the following approximate relationships:

$$\ln \hat{\beta_{ij}} = p_{i1} + p_{i2}\hat{\alpha_{ij}}, \quad \hat{\alpha_{ij}} \in (\alpha_{iL}, \alpha_{iR}),$$

where

$$(p_{i1}, p_{i2}) = \begin{cases} (-0.5914, -10.2371), & \text{for } i\text{=}1, \\ (-0.4898, -9.8540), & \text{for } i\text{=}2, \\ (-0.7635, -8.9020), & \text{for } i\text{=}3, \end{cases}$$

and

$$(\alpha_{iL}, \alpha_{iR}) = \begin{cases} (0.3127, 0.8065), & \text{for } i\text{=}1, \\ (0.4636, 0.6328), & \text{for } i\text{=}2, \\ (0.2927, 0.4490), & \text{for } i\text{=}3. \end{cases}$$

In addition, the R^2 values for these three models are 0.9974, 0.9807 and 0.9875, respectively. Thus, the following model is appropriate for describing the relationship between α_{ij} and β_{ij} :

$$\ln \beta_{ij} = p_{i1} + p_{i2}\alpha_{ij} + \eta_{ij}, \quad \alpha_{ij} \in (\alpha_{iL}, \alpha_{iR}), \tag{11}$$

where η_{ij} is $N(0, \sigma_\eta^2)$. From Section 4, we obtain $\sigma_\epsilon \approx 0.01$ and $\sigma_\eta \approx 0.2563$. Thus, we choose various combinations of $\sigma_\eta = (1 + \delta_1) * 0.2563$ and $\sigma_\epsilon = (1 + \delta_2) * 0.01$ (where $-5\% \le \delta_1 \le 5\%$ and $-20\% \le \delta_2 \le 20\%$) for the simulation study. Set $n_1 = 16$, $n_2 = 14$, and $n_3 = 18$, the sample sizes used in the example of Section 4. Now, the simulation procedure is summarized as follows:

For $1 \le j \le n_i$, $1 \le i \le 3$,

1. Generate $(\alpha_{ij}, \beta_{ij})$ from Equation (11).
2. Generate a degradation path $\{LP_{ij}(t_k)\}_{k=1}^{58}$ from Equation (4).
3. Use the procedure given in Section 3 to estimate $\{\tau_{ij}\}$ and the corresponding MTTF under normal use conditions.
4. Determine the termination time t_l^* and the corresponding asymptotic mean lifetime $\hat{f}_l(\infty)$ with a tolerance error $\varepsilon = 0.01$.

For each cell of (δ_1, δ_2), we conduct 100 trials and the following quantities are computed:

M_f : the sample mean of asymptotic mean lifetime $\{\hat{f}_l(\infty)\}$;

S_f : the standard error of asymptotic mean lifetime $\{\hat{f}_l(\infty)\}$;

ϕ_{t_l} : the sample mean of termination time $\{t_l^*\}$.

These values are given in Table 2.

From the results, it is seen that:

1. The value of M_f in each cell is very close to 63548.85 hours (the asymptotic mean lifetime which was obtained in Section 4). The largest absolute error is less than 3.5%. It shows the proposed stopping rule is quite robust to variation of δ_1 and δ_2.
2. The values of S_f are moderately affected by the values of δ_1 and δ_2. Thus, the values of σ_ϵ and σ_η have a moderate impact on the precision of the asymptotic mean lifetime.
3. The values of ϕ_{t_l} are less than 7480 hours (the termination time which was obtained in Section 4). It means the termination time of the simulation data is shorter than that of the real LED data. This may be due to the reason that the real LED data in Section 4 fluctuate more irregularly than our simulation data.

Table 2. The values of M_f, S_f, and ϕ_{t_l} under various combinations of $(1+\delta_1)*0.2563$ and $(1+\delta_2)*0.01$

δ_2 \ δ_1	-5%	0%	+5%
-20%	$M_f = 64779.40$ $S_f = 5262.235$ $\phi_{t_l} = 5181.46$	$M_f = 65771.43$ $S_f = 5684.418$ $\phi_{t_l} = 5335.48$	$M_f = 64770.12$ $S_f = 6254.110$ $\phi_{t_l} = 5423.00$
0%	$M_f = 65172.55$ $S_f = 5855.343$ $\phi_{t_l} = 5424.49$	$M_f = 64862.06$ $S_f = 6356.580$ $\phi_{t_l} = 5526.97$	$M_f = 64201.84$ $S_f = 6506.734$ $\phi_{t_l} = 5553.90$
+20%	$M_f = 64048.70$ $S_f = 6026.102$ $\phi_{t_l} = 5618.76$	$M_f = 63471.92$ $S_f = 6381.975$ $\phi_{t_l} = 5761.09$	$M_f = 64674.68$ $S_f = 6758.642$ $\phi_{t_l} = 5888.25$

6. Concluding Remarks

Determining an appropriate termination time for conducting an ADT is an important decision problem for experimenters. By modifying Tseng and Yu (1997), we propose an intuitive method to achieve the above goal. The method consists of using the traditional ALT and ML procedures to estimate the unknown parameters and MTTF of the device under a normal use condition. Finally, an appropriate termination time is determined by using the limiting property of the estimator of MTTF.

Finally, some concluding remarks about the method are as follows:

(1) The proposed method provides the decision maker an on-line real-time information about the product lifetime. It assesses the lifetime distribution of the product at each testing time. Thus, some important reliability measures, such as MTTF, hazard function and pth percentile under the normal use conditions can be easily obtained. Taking the LED data mentioned above, for example,

if the experiment is terminated at $t_{46} = 7984$ hours and the decision-maker wishes to estimate the 5th percentile of the product's lifetime, then, from Table 1, we have $\hat{\mu}_{46} = 11.13094$ and $\hat{\sigma}_{46} = 0.1957814$. Thus, the $5th$ percentile of the product's lifetime is 49459.44 hours.

(2) This method also provides the decision-maker with a simple criterion to measure the difference between the estimated MTTF and the asymptotic mean lifetime. It can be expressed as follows:

$$\rho^*(l) = \left| 1 - \frac{\hat{\text{MTTF}}(l)}{\hat{f}_l(\infty)} \right|. \tag{12}$$

Column 8 of Table 1 lists the values of $\rho^*(l)$. It shows that the differences are not significant (less than 3%) if the experiment is conducted over 7816 hours.

(3) Although there is no analytical support for the proposed stopping rule, we conducted a simulation study to assess its performance. The results in Table 2 indicate that the proposed rule is quite robust in estimating the asymptotic mean lifetime.

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