# CORRECTION: DERIVATIVE PRINCIPAL COMPONENTS FOR REPRESENTING THE TIME DYNAMICS OF LONGITUDINAL AND FUNCTIONAL DATA 

Xiongtao Dai, Hans-Georg Müller and Wenwen Tao<br>Iowa State University, University of California, Davis and Quora


#### Abstract

We provide corrections to Dai, X., Tao, W., Müller, H.G. (2018). Derivative principal components for representing the time dynamics of longitudinal and functional data in the rates of convergence of the local polynomial estimates of the derivative mean and the covariance functions.


Key words and phrases: Best linear unbiased prediction, derivatives, empirical dynamics, functional principal component analysis, growth curves.

We are grateful to Dr. Hassan Sharghi (Shahid Beheshti University) for pointing out some errors in this paper, for which we provide corrections in the following.

The definitions of $a_{n}$ in 4.1 -4.2 missed some factors $h_{\mu}$ and $h_{G}$ in the denominators. The corrected version is as follows:

$$
\begin{align*}
& a_{n 1}=h_{\mu}^{2}+\sqrt{\frac{\log (n)}{n h_{\mu}^{3}}}, \quad b_{n 1}=h_{G}^{2}+\sqrt{\frac{\log (n)}{n h_{G}^{6}}},  \tag{4.1}\\
& a_{n 2}=h_{\mu}^{2}+\sqrt{\left(1+\frac{1}{m h_{\mu}}\right) \frac{\log (n)}{n h_{\mu}^{2}}}, \quad b_{n 2}=h_{G}^{2}+\left(1+\frac{1}{m h_{G}}\right) \sqrt{\frac{\log (n)}{n h_{G}^{4}}} . \tag{4.2}
\end{align*}
$$

As a result, conditions (A5) and (A7) change
(A5) $h_{\mu} \rightarrow 0$ and $\log (n) \sum_{i=1}^{n} N_{i} w_{i}^{2} / h_{\mu}^{3} \rightarrow 0, \log (n) \sum_{i=1}^{n} N_{i}\left(N_{i}-1\right) w_{i}^{2} / h_{\mu}^{2} \rightarrow 0$.
(A7) $h_{G} \rightarrow 0, \log (n) \sum_{i=1}^{n} N_{i}\left(N_{i}-1\right) v_{i}^{2} / h_{G}^{6} \rightarrow 0, \log (n) \sum_{i=1}^{n} N_{i}\left(N_{i}-1\right)\left(N_{i}-\right.$ $2)\left(N_{i}-3\right) v_{i}^{2} / h_{G}^{4} \rightarrow 0$.

In the first paragraph after Theorem 1, the optimal rate of convergence for $\hat{G}^{(1,1)}$ and $\hat{\phi}_{k, 1}$ should be $O\left((\log (n) / n)^{1 / 5}\right)$, almost surely, achieved, for example, if $h_{\mu} \asymp(\log (n) / n)^{1 / 7}, h_{G} \asymp(\log (n) / n)^{1 / 10}, \alpha>14 / 5$, and $\beta>10 / 3$, as in (A6)

[^0]and (A8). In the dense case, the root- $n$ rate can no longer be achieved because of the variance term $b_{n 2}$.

In the proof of Theorem 1, for $h_{\mu} \hat{\mu}^{(1)}(t)=h_{\mu} \hat{\alpha}_{1}=|\mathbf{S}|^{-1}\left(C_{12} R_{0}+C_{22} R_{1}+\right.$ $C_{32} R_{2}$ ), the cofactors of $[\mathbf{S}]_{1,2},[\mathbf{S}]_{2,2}$, and $[\mathbf{S}]_{3,2}$ should instead be defined as

$$
C_{12}=-\left|\begin{array}{ll}
S_{1} & S_{3} \\
S_{2} & S_{4}
\end{array}\right|, \quad C_{22}=\left|\begin{array}{ll}
S_{0} & S_{2} \\
S_{2} & S_{4}
\end{array}\right|, \quad C_{23}=-\left|\begin{array}{ll}
S_{0} S_{2} \\
S_{1} & S_{3}
\end{array}\right|,
$$

respectively. The second-order terms in the Taylor expansion of $h_{\mu}\left(\hat{\alpha}_{1}-\mu^{(1)}(t)\right)$ should be divided by an additional factor of 2 . The asymptotic variance terms in the last equation on page 1604 and the first expression on page 1605 should be divided by an additional factor of $h_{\mu}$.

Xiongtao Dai
Department of Statistics, Iowa State University, Ames, IA 50011, USA.
E-mail: xdai@iastate.edu
Hans-Georg Müller
Department of Statistics, University of California, Davis, CA 95616, USA.
E-mail: hgmueller@ucdavis.edu
Wenwen Tao
Quora, Mountain View, CA 94041, USA.
E-mail: wenwen@quora.com
(Received May 2020; accepted June 2020)


[^0]:    Corresponding author: Xiongtao Dai, Department of Statistics, Iowa State University, Ames, IA 50011, USA. E-mail: xdai@iastate.edu

