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# ON SEMIPARAMETRIC INSTRUMENTAL VARIABLE ESTIMATION OF AVERAGE TREATMENT EFFECTS THROUGH DATA FUSION

BaoLuo Sun and Wang Miao

*National University of Singapore and Peking University*

*Abstract:* Suppose one is interested in estimating causal effects, in the presence of potentially unmeasured confounding using a valid instrumental variable. This study investigates the problem of making inferences about the average treatment effect when data are fused from two separate sources. Here, one data source contains information on the treatment and the other contains information on the outcome, while values for the instrument and a vector of baseline covariates are recorded in both. We provide a general set of sufficient conditions under which the average treatment effect is nonparametrically identified from the observed data law induced by data fusion, even when the data are from two heterogeneous populations, and derive the efficiency bound for estimating this causal parameter. For inference, we develop both parametric and semiparametric methods, including a multiply robust and locally efficient estimator that is consistent, even under partial misspecification of the observed data model. We illustrate the methods using simulations and an application on public housing projects.

*Key words and phrases:* Multiple robustness, two-sample inference, unmeasured confounding.

## 1. Introduction

The instrumental variable method is widely used in the health and social sciences for the identification and estimation of causal effects in the presence of potentially unmeasured confounding (Bowden and Turkington (1990); Robins (1994); Angrist, Imbens and Rubin (1996); Greenland (2000); Wooldridge (2010); Hernán and Robins (2006); Didelez, Meng and Sheehan (2010)). A valid instrumental variable $Z$ is a pre-exposure variable that (a) is associated with treatment $D$, (b) is independent of any unmeasured confounder $U$ of the exposure-outcome association, and (c) has no direct causal effect on the outcome $Y$, conditional on a set of measured baseline covariates $X$. The instrumental variable approach has a longstanding tradition in econometrics, going back to the original works

Corresponding author: BaoLuo Sun, Department of Statistics and Applied Probability, National University of Singapore, Singapore 119077. E-mail: stasb@nus.edu.sg.

of Wright (1928) and Goldberger (1972) in the context of linear structural modeling; see Wooldridge (2010), Clarke and Windmeijer (2012), Baiocchi, Cheng and Small (2014), and Swanson et al. (2018) for more recent reviews. Under a correct specification of the linear structural equation models, and assuming an absence of baseline covariates, the conventional instrumental variable estimand of the average treatment effect is the population moment ratio $\mathrm{cov}(Z, Y)/\mathrm{cov}(Z, D)$.

However, in many empirical scenarios, only information on $(Y, Z, X)$ is available from the primary population of interest. Angrist and Krueger (1992) and Arellano and Meghir (1992) showed that the two sets of moments can be estimated from two separate sources by leveraging information on $(D, Z, X)$ from an auxiliary population, a method known as two-sample instrumental variable estimation. Furthermore, Klevmarken (1982) and Angrist and Krueger (1995) introduced a two-sample two-stage least squares estimation with a first-stage regression for the treatment model based on the auxiliary sample; see Ridder and Moffitt (2007) and Angrist and Pischke (2008) for reviews. This methodology has since been widely applied in econometrics and social sciences (Inoue and Solon (2010)), and more recently in two-sample Mendelian randomization studies to estimate causal relationships using genetic factors as instruments (Pierce and Burgess (2013); Gamazon et al. (2015); Lawlor (2016); Zhao et al. (2018, 2019)). As noted by Zhao et al. (2019), the aforementioned methods typically assume that the auxiliary data are also sampled from the primary population. In addition, linear structural models impose strong homogeneity assumptions on the treatment effect. Thus, a robust analytic framework for the instrumental identification and estimation of causal effects under data fusion remains of keen interest in observational studies. Graham, Pinto and Egel (2016) identified the two-sample instrumental variable problem as a specific example of a general class of data combination models, and extended the semiparametric efficiency theory of Hahn (1998) and Chen, Hong and Tarozzi (2008) to this class of models. Recent works have also made significant strides toward relaxing the assumptions for identifying causal effects under data fusion (Pacini and Windmeijer (2016); Choi, Gu and Shen (2018); Zhao et al. (2018); Buchinsky, Li and Liao (2018); Shu and Tan (2019); Zhao et al. (2019); Pacini (2019)).

When full data on $L = (Y, D, Z, X)$ are available from the primary population of interest, Robins (1994), Imbens and Angrist (1994), Angrist, Imbens and Rubin (1996), and Heckman (1997) formalized the instrumental variable approach under the potential outcome framework (Neyman (1923); Rubin (1974)), which allows one to nonparametrically define the causal estimands of interest. In this paper, we propose novel assumptions under which the average treatment effect of $D$ on $Y$ in the primary population of interest can be uniquely and nonparametrically identified from the observed data law induced by data fusion. To estimate this identifying statistical functional, we develop a suite of parametric and semiparametric estimators, including a multiply robust and locally efficient one that remains consistent, even if the observed data model is partially misspecified. We compare the proposed estimators both in theory and using simulations, and investigate the efficiency and robustness of existing estimators.

## 2. Model

Suppose we are interested in estimating the average treatment effect of a binary treatment $D$ on outcome $Y$ in a primary population of interest, which is confounded by measured covariates $X$ and unmeasured covariates $U$, using a binary instrumental variable $Z$. However, we only observe $\{(Y_i, Z_i, X_i)^T,\ i = 1, \ldots, n_p\}$ from this population. As a remedy, suppose an additional sample $\{(D_i, Z_i, X_i)^T,\ i = 1, \ldots, n_a\}$ is available from an auxiliary population, possibly different from the primary population. Similarly to Graham, Pinto and Egel (2016) and Shu and Tan (2019), we assume the following about the data source mechanism.

**Assumption 1** (Binomial sampling)**.** *The combined set of $n = n_p + n_a$ units are independent, and are drawn from either the primary population with a fixed probability $Q_0 \in (0, 1)$ or the auxiliary population with probability $1 - Q_0$.*

Let $R_i$ be an indicator variable, equal to one if the $i$th unit is drawn from the primary population, and zero otherwise. By assumption 1, the combined set of observed data $\{O_i = (R_i, R_iY_i, (1 - R_i)D_i, Z_i, X_i)^T,\ i = 1, \ldots, n\}$ can be treated as a random sample from a synthetic merged population. Let $F(O)$ denote the distribution of $O$, with density with respect to some dominating measure given by

$$\begin{aligned} f(O) = q^{\dagger R}(1 - q^\dagger)^{1-R} f(V|R = 1)^R f(V|R = 0)^{1-R} \times \\ f(Y|V, R = 1)^R f(D|V, R = 0)^{1-R}, \end{aligned} \tag{2.1}$$

where $V = (Z, X)$ and $q^\dagger = \mathrm{pr}(R = 1)$. Let $E(\cdot)$ denote the expectation taken with respect to this mixture distribution, and let $\pi(z, x) = E(R|Z = z, X = x)$. By Bayes' rule,

$$f(z, x|R = 1) = f(z, x|R = 0) \left\{ \frac{1 - q^\dagger}{q^\dagger} \frac{\pi(z, x)}{1 - \pi(z, x)} \right\}.$$

Let $Y_d$, for $d \in \{0,1\}$, denote the potential outcome that would be observed if $D$ were set to $d$, which is related to the observed data via the consistency assumption $Y_d = Y$ if $D = d$. To achieve identification of $\Delta \equiv E(Y_1 - Y_0 | R = 1)$ based on the observed data law $F(O)$ induced by data fusion, we make several assumptions about the primary and auxiliary populations, discussed below.

### 2.1. Primary population

Suppose $Z$ is a valid binary instrument that satisfies the following assumptions (Didelez and Sheehan (2007); Pearl (2009); Clarke and Windmeijer (2012)):

**Assumption 2** (Instrument Relevance). $Z \not\perp\!\!\!\perp D | X, R = 1$.

**Assumption 3** (Instrument Independence). $Z \perp\!\!\!\perp U | X, R = 1$.

**Assumption 4** (Exclusion Restriction). $Y \perp\!\!\!\perp Z | D, X, U, R = 1$.

Here, $A \perp\!\!\!\perp B | C$ indicates conditional independence of $A$ and $B$, given $C$ (Dawid (1979)). Instrument relevance ensures that $Z$ is a correlate of the exposure, even after conditioning on $X$, and instrument independence states that $Z$ is independent of all unmeasured confounders of the exposure-outcome association. Exclusion restriction formalizes the assumption of no direct effect of $Z$ on $Y$ not mediated by $D$. Furthermore, the assumption of no unmeasured confounding given $(X, U)$ can be stated as follows.

**Assumption 5** (Latent Ignorability). $Y_d \perp\!\!\!\perp D | X, U, R = 1$, *for* $d \in \{0,1\}$ *(Robins (*1994*)).*

Assumptions 2–5 may be known to hold at the design stage when the investigator controls the treatment allocation, conditional on baseline covariates, in double blind randomized trials. In observational studies, the potential instrumental variable may be viewed as being randomized through some natural or quasi-experiment within levels of the observed covariates (Hernán and Robins (2006)), although these assumptions are typically untestable without further conditions. The exclusion restriction assumption 4 implies the following semiparametric structural models:

$$\begin{aligned} E(D \mid Z, X, U, R = 1) &= g_0(X, U) + g_1(X, U)Z \\ E(Y \mid D, Z, X, U, R = 1) &= h_0(X, U) + h_1(X, U)D, \end{aligned} \tag{2.2}$$

where for $k \in \{0,1\}$, $g_k(\cdot)$ and $h_k(\cdot)$ are arbitrary square-integrable functions of $(X, U)$ that are only restricted by natural features of the model, for example, such that the exposure mean is bounded between zero and one. Note that for binary $(Z, D)$, model (2.2) is saturated, because there are no restrictions on the corresponding data laws $f(D|Z, X, U, R = 1)$ and $f(Y|D, Z, X, U, R = 1)$, except for the implications of assumption 4. Under assumptions 4 and 5, $h_1(x, u) = E(Y_1 - Y_0|X = x, U = u, R = 1)$ encodes the conditional average treatment effect within levels of $(X, U)$; hence, $\Delta = E\{h_1(X, U)|R = 1\}$. The linear structural equation model (Wright (1928); Goldberger (1972))

$$\begin{aligned} E(D \mid Z, X, U, R = 1) &= \theta_0 + \theta_1 X + \theta_2 U + \theta_3 Z \\ E(Y \mid D, Z, X, U, R = 1) &= \beta_0 + \beta_1 X + \beta_2 U + \Delta D \end{aligned} \tag{2.3}$$

is a special case of (2.2), where the function $h_1(X, U)$ is reduced to the scalar parameter of interest $\Delta$ encoding the homogeneous average treatment effect within levels of $(X, U)$.

Even when full data on $L = (Y, D, Z, X)$ are available from the primary population, it is well known that while a valid instrumental variable satisfying assumptions 2–5 suffices to obtain a valid statistical test of the sharp null hypothesis of no individual causal effect, the population average treatment effect $\Delta$ is itself not uniquely identified from the law $F(L|R = 1)$ (Balke and Pearl (1997)). Adding a further monotonicity assumption about the effect of $Z$ on $D$, Angrist, Imbens and Rubin (1996) showed that the local average treatment effect (LATE) among compliers can be identified nonparametrically. This framework has been further generalized in recent years by Abadie, Angrist and Imbens (2002), Abadie (2003), Carneiro, Heckman and Vytlacil (2003), Tan (2010a), Ogburn, Rotnitzky and Robins (2015), and Kennedy, Lorch and Small (2019). Zhao et al. (2019) discussed the identification of LATE in two-sample instrumental variable analyses. However, because the population of compliers is itself nonidentifiable in general, $\Delta$ is arguably still a causal parameter of interest in many observational studies (Robins and Greenland (1996); Imbens (2010)). Wang and Tchetgen Tchetgen (2018) proved the identifiability of $\Delta$ from the law $F(L|R = 1)$ under the additional assumption

$$g_1(X, U) = g_1(X) \quad \text{or} \quad h_1(X, U) = h_1(X), \qquad \text{almost surely;} \tag{2.4}$$

that is, at least one of these effects is not allowed to vary with $U$. We show that $\Delta$ can be identified from $F(O)$, provided that $X$ is sufficiently rich that the effect of exposure on the outcome is uncorrelated with the effect of the instrument on the exposure conditional on $X$ (Cui and Tchetgen Tchetgen (2019)). This can be achieved, even if $X$ does not include all confounders of the effect of $D$ on $Y$.

**Assumption 6** (Orthogonality). *$cov\{g_1(X,U), h_1(X,U)|X, R=1\} = 0$, almost surely.*

Assumption 6 may hold under certain data-generating mechanisms, even if (2.4) does not, and is guaranteed to hold under the sharp causal null effect. In addition, we require that every unit within levels of the observed covariates have some chance of receiving each level $z \in \{0,1\}$ of the instrument.

**Assumption 7** (Positivity). *$0 < pr(Z=1|X, R=1) < 1$, almost surely.*

### 2.2. Auxiliary population

We make the following assumptions about the auxiliary population.

**Assumption 8** (Support overlap). *$0 < \pi(Z,X) < 1$, almost surely.*

**Assumption 9** (Propensity score equality). *$pr(D=1|Z,X,R=0) = pr(D=1|Z,X,R=1)$, almost surely.*

Assumption 8 ensures that the support of the common variables $(Z,X)$ in the primary population is contained within that in the auxiliary population and, together with assumption 9, allows us to identify the treatment propensity score $\tau(z,x) = \mathrm{pr}(D=1|Z=z, X=x, R=1)$ based on $F(O)$. Assumption 9 requires only predictive invariance for the treatment between the two heterogeneous populations, and we do not require the stronger condition of "structural invariance" (e.g., assumptions 3–6 also hold in the auxiliary population), which is related to the notions of "invariant prediction" (Peters, Bühlmann and Meinshausen (2016)), "autonomy" (Haavelmo (1944)), and "stability" (Pearl (2009)) as discussed in Zhao et al. (2019).

### 2.3. Nonparametric identification

We show that under assumptions 1–9, $\Delta$ is a functional on the nonparametric observed data statistical model $\mathcal{M}_{\mathrm{np}} = \{F(O) : F(O) \text{ unrestricted}\}$ of all regular laws $F(O)$ that satisfy the positivity and support overlap assumptions. In the following, let $\lambda(z|x) = \mathrm{pr}(Z=z|X=x, R=1)$ denote the probability density or mass function of $Z$ given $X$ in the primary population.

**Theorem 1.** *Under assumptions 1–9,*

$$\Delta = E\left\{\frac{R}{q^\dagger}\frac{(-1)^{1-Z}}{\lambda(Z|X)}\frac{Y}{[\tau(1,X)-\tau(0,X)]}\right\}. \tag{2.5}$$

**Remark 1.** When $Y$ is continuous and $D$ and $Z$ are discrete of finite domain, the canonical instrumental variable assumptions 3 and 4 impose no constraints on the law $F(L|R=1)$ (Bonet (2001)). In addition, assumption 9 is akin to coarsening at random, which leaves the observed data law $F(O)$ unrestricted (Robins (1997); van der Laan and Robins (2003)). When $Y$ is also discrete, assumptions 3 and 4 impose inequality constraints that do not restrict the parameter space of $F(L|R=1)$ locally if the true observed data law lies in the interior of the space defined by these constraints (Wang, Robins and Richardson (2017); Wang and Tchetgen Tchetgen (2018)).

**Remark 2.** Although nuisance parameters such as $\{\lambda(\cdot), \tau(\cdot)\}$ can, in principle, be estimated nonparametrically using methods such as sieve estimation (Hahn (1998); Hirano, Imbens and Ridder (2003); Chen, Hong and Tarozzi (2008)), we focus on parametric working models, owing to the curse of dimensionality when $X$ is of moderate or high dimension (Robins and Ritov (1997)). Because one cannot be confident that any of these models is correctly specified, we also propose an estimator of $\Delta$ that is robust to misspecifications of these models.

**Remark 3.** The form of the identification formula (2.5) in Theorem 1 suggests that $\Delta$ may be identified as long as one has access to consistent estimators of the propensity score $\tau(z,x)$ in the primary population. The utility of the sample from the auxiliary population lies in the estimation of $\tau(z,x)$ under Assumption 9, which is not testable because $D$ is not observed in the sample from the primary population. On the other hand, $\Delta$ may be identified without the need for an auxiliary sample if the propensity score is known by design in the primary population.

## 3. Estimation

### 3.1. Maximum likelihood estimation

Let $\hat{E}(\cdot)$ denote the empirical mean operator $\hat{E}\{h(O)\} = n^{-1}\sum_{i=1}^{n} h(O_i)$, and let $(\hat{\alpha}, \hat{\psi}, \hat{\xi}, \hat{\theta})$ denote the maximum likelihood estimators of $(\alpha, \psi, \xi, \theta)$ that index the parametric models $\pi(z,x;\alpha)$, $\lambda(z|x;\psi)$, $\tau(z,x;\xi)$, and $f(y|z,x,R=1;\theta) = f(Y=y|Z=z,X=x,R=1;\theta)$ for the outcome conditional density specified by the analyst. Note that under assumption 9, $\tau(z,x) = \mathrm{pr}(D=1|Z=z,X=x,R=0)$ so that inferences on $\xi$ can be based on the auxiliary sample. By taking an iterated expectation of (2.5) with respect to $(Z,X)$, the plug-in estimator of $\Delta$ is

$$\hat{\Delta}_{\mathrm{mle}} = \hat{E}\left\{\frac{1}{\hat{q}}\frac{(-1)^{1-Z}}{\lambda(Z|X;\hat{\psi})}\frac{\pi(Z,X;\hat{\alpha})E(Y|Z,X,R=1;\hat{\theta})}{\tau(1,X;\hat{\xi})-\tau(0,X;\hat{\xi})}\right\}, \tag{3.1}$$

where the distribution of $(Z, X)$ is estimated using its empirical distribution and $\hat{q} = \hat{E}(R)$. It is clear that the consistency of $\hat{\Delta}_{\text{mle}}$ relies on correct specifications of the models $\pi(z, x; \alpha)$, $\lambda(z|x; \psi)$, $\tau(z, x; \xi)$, and $f(y|z, x, R = 1; \theta)$. In the following, we propose several semiparametric estimators of $\Delta$ that do not require these models to be fully specified. We proceed by first noting the following decomposition of the outcome conditional mean model.

**Lemma 1.** *Under assumptions* 2–6*,*

$$E(Y|Z = z, X = x, R = 1) = \mathcal{H}(x)\tau(z, x) + \omega(x), \tag{3.2}$$

*where* $\omega(x) \equiv$ *cov*$[g_1(X, U), h_1(X, U) |X = x, R = 1] + E[h_0(X, U)|X = x, R = 1]$ *and* $\mathcal{H}(x) \equiv E[h_1(U, X)|X = x, R = 1]$ *is the treatment effect curve conditional on the observed covariates. Therefore,* $\Delta = E\{\mathcal{H}(X)|R = 1\}$*.*

### 3.2. Semiparametric estimation

Consider the following submodels of $\mathcal{M}_{\text{np}}$, in which smooth parametric models (indexed by finite-dimensional parameters) for certain components of the observed data law $F(O)$ are correctly specified:

**Definition 1.**

- $\mathcal{M}_1$: The models $\lambda(z|x; \psi)$ and $\tau(z, x; \xi)$ are correctly specified, such that $\lambda(z|x; \psi^\dagger) = \lambda(z|x)$ and $\tau(z, x; \xi^\dagger) = \tau(z, x)$, for some unknown values $(\psi^\dagger, \xi^\dagger)$;
- $\mathcal{M}_2$: The models $\mathcal{H}(x; \gamma)$, $\omega(x; \eta)$ and $\tau(z, x; \xi)$ are correctly specified, such that $\mathcal{H}(x; \gamma^\dagger) = \mathcal{H}(x)$, $\omega(x; \eta^\dagger) = \omega(x)$ and $\tau(z, x; \xi^\dagger) = \tau(z, x)$, for some unknown values $(\gamma^\dagger, \eta^\dagger, \xi^\dagger)$;
- $\mathcal{M}_3$: The models $\mathcal{H}(x; \gamma)$, $\omega(x; \eta)$ and $\pi(z, x; \alpha)$ are correctly specified, such that $\mathcal{H}(x; \gamma^\dagger) = \mathcal{H}(x)$, $\omega(x; \eta^\dagger) = \omega(x)$ and $\pi(z, x; \alpha^\dagger) = \pi(z, x)$, for some unknown values $(\gamma^\dagger, \eta^\dagger, \alpha^\dagger)$.

We propose semiparametric estimators for $\Delta$ that are consistent and asymptotically normal in each of the above submodels. Our first estimator $\hat{\Delta}_1$ of $\Delta$ is motivated by identification formula (2.5), which does not require a specification of an outcome model for $f(y|z, x, R = 1)$, and solves

$$\begin{aligned} 0 &= \hat{E}\left\{\mu_1(O; \Delta, \hat{\psi}, \hat{\xi}, \hat{q})\right\} \\ &\equiv \hat{E}\left\{\frac{R}{\hat{q}} \frac{(-1)^{1-Z}}{\lambda(Z|X; \hat{\psi})} \frac{Y}{[\tau(1, X; \hat{\xi}) - \tau(0, X; \hat{\xi})]} - \Delta\right\}. \end{aligned} \tag{3.3}$$

**Remark 4.** The models for $\{\lambda(\cdot), \tau(\cdot)\}$ can be specified and estimated without access to the outcome data. Estimating $\Delta$ using $\hat{\Delta}_1$ could therefore be considered part of a more objective analysis design in the sense that it mitigates the potential for "data-dredging" exercises when the outcome model is fully specified (Rubin (2007)).

We propose two additional estimators of $\Delta$, which do not require a model for $\lambda(\cdot)$, but instead posit models $\mathcal{H}(X;\gamma)$ and $\omega(X;\eta)$ for the components of the outcome conditional mean (3.2). Consider the semiparametric estimators $\hat{\Delta}_2$ and $\hat{\Delta}_3$ that solve

$$0 = \hat{E}\left\{\mu_j(O;\Delta,\hat{\gamma}_j,\hat{q})\right\} \equiv \hat{E}\left\{\frac{R}{\hat{q}}[\mathcal{H}(X;\hat{\gamma}_j) - \Delta]\right\}, \tag{3.4}$$

for $j = 1, 2$, respectively, where the estimators $\hat{\gamma}_2$ and $\hat{\gamma}_3$ are constructed in such a way such that they are consistent in the submodels $\mathcal{M}_2$ and $\mathcal{M}_3$, respectively, as follows. Let $v(X)$ and $w(X)$ be analyst-specified vector functions of the same dimensions as $\gamma$ and $\eta$, respectively, for example $\{v(X), w(X)\} = \{\partial\mathcal{H}(X;\gamma)/\partial\gamma, \partial\omega(X;\eta)/\partial\eta\}$, and let $\mathcal{G}_{v,w}(X,Z) = \{v^T(X)Z, w^T(X)\}^T$, where $A^T$ denotes the transpose of $A$. Then, let $(\hat{\gamma}_2, \hat{\eta}_2)$ be the joint solution to the estimating equation

$$\begin{aligned} 0 = \hat{E}\big\{\mathcal{G}_{v,w}(X,Z)\big\{&R[Y - \mathcal{H}(X;\gamma)\tau(Z,X;\hat{\xi}) - \omega(X;\eta)] \\ &-(1-R)\mathcal{H}(X;\gamma)[D - \tau(Z,X;\hat{\xi})]\big\}\big\}, \end{aligned}$$

and let $(\hat{\gamma}_3, \hat{\eta}_3)$ jointly solve

$$0 = \hat{E}\left\{\mathcal{G}_{v,w}(X,Z)\left\{R[Y - \omega(X;\eta)] - \frac{(1-R)\pi(Z,X;\hat{\alpha})}{1-\pi(Z,X;\hat{\alpha})}\mathcal{H}(X;\gamma)D\right\}\right\}.$$

**Lemma 2.** *Under standard regularity conditions (Newey and McFadden (*1994*)), the estimators* $\hat{\Delta}_1$*,* $\hat{\Delta}_2$*, and* $\hat{\Delta}_3$ *are consistent and asymptotically normal in submodels* $\mathcal{M}_1$*,* $\mathcal{M}_2$*, and* $\mathcal{M}_3$*, respectively.*

**Remark 5.** To ensure that the proposed estimators of $\Delta$ lie between $-1$ and $1$ in the case of binary $Y$, following Wang and Tchetgen Tchetgen (2018), we specify a model such as

$$\mathcal{H}(X;\gamma) = \tanh(\gamma^T X) = \frac{\exp(2\gamma^T X) - 1}{\exp(2\gamma^T X) + 1},$$

which guarantees that $\mathcal{H}(X;\gamma) \in [-1,1]$. In addition, instead of the decomposition (3.2) for continuous $Y$, Wang and Tchetgen Tchetgen (2018) provide a variation-independent decomposition of the components in the likelihood $\{\mathrm{pr}(Y = 1|Z, X, R = 1), \mathrm{pr}(D = 1|Z, X, R = 1)\}$ for binary $Y$; we adopt a similar estimation strategy for these components.

### 3.3. Multiply robust estimation

To motivate the multiply robust estimator, we consider the efficient estimation of $\Delta$ in $\mathcal{M}_{\mathrm{np}}$. Any regular and asymptotically linear estimator $\hat{\Delta}$ has an associated influence function $\mu(O; \Delta)$, such that $\hat{\Delta} - \Delta = \hat{E}\{\mu(O; \Delta)\} + o_p(n^{-1/2})$ (Bickel et al. (1993)). Therefore, it suffices to identify $\mu(O; \Delta)$ with the lowest variance, which is the efficient influence function.

**Theorem 2.** *The efficient influence function for $\Delta$ in $\mathcal{M}_{np}$ is*

$$\begin{aligned}&\mu_{\mathrm{eff}}(O;\Delta) = \\ &\frac{(-1)^{1-Z}\left\{\begin{array}{c}(R/q^{\dagger})[Y - \mathcal{H}(X)\tau(Z,X) - \omega(X)] \\ -((1-R)/q^{\dagger})(\pi(Z,X)/(1-\pi(Z,X)))\mathcal{H}(X)[D - \tau(Z,X)]\end{array}\right\}}{\lambda(Z|X)[\tau(1,X) - \tau(0,X)]} \\ &+ \frac{R}{q^{\dagger}}\{\mathcal{H}(X) - \Delta\},\end{aligned} \tag{3.5}$$

*so that the semiparametric efficiency bound for estimating $\Delta$ in $\mathcal{M}_{np}$ is $E\{\mu^2_{\mathrm{eff}}(O; \Delta)\}$.*

We use $\mu_{\mathrm{eff}}(\cdot)$ as an estimating function and substitute in estimates of the nuisance parameters to estimate the causal effect $\Delta$. This method of constructing estimating equations from influence functions is used widely; see, for example, Bang and Robins (2005), Tan (2006b), Tchetgen Tchetgen, Robins and Rotnitzky (2009), Sun et al. (2018), Sun and Tchetgen Tchetgen (2018), and Wang and Tchetgen Tchetgen (2018). Consider $(\tilde{\gamma}, \tilde{\eta})$, which jointly solve

$$\begin{aligned}\mathbf{0} = \hat{E}\Bigg\{\mathcal{G}_{v,w}(X,Z)\Bigg\{&R[Y - \mathcal{H}(X;\gamma)\tau(Z,X;\hat{\xi}) - \omega(X;\eta)] \\ &- \frac{(1-R)\pi(Z,X;\hat{\alpha})}{1-\pi(Z,X;\hat{\alpha})}\mathcal{H}(X;\gamma)[D - \tau(Z,X;\hat{\xi})]\Bigg\}\Bigg\}.\end{aligned} \tag{3.6}$$

Note that the estimator $\tilde{\gamma}$ is doubly robust in the sense that it is consistent for $\gamma^{\dagger}$ in the model $\mathcal{M}_2 \cup \mathcal{M}_3$, which is necessary for the multiply robust result stated below.

**Lemma 3.** *Under standard regularity conditions (Newey and McFadden (*1994*)),* *the estimator $\hat{\Delta}_{\text{mul}}$ that solves*

$$0 = \hat{E}\left\{\mu_{\text{eff}}(O; \Delta, \tilde{\eta}, \tilde{\gamma}, \hat{\psi}, \hat{\xi}, \hat{\alpha}, \hat{q})\right\} \tag{3.7}$$

*is consistent and asymptotically normal in the union model $\mathcal{M}_{\text{union}} = \cup_{j=1}^{3}\mathcal{M}_j$ (multiply robust). Moreover, $\hat{\Delta}_{\text{mul}}$ attains the semiparametric efficiency bound in $\mathcal{M}_{np}$ (and, following the general results of Robins and Rotnitzky (*2001*), also in $\mathcal{M}_{\text{union}}$) at the intersection submodel $\{\cap_{j=1}^{3}\mathcal{M}_j\}$, where all working models are correctly specified (locally efficient).*

The asymptotic variance formula of each estimator described in this section follows from standard M-estimation theory (Newey and McFadden (1994)). For inferences based on the proposed semiparametric estimators of $\Delta$ in both the simulation study and the application (sections 5 and 6, respectively), the consistent estimation of the asymptotic variance is described in the Supplementary Material.

## 4. Comparison with Existing Estimators

Suppose that $E(U|Z=z, X=x, R=1) = E(U|X=x, R=1)$ is linear in $x$; then, the linear structural models (2.3) yield the observed data models

$$\begin{aligned}\tau_{\text{linear}}(Z, X; \xi) &= \xi^T(1, Z, X)^T;\\ \omega_{\text{linear}}(X; \eta) &= \eta^T(1, X)^T;\\ E(Y \mid Z, X, R=1) &= \Delta\tau_{\text{linear}}(Z, X; \xi) + \omega_{\text{linear}}(X; \eta).\end{aligned}$$

We also have that $\mathcal{H}(X)$ is indexed by the scalar parameter of interest $\Delta$. Using the notation in section 3, it can be shown that the two-sample instrumental variable estimator (Inoue and Solon (2010)) $(\hat{\Delta}_{\text{tsiv}}, \hat{\eta}_{\text{tsiv}})$ solves

$$0 = \hat{E}\left\{\mathcal{G}_{v,w}(X, Z)\left\{R[Y - \omega_{\text{linear}}(X; \eta)] - \frac{(1-R)\hat{q}}{1-\hat{q}}\Delta D\right\}\right\}.$$

Inferences based on the two-sample instrumental variable estimator can be viewed as special instances of inferences obtained under a particular specification of submodel $\mathcal{M}_3$, with the above parametric models for $\{\mathcal{H}(\cdot), \omega(\cdot)\}$ and $\pi(z, x; \alpha) = q$, where $q \in \mathbb{R}$; for example, the marginal distribution of $(Z, X)$ is the same in the primary and auxiliary populations. Therefore, $\hat{\Delta}_{\text{tsiv}}$ will fail to be consistent for $\Delta$ if any of the parametric models in $\mathcal{M}_3$ is incorrectly specified. Furthermore, note that the two-sample two-stage least squares estimator $(\hat{\Delta}_{\text{ts2sls}}, \hat{\eta}_{\text{ts2sls}})$ solves

$$
\begin{aligned}
0 = \hat{E}\bigg\{\mathcal{G}_{v,w}(X,Z)\bigg\{ & R[Y - \Delta\tau_{\text{linear}}(Z,X;\hat{\xi}) - \omega_{\text{linear}}(X;\eta)] \\
& -\frac{(1-R)\hat{q}}{1-\hat{q}}\Delta[D - \tau_{\text{linear}}(Z,X;\hat{\xi})]\bigg\}\bigg\},
\end{aligned}
$$

which is a special case of the doubly robust estimating equation (3.6). It follows that $\hat{\Delta}_{\text{ts2sls}}$ is consistent for $\Delta$ in $\mathcal{M}_2 \cup \mathcal{M}_3$. Even when the true marginal distribution of $(Z, X)$ differs between the primary and the auxiliary populations, $\hat{\Delta}_{\text{ts2sls}}$ is consistent, provided that the linear propensity score model $\tau_{\text{linear}}(\cdot)$ is correctly specified. We can also show via semiparametric efficiency theory that $\hat{\Delta}_{\text{ts2sls}}$ is asymptotically more efficient than its nondoubly robust counterpart $\hat{\Delta}_{\text{tsiv}}$ at the intersection submodel $\mathcal{M}_2 \cap \mathcal{M}_3$ (Tan (2007); Tsiatis (2007)). The above properties are noted by Inoue and Solon (2010).

Shu and Tan (2019) proposed a class of doubly robust estimators $(\hat{\Delta}_{\text{dr}}, \hat{\eta}_{\text{dr}})^T$ that solve

$$
\begin{aligned}
0 = \hat{E}\bigg\{\mathcal{G}_{v,w}(X,Z)\bigg\{ & R[Y - \Delta\tau(Z,X;\hat{\xi}) - \omega_{\text{linear}}(X;\eta)] \\
& -\frac{(1-R)\pi(Z,X;\hat{\alpha})}{1-\pi(Z,X;\hat{\alpha})}\Delta[D - \tau(Z,X;\hat{\xi})]\bigg\}\bigg\},
\end{aligned}
$$

where users can freely specify models for $\{\tau(\cdot), \pi(\cdot)\}$. Graham, Pinto and Egel (2016) introduced a doubly robust auxiliary-to-study tilting estimator under restricted nuisance model specifications for the efficient estimation of data combination models. Inferences based on $\hat{\Delta}_{\text{dr}}$ can be viewed as special instances of inferences obtained under a particular specification of submodel $\mathcal{M}_2 \cup \mathcal{M}_3$, with $\mathcal{H}(X) = \Delta$ and $\omega_{\text{linear}}(\cdot)$. In constrast to $\hat{\Delta}_{\text{mul}}$, $\hat{\Delta}_{\text{dr}}$ will generally fail to be consistent for $\Delta$ outside the union model $\mathcal{M}_2 \cup \mathcal{M}_3$. Note that a generalized version of $\hat{\Delta}_{\text{dr}}$ that accommodates arbitrary parametric model specifications in $\mathcal{M}_2 \cup \mathcal{M}_3$ is given by

$$
\hat{\Delta}_{\text{dr2}} = \hat{E}\left\{\frac{R\mathcal{H}(X;\tilde{\gamma})}{\hat{q}}\right\}, \tag{4.1}
$$

where $\tilde{\gamma}$ solves (3.6).

## 5. Simulation Study

We investigate the finite-sample properties of the proposed semiparametric estimators under a variety of settings. For the primary population, the baseline covariates $X = (X_1, X_2, X_3)^T$ are mutually independent and marginally distributed as $\mathrm{U}(0,1)$; $(Y, A, Z, U)$ is distributed as follows:

$$\begin{aligned} U|X &\sim \mathrm{TN}\{\vartheta^T X, 1, (\vartheta^T X - 1, \vartheta^T X + 1)\}; \\ Z|X &\sim \text{Bernoulli } \{p = \{1 + \exp{[-\psi^T(1, X^T)^T]}\}^{-1}\}; \\ D|Z, X, U &\sim \text{Bernoulli } \{p = \{1 + \exp{[-\xi^T(1, Z, X^T)^T]}\}^{-1} + 0.2[U - \vartheta^T X]\}; \\ Y|D, X, U &\sim \mathrm{N}\{\gamma^T(1, X^T)^T D + 1.25 \times \vec{1}^T X + 6U, 1\}, \end{aligned}$$

where $\mathrm{TN}\{\mu, \sigma^2, (l, u)\}$ denotes a truncated normal distribution with support $[l, u]$, $\vartheta = (0.5, -0.5, 0)^T$, $\psi = (-1, 0.5, 0.5, 0.5)^T$, $\xi = (-1.3, 1.2, 0.5, -0.25 - 0.25)^T$, $\gamma = (2, 0.5, 0.5, 0.5)^T$, and $\vec{1} = (1, 1, 1)^T$. For the auxiliary population, $X = (X_1, X_2, X_3)^T$ are mutually independent and marginally distributed as $\mathrm{TN}\{0.5, 1, (0, 1)\}$, $Z|X \sim$ Bernoulli $\{p = \{1 + \exp{[-\psi^T(1, X^T)^T]}\}^{-1}\}$, and $D|Z, X \sim$ Bernoulli $\{p = \{1 + \exp{[-\xi^T(1, Z, X^T)^T]}\}^{-1}\}$; the remaining parts of the data law are left unrestricted. For each simulation replicate of total sample size $n$, we generate $n_p \sim \text{binomial}(n, p = 0.7)$, followed by an independent and identically distributed (i.i.d.) sample of size $n_p$ from the primary population with only realizations of $(Y, Z, X)$ recorded, and another i.i.d. sample of size $n_a = n - n_p$ from the auxiliary population with only realizations of $(D, Z, X)$ recorded. The two samples are then merged, and an indicator variable $R$ is introduced, equal to one or zero if the unit is drawn from the primary or auxiliary population, respectively. It can be verified that the above data-generating mechanism satisfies assumptions 1–9, and that the corresponding true observed data models are $\lambda(1|x; \psi) = \{1 + \exp{[-\psi^T(1, x^T)^T]}\}^{-1}$, $\tau(z, x; \xi) = \{1 + \exp{[-\xi^T(1, z, x^T)^T]}\}^{-1}$, $\mathcal{H}(x; \gamma) = \gamma^T(1, x^T)^T$, $\omega(x; \eta) = \eta^T(1, x^T)^T$, and $\pi(z, x; \alpha) = \{1 + \exp{[-\alpha^T(1, z, x^T, x^{2T})^T]}\}^{-1}$, where $x^2 = (x_1^2, x_2^2, x_3^2)^T$ (by Bayes' rule). We are interested in estimating the average treatment effect $\Delta = E\{\gamma^T(1, X^T)^T | R = 1\} = 2.75$. The four semiparametric estimators $\hat{\Delta}_1$, $\hat{\Delta}_2$, $\hat{\Delta}_3$, and $\hat{\Delta}_{\text{mul}}$ are implemented using $v(x) = w(x) = (1, x^T)^T$ as index functions.

Similarly to Kang and Schafer (2007), we evaluate the performance of the proposed estimators in situations where some models may be misspecified by considering the transformed variables $V^* = (Z^*, X_1^*, X_2^*, X_3^*)^T$, where $Z^* \sim \text{Bernoulli}\{p = \Phi(-2 + 3Z)\}$, $X_1^* = \exp(-0.5X_1) + \epsilon_1$, $X_2^* = X_2/[1 + \exp(Z)] + \epsilon_2$, and $X_3^* = (X_1 X_3)^3 + \epsilon_3$; $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution, and the error terms are generated as $(\epsilon_1, \epsilon_2, \epsilon_3)^T \sim N(0, I_3)$. Then, a particular component model is misspecified when the analyst uses $V^*$ instead of $V$ in the working model. Specifically, we report the results from the following five scenarios:

Table 1. Monte Carlo results of the proposed semiparametric estimators under different scenarios.

| Model | Estimator | | | |
|---|---|---|---|---|
| | $\hat{\Delta}_1$ | $\hat{\Delta}_2$ | $\hat{\Delta}_3$ | $\hat{\Delta}_{\rm mul}$ |
| \|Bias\| (SE) | | | | |
| $\mathcal{M}'_0$ | 0.01 (0.29) | 0.01 (0.29) | 0.08 (0.33) | 0.04 (0.31) |
| $\mathcal{M}'_1$ | 0.01 (0.29) | 0.65 (0.34) | 0.74 (0.37) | 0.05 (0.30) |
| $\mathcal{M}'_2$ | 0.67 (0.36) | 0.01 (0.32) | 0.11 (0.41) | 0.05 (0.33) |
| $\mathcal{M}'_3$ | 1.10 (0.46) | 1.20 (0.48) | 0.09 (0.34) | 0.06 (0.33) |
| $\mathcal{M}'_4$ | 1.30 (0.47) | 2.20 (0.57) | 0.77 (0.44) | 0.72 (0.39) |
| RMSE | | | | |
| $\mathcal{M}'_0$ | 0.09 | 0.09 | 0.11 | 0.10 |
| $\mathcal{M}'_1$ | 0.08 | 0.54 | 0.68 | 0.09 |
| $\mathcal{M}'_2$ | 0.58 | 0.10 | 0.18 | 0.11 |
| $\mathcal{M}'_3$ | 1.50 | 1.70 | 0.12 | 0.11 |
| $\mathcal{M}'_4$ | 1.80 | 5.00 | 0.78 | 0.67 |

$\mathcal{M}'_0$: All models are correct;

$\mathcal{M}'_1$: Only models $\lambda(z|x;\psi)$ and $\tau(z,x;\xi)$ are correct;

$\mathcal{M}'_2$: Only models $\tau(z,x;\xi)$, $\mathcal{H}(x;\gamma)$ and $\omega(x;\eta)$ are correct;

$\mathcal{M}'_3$: Only models $\pi(z,x;\alpha)$, $\mathcal{H}(x;\gamma)$ and $\omega(x;\eta)$ are correct;

$\mathcal{M}'_4$: All models are incorrect.

All simulation results are based on 1,000 Monte Carlo runs of $n = 10{,}000$ units each. Table 1 summarizes the simulation results. In agreement with theory, $\hat{\Delta}_1$ has small bias in $\mathcal{M}'_0$ and $\mathcal{M}'_1$, $\hat{\Delta}_2$ has small bias in $\mathcal{M}'_0$ and $\mathcal{M}'_2$, $\hat{\Delta}_3$ has small bias in $\mathcal{M}'_0$ and $\mathcal{M}'_3$, and $\hat{\Delta}_{\rm mul}$ has small bias in $\mathcal{M}'_l$, for $l = 0,1,2,3$. In $\mathcal{M}'_0$, where all models are correct, $\hat{\Delta}_1$ and $\hat{\Delta}_2$ have smaller Monte Carlo standard errors than that of $\hat{\Delta}_3$, which involves weighting through the data source propensity score $\pi(z,x)$.

## 6. Application

Currie and Yelowitz (2000) studied the effect of public housing participation on housing quality and educational attainment, showing that project participation is associated with poorer outcomes, based on data from the Survey of Income and Program Participation (SIPP). However, many unobserved factors, such as social ties, are likely to affect both project participation and the outcomes. As such, the authors suspect that failing to control for this source of endogeneity would bias the estimated causal effects of living in projects downwards, because families in projects may be more likely to live in substandard housing in any case, and their children may be more likely to experience negative outcomes. Leveraging on the sex composition of children as an instrumental variable for project participation, Currie and Yelowitz (2000) use two-sample instrumental variable methods to combine information from the 1990 Census and the 1990—1995 waves of the March Current Population Survey (CPS). Their findings show that project households are less likely to suffer from overcrowding or live in high-density complexes, and project children are less likely to have been held back. Their study is important, because the results overturn the stereotype that project participation is harmful in terms of living conditions and children's educational attainment.

In this analysis, we apply the proposed methods to estimate the causal effect of project participation ($D$) on reported monthly rental payments ($Y$) in the SIPP population; here reported rent is viewed as a proxy for housing quality (Currie and Yelowitz (2000)). The binary instrumental variable $Z$ takes the value one if a family has a boy and a girl, and zero if both children are boys or girls. Families with two children of opposite genders are eligible for three-bedroom apartments as opposed to two-bedroom apartments, and therefore are more likely to participate in the housing project, although there is little reason to expect that the children's sex composition will directly affect $Y$. In line with the Currie and Yelowitz (2000) study, the vector of baseline covariates $X$ includes the household head's gender, age, race, education, marital status, and the number of boys in the family. We specify main effects models for $\{\lambda(\cdot), \tau(\cdot), \pi(\cdot)\}$ with logistic links. In addition, following Shu and Tan (2019), we add an additional interaction term involving household head information to the linear predictor function of the model for $\pi(\cdot)$ in order to improve the covariate balance, and specify $\omega(x;\eta) = \eta^T(1, x^T)^T$, $\mathcal{H}(x;\gamma) = \Delta$. The analysis results based on $n_1$ = 116,901 renters' complete records for $(Y, Z, X)$ from the 1990 Census of SIPP ($R = 1$) and $n_0$ = 10,382 renters' complete records for $(D, Z, X)$ from CPS ($R = 0$), for a total sample size of $n$ = 127,283, are summarized in Table 2.

The two-sample two-stage least squares estimate of 0.3717 agrees with the point estimate presented in Table 4 of Currie and Yelowitz (2000), although the analytic standard error of 0.1124 is larger than the value of 0.0589 reported by the original study, because the former takes into account the variability associated with the first-stage estimation. While the point estimates of the proposed estimators are all larger than 0.3717, the point estimate of $\hat{\Delta}_{\text{mul}}$ is closest to that of $\hat{\Delta}_1$, which suggests that the models for $\{\lambda(\cdot), \tau(\cdot)\}$ in this illustrative analysis may be specified nearly correctly; Tchetgen Tchetgen and Robins (2010) describe

Table 2. Estimates of the effect of public housing project participation on reported monthly rental (divided by 1,000 US dollars).

| | point estimate | standard error | 95% Wald CI |
|---|---|---|---|
| $\hat{\Delta}_{\mathrm{ts2sls}}$ | 0.3717 | 0.1124 | (0.1513, 0.5920) |
| $\hat{\Delta}_1$ | 0.7650 | 0.3442 | (0.0903, 1.4397) |
| $\hat{\Delta}_2$ | 0.3790 | 0.1162 | (0.1513, 0.6068) |
| $\hat{\Delta}_3$ | 0.4999 | 0.2533 | (0.0034, 0.9964) |
| $\hat{\Delta}_{\mathrm{mul}}$ | 0.9155 | 0.4126 | (0.1069, 1.7242) |

a formal specification test to detect which of the baseline models is correct under the union model $\mathcal{M}_{\mathrm{union}}$. The point estimate of 0.9155 for $\hat{\Delta}_{\mathrm{mul}}$ also suggests that the causal effect of housing project participation on improving household living conditions is probably larger than the value reported in Currie and Yelowitz (2000), because $\hat{\Delta}_{\mathrm{ts2sls}}$ is, in general, no longer consistent outside the union model $\mathcal{M}_2 \cup \mathcal{M}_3$.

## 7. Discussion

Suppose we observe data on $(D, Z, X)$ from the primary population of interest and fuse it with data on $(Y, Z, X)$ from an auxiliary source; here, $R_i$ is equal to either zero or one, depending on whether the $i$th unit is drawn from the primary or the auxiliary population, respectively. In this case, it is clear that an inference about the identifying functional

$$\Delta = E\left\{\frac{1-R}{1-q^{\dagger}}\frac{(-1)^{1-Z}}{\lambda(Z|X)}\frac{Y}{[\tau(1,X)-\tau(0,X)]}\right\}$$

is not possible under submodel $\mathcal{M}_1$, because $Y$ is not observed from the primary population. Nonetheless, an inference for $\Delta$ is still possible under $\mathcal{M}_2 \cup \mathcal{M}_3$ if we replace assumption 9 with predictive invariance for the outcome.

**Assumption 10.** $E(Y|Z, X, R = 0) = E(Y|Z, X, R = 1)$, *almost surely.*

Indeed, it can be shown that under assumptions 1–8 and 10, the estimator

$$\tilde{\Delta}_{\mathrm{dr3}} = \hat{E}\left\{\frac{(1-R)\mathcal{H}(X;\tilde{\gamma})}{1-\hat{q}}\right\}, \tag{7.1}$$

where $\tilde{\gamma}$ solving (3.6) is consistent and asymptotically normal in the union model $\mathcal{M}_2 \cup \mathcal{M}_3$. Note that because $\hat{\Delta}_{\mathrm{tsiv}}$, $\hat{\Delta}_{\mathrm{ts2sls}}$, and $\hat{\Delta}_{\mathrm{dr}}$ typically specify $\mathcal{H}(x;\gamma) = \Delta$, which does not depend on values for the baseline covariates, one can be agnostic as to which of the two samples is drawn from the primary population, as long as assumptions 1–10 all hold.

There are several improvements and extensions for future work. Multiple valid instrumental variables can be incorporated by adopting a standard generalized method of moments approach (Hansen (1982)), and the proposed estimators can be improved in terms of efficiency (Tan (2006a, 2010b)) and bias (Vermeulen and Vansteelandt (2015)). In this study, we focused on the canonical case of binary $Z$ and $D$. Thus, an extension of the proposed methodology to the case of general $Z$ or $D$ is an interesting topic for future research. It will also be of interest to investigate the use of negative controls under data fusion to mitigate unmeasured confounding and identify causal effects, which has gained increasing recognition and popularity in recent years (Miao and Tchetgen Tchetgen (2017); Shi, Miao and Tchetgen Tchetgen (2018)).

A multiply robust estimation typically entails postulating various parametric models for the nuisance parameters (Molina et al. (2017)). Chernozhukov et al. (2018) showed that a $n^{-1/2}$-consistent estimation of low-dimensional parameters of interest, based on nonparametric efficient scores such as $\mu_{\text{eff}}(O; \Delta)$, is possible when all the nuisance parameters are estimated consistently with sufficiently fast rates, even when the complexity of the nuisance model space is no longer limited by classical settings. In future research, we plan to investigate estimation and inference for the average treatment effect under data fusion when various flexible and highly data-adaptive machine learning methods are used to estimate the nuisance parameters.

## Supplementary Material

The online Supplementary Material includes the proofs of the lemmas and theorems, as well as details on asymptotic variance estimation for the proposed estimators.

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BaoLuo Sun
Department of Statistics and Applied Probability, National University of Singapore.
E-mail: stasb@nus.edu.sg
Wang Miao
Department of Probability and Statistics, Peking University.
E-mail: mwfy@pku.edu.cn

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