BAYESIAN MODELS FOR DETECTING DIFFERENCE BOUNDARIES IN AREAL DATA

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Supplementary Material

S1 Computational details

Posterior inference for our models are based on MCMC posterior simulations. There are two main strategies used. The first avoids computing parameters characterizing G by marginalizing it out and relying on the Polya urn scheme of Blackwell and MacQueen (1973). A limitation of this approach is that it is only applicable when the prior can be characterized by a generalized Polyn urn mechanism. Ishwaran and James (2001) proposed the blocked Gibbs Sampler that directly sampled from the posterior of the random measure, avoiding the marginalization over G. We use the blocked Gibbs Sampler with some Metropolis-Hasting steps nested in it to update all random parameters in our model. We truncate the infinite sum in G by the first m terms. We only provide details for the ARDP model. That for the ARSB model is similar (and even simpler), while algorithms for the DPM model may be found in Escobar and West (1995).

We place a flat prior on parameter β and reparameterize the variance parameters with its inverse, $\tau_s = \sigma_s^{-2}, \tau_\gamma = \sigma_\gamma^{-2}$, then place a conjugate gamma prior of the precision parameters τ_s and τ_γ. The likelihood of the model is expressed as

$$L = \prod_{i=1}^{n} Poisson(Y_i \mid \mathbf{x}_i'\boldsymbol{\beta} + \phi_i) \tag{S1.1}$$

The posterior density given the data $\mathbf{Y} = \{Y_i\}$ is proportional to the likelihood multiplied by all the prior distributions: $L(Y_i \mid \boldsymbol{\beta}, \{\phi_i\})p(\boldsymbol{\beta})p(\boldsymbol{\phi} \mid \tau_s)p(\gamma \mid \tau_\gamma)p(\mathbf{V})p(\tau_s)p(\tau_\gamma)$. Note that $\phi_i = \theta_{u_i}$ and we updated ϕ_i by updating $\boldsymbol{\theta}$ and u_i. The MCMC algorithm proceeds as follows.

Step 1: update $\boldsymbol{\beta} \mid \boldsymbol{\theta}, \boldsymbol{\gamma}, \mathbf{V}, \tau_s, \tau_\gamma$: The full conditional distribution only depends on the likelihood due to the flat prior. Sample candidate β^* from $N(\beta, K_\beta I)$ ($K_\beta = 0.05$

worked well),then accept the candidate * with probability

$$\min\left\{1, \frac{\exp(\sum_{i=1}^{n}(-E_i \exp(\mathbf{x}_i'\boldsymbol{\beta}^* + \phi_i) + y_i(\mathbf{x}_i'\boldsymbol{\beta}^* + \phi_i)))}{\exp(\sum_{i=1}^{n}(-E_i \exp(\mathbf{x}_i'\boldsymbol{\beta} + \phi_i) + y_i(\mathbf{x}_i'\boldsymbol{\beta} + \phi_i)))}\right\} \tag{S1.2}$$

Step 2: update $\theta_j \,|\, \boldsymbol{\beta}, \boldsymbol{\gamma}, \mathbf{V}, \tau_s, \tau_\gamma$: Sample candidate θ_j^* from $N(\theta_j, K_\theta)$ ($K_\theta = 0.05$ worked well), then accept the candidate $\theta_{\mathbf{j}}{}^*$ with probability

$$\min\left\{1, \frac{\exp(\sum_{i:u_i=j}(-E_i \exp(\mathbf{x}_i'\boldsymbol{\beta} + \theta_j^*) + y_i(\mathbf{x}_i'\boldsymbol{\beta} + \theta_j^*)) - \frac{\tau_s}{2}\theta_j^{*2})}{\exp(\sum_{i:u_i=j}(-E_i \exp(\mathbf{x}_i'\boldsymbol{\beta} + \theta_j) + y_i(\mathbf{x}_i'\boldsymbol{\beta} + \theta_j)) - \frac{\tau_s}{2}\theta_j^{2})}\right\} \tag{S1.3}$$

Step 3: update $\boldsymbol{\gamma_i} \,|\, \boldsymbol{\beta}, \boldsymbol{\theta}, \mathbf{V}, \tau_s, \tau_\gamma$: Sample candidate γ^* from $N(\gamma_{\mathbf{i}}, K_\gamma)$ ($K_\gamma = 0.01$ worked well), compute the coresponding candidate $\mathbf{u}^*$ through $u_i^* = \sum_{j=1}^{n} jI(\Sigma_{k=1}^{j-1}p_k < F^{(i)}(\gamma_i) < \Sigma_{k=1}^{j}p_k)$, then accept the candidate γ^* with probability

$$\min\left\{1, \frac{\exp\left(-\frac{1}{2}\boldsymbol{\gamma^*}'\Sigma_2^{-1}\boldsymbol{\gamma^*}\right)\exp(-E_i \exp(\mathbf{x}_i'\boldsymbol{\beta} + \theta_{u_i^*}) + y_i(\mathbf{x}_i'\boldsymbol{\beta} + \theta_{u_i^*}))}{\exp\left(-\frac{1}{2}\boldsymbol{\gamma}'\Sigma_2^{-1}\boldsymbol{\gamma}\right) exp(-E_i \exp(\mathbf{x}_i'\boldsymbol{\beta} + \theta_{u_i}) + y_i(\mathbf{x}_i'\boldsymbol{\beta} + \theta_{u_i}))}\right\} \tag{S1.4}$$

Step 4: update $\mathbf{V} \,|\, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\gamma}, \tau_s, \tau_\gamma$: Sample candidate $\mathbf{V}^*$ from $N(\mathbf{V}, 0.01I_m)$, compute the corresponding $\mathbf{p}^*$ and $\mathbf{u}^*$. Accept the candidate $\mathbf{V}^*$ with probability

$$\min\left\{1, \frac{\prod_{k=1}^{m}(1 - V_k^*)^{\alpha-1}\prod_{i=1}^{n}\exp(-E_i \exp(\mathbf{x}_i'\boldsymbol{\beta} + \theta_{u_i^*}) + y_i(\mathbf{x}_i'\boldsymbol{\beta} + \theta_{u_i^*}))}{\prod_{k=1}^{m}(1 - V_k)^{\alpha-1}\prod_{i=1}^{n}\exp(-E_i \exp(\mathbf{x}_i'\boldsymbol{\beta} + \theta_{u_i}) + y_i(\mathbf{x}_i'\boldsymbol{\beta} + \theta_{u_i}))}\right\} \tag{S1.5}$$

Step 5: update $\tau_s \,|\, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\gamma}, \mathbf{V}, \tau_\gamma$: Sample from $Gamma\left(\frac{n}{2} + a, \frac{\Sigma_{i=1}^{n}\phi_i^2}{2} + b\right)$, where $a = b = 0.01$, which is the conjugate gamma full conditional distribution for τ_s.

Step 6: update $\tau_\gamma \,|\, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\gamma}, \mathbf{V}, \tau_s$: Sample from $Gamma\left(\frac{n}{2} + c, \frac{\Sigma_{i=1}^{n}\boldsymbol{\gamma}(D - \rho W)\boldsymbol{\gamma}}{2} + d\right)$, where $c = d = 0.01$, which is the conjugate gamma full conditional distribution for τ_γ.