

Let us use distinct and well-recognizable notations and abbreviations for writings in probability and statistics

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Communicated by the Editor

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Among the main activities in our profession is to read, be familiar with the current state of art in our area, work on new problems, write, publish and share knowledge and experience with others. In order to keep good level of mutual understanding, we need to follow more or less the same basic requirements for the way we write and speak.

In the literature, most of the used notations and abbreviations are appropriate. However, not all. There are some, which are distracting and/or misleading, or are just tasteless. Occasionally, there are exotic 'inventions', too far from any common sense.

Based on my more than 50 years experience as a learner, reader, author, teacher, advisor, reviewer and editor, I have been making brief notes about good practices followed by leading specialists. Naturally, there are differences, and this brings expected and unexpected challenges. Thus I arrive at the idea to prepare a short material, different versions of which were distributed around the world over the last couple of years. Some colleagues, authors and/or editors, expressed a positive opinion and supported the idea to look at how we write and what eventually can be done for the better image and aesthetic view of our works. Publishers and Editors (believe they) are the 'commanders' deciding what is the 'right' policy. Still, there is always some freedom.

So, essential is to think not only of the content, but also of the general 'quality' of the outcome of our works from the point of view of their easy accessibility by potential readers. There is a common illusion, everybody believes that she/he is an excellent writer, speaker, etc. Yes, there are many such cases. And ... there are counterexamples.

It was and is not clear how to find an appropriate platform for discussions among the scientific community with the goal to arrive at a compromise for at least a minimum of well selected suggestions to be rec-

ommended for implementation. Surely there are colleagues who are willing to come up with new valuable suggestions. This would confirm the statement that 'two heads are better than one'. And ... a hundred of heads is even better. This is why the appeal is to the members of professional organizations. Just to mention that a short Essay of 2 pages has been published in the Bulletin of the IMS with quite positive feedback. A version of my short notes was distributed among all members of the Italian Statistical Association and also of the Greek Statistical Institute. Not surprisingly, some colleagues are positive and enthusiastic about the necessity to do something, however, which is normal, there are sceptics.

I am grateful to the Editors of Bernoulli New for the opportunity given to me to share my concerns and specific suggestions with the members of Bernoulli Society.

My appeal is addressed to *Probabilists, Statisticians, Mathematicians*, to *Publishers, Editors, Authors, Reviewers*, to look at the current situation. If a wish, with some efforts one can arrive at a common opinion: let us try and do something useful for improving the quality and the image of all papers, books, etc., in Probability and Statistics.

Here are three reasonable and well-motivated 'rules':

Rule 1. *Use one letter/symbol as the notation for only one item/object.*

Rule 2. *For each specific group of objects, use the same font.*

Rule 3. *For different groups of objects use different fonts.*

In my opinion, shared by many others, these are a kind of indisputable rules. They can be slightly modified, perhaps new rules added. For now, let us stick to these three.

Question: Are you comfortable with the following *terrible examples* in the literature?

Example 1: Let the IID $\{X_n\}$ be ID with DD ... (That author meant: IID = 'independent and identically distributed', ID = 'infinitely divisible', DD = 'decreasing dependence'.)

Example 2: The probability $\Pr(\xi \leq x)$, its expectation $\mathbb{E}\xi$ and variance $\text{var}\xi$... (Here 'Pr', 'E', 'var', are in one group of objects, however they are written in three different ways. So, let us use one font, 'sans serif' is the most appropriate: $P(\xi \leq x)$, $E[\xi]$, $\text{Var}[\xi]$.)

Example 3: For a RV \mathbb{X} on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with values in \mathbb{R} , denote

$$\mathbb{E}[\mathbb{X}] = \int_{\Omega} \mathbb{X}(\omega) d\mathbb{P}; \quad \mathbb{P}[\mathbb{X} \leq x] = \mathbb{F}(x), \quad x \in \mathbb{R}.$$

(Here, the same font 'bb' is used for five different objects. Terrible! What a taste?!)

Abbreviations: Some publishers do not allow abbreviations. If allowed, here is a set of a good selection. The meaning of each is absolutely clear at one glance:

i.i.d., r.v., d.f., a.s., m.g.f., p.g.f., ch.f., inf.div. (with no space between).

It is old-fashionable to write RV, PDF, CDF. Density is density, why PDF? Why capitals? Why CDF? It is cumulative by definition! Write simply 'density' and 'd.f.'.

Mathematics Subject Classification (MSC): Use the last **MSC 2020**. Strangely, there are still papers in journals (not by the IMS and BS) referring to MSC 2010.

Traditionally, we look for works of good content in our specific area. Next, we do pay attention to how appropriate, clear and distinct notations are used. Good notations make the works easier to read! This observation does not apply to people who only write but do not read. The worst is, there are authors who do not care of it at all.

Below is a brief list of basic notations (Many of them are widely used in Mathematics):

- For the 'standard normal density' use φ , not ϕ ('varphi' not 'phi').
- Use 'cal' font for the normal distribution, $\mathcal{N}(0, 1)$, $\mathcal{N}(\mu, \sigma^2)$, standard, with parameters μ, σ^2 .
- For spaces and sets, use 'bb' font: $\mathbb{R} = (-\infty, +\infty)$, \mathbb{R}^n , $\mathbb{R}_+ = [0, \infty)$, \mathbb{Q} for the rationals, \mathbb{C} for the complex plane, $\mathbb{N} = \{1, 2, \dots\}$ or \mathbb{Z}_+ , $\mathbb{N}_0 = \{0, 1, 2, \dots\}$ or \mathbb{Z}_0 , $\overline{\mathbb{N}} = \{\dots, -1, 0, 1, 2, \dots\}$ or \mathbb{Z} . Remember, 'bf' font is reserved, see the next item.
- Keep 'bf' font for matrices and vectors, **A, B, ...**,

a, b, ...

- Font 'cal' is good for families, σ -algebras, sets of functions: $\mathcal{A}, \mathcal{B}, \mathcal{F}, \mathcal{C}, \mathcal{P}, \mathcal{M}$, etc.
- Use Roman font, 'rm', for 'e', 'd' 'exp', 'Re' and 'Im', respectively (Napier constant, differentials, exponent, real and imaginary parts of a complex number). Not e, d, exp , and not $\mathcal{R}e$ and $\mathcal{I}m$. Also write ε , not ϵ .
- To use 'parentheses' in $\max\{a, b\}$, $\min\{a, b\}$, is better than $\max(a, b)$, $\min(a, b)$, etc.
- For functions (= mappings), use \mapsto not \rightarrow ; the latter is 'reserved' for convergence.
- For specified, say s -convergence, use \xrightarrow{s} , but not \rightarrow_s and not \rightarrow^s .
- For convergence, instead of $\xrightarrow{n \rightarrow \infty}$, it is better to avoid overloading and write $n \rightarrow \infty$ after.
- Two symbols: $X \perp Y$ for orthogonal (uncorrelated) r.v.s; $X \perp\!\!\!\perp Y$ for independent r.v.s.
- Write $X \sim F$, or $X \sim f$, for a r.v. X with d.f. F , or density f .
- Keep $\Gamma(\cdot)$ and $B(\cdot, \cdot)$ for the classical Euler's Gamma function and Beta function.
- Use $\gamma(a, b)$ or $\text{Gamma}(a, b)$, gamma distribution with parameters a, b . Not $\Gamma(\alpha, \beta)$.
- Use $\beta(a, b)$, or $\text{Beta}(a, b)$, for the beta distribution with parameters a, b . Not $B(\alpha, \beta)$.
- Use $\text{Exp}(\lambda)$ for the exponential distribution with parameter λ . But not \exp_λ .
- Use $\text{Bin}(n, p)$ for the binomial distribution with parameters n, p .
- Use $\text{Poi}(\lambda)$ for the Poisson distribution with parameter λ .
- Use $\text{Ge}(p)$ or $\text{Geo}(p)$ for the geometrical distribution with parameter p .
- Use 'sans serif' font for the following objects/items, they are all in one group: \mathbb{P} , \mathbb{E} , \mathbb{V} or Var , $\text{Cov}(\cdot, \cdot)$, $\text{Corr}(\cdot, \cdot)$, for probability, expectation, variance, covariance, correlation.

It is remarkable, 'sans serif' font was used by A.N. Kolmogorov in his book *Grundbegriffe der Wahrscheinlichkeitsrechnung*, Julius Springer, 1933, and later in any of the many translations and editions, including in *Foundations of Probability Theory*, Chelsea Publ. Co., 1956.

Several outstanding scientists have used in their books 'sans serif' font. Here are only a few names: K.L. Chung,

A.N. Shiryaev, S. Foss, V.G. Kulkarni, R. Gatto, I. Song.

Here are just a few names among the many authors using ‘sans serif’: S. Molchanov, V. Panov, A. Bulinskii, I. Pinelis, V. Abramov, L. Bogachev, Ph. Ernst, G. Peskir, C. Genest, F. Ouimet, V. Ulyanov, D. Dai, Zh. Su, H. Wang, A. Veretennikov, V. Malinovsky.

It is pity, fonts like p , **P**, Pr, for ‘probability’, are still in use as 100 years ago. They should be forgotten by smoothly switching to ‘sans serif’ as an excellent option.

Final words: While the mathematical content is a priority number one, not less important are things of technical nature, like those discussed above. To use systematically good notations and abbreviations will lead to an improved ‘face’ of the writings in Mathematics, hence in Probability and Statistics.

There are masterly written books and masterly designed journals. This is when the author’s good taste coincides with good journal rules. It is enjoyable to see notions, notations and abbreviations which are indeed ‘distinct and well-recognizable’. Two conclusions:

- **Let us follow the Masters!**
- **Make your own works attractive and a source of intellectual pleasure!**

It is appropriate to give here the following relevant citation:

A.N. Kolmogorov: *Mathematics is a synthesis of the two ideals making meaningful human life, the ideal of truth and the ideal of beauty.*

I do not attempt to discuss here on other important

aspects such as ‘Citation Style Recommendations’ and ‘References style’. These are up to Publishers, journal Editors, etc.

Among the large number of available sources, below are just a few works by Masters.

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