Supplementary materials for Tsai and Gilmour (2015)

This supplementary file contains (1) the relative efficiencies for an example of 10-run designs with different number of level-balanced factors, (2) an example discussing the secondary criterion for Q_B-optimal designs obtained from conference matrices, and (3) the conference matrices that are required for generating the Q_B-optimal designs in Table 1 of Tsai and Gilmour (2015).

Relative efficiencies for the five 10-run Q_B-optimal designs with different numbers of level-balanced factors

For $N = 10$ the A_s-optimal design has 5 level-balanced factors and 4 non-level-balanced factors, and the p-efficient design has 9 level-balanced factors. We obtain three other designs with 6, 7 and 8 level-balanced factors, respectively. Figure 1 gives the plot of the relative efficiencies for the five 10-run Q_B-optimal designs for $\pi \in (0, 0.5]$. It demonstrates that, depending on the experimenters' prior beliefs, different designs are better and should be recommended. For example, if the expected number of active factors is about 1 or 2, then the design with 6 level-balanced factors would be better than the other designs for $\pi \in (1/8, 1/4)$, and when π is less than 1/8, this design is not the best, but it is better than the A_s-optimal design.

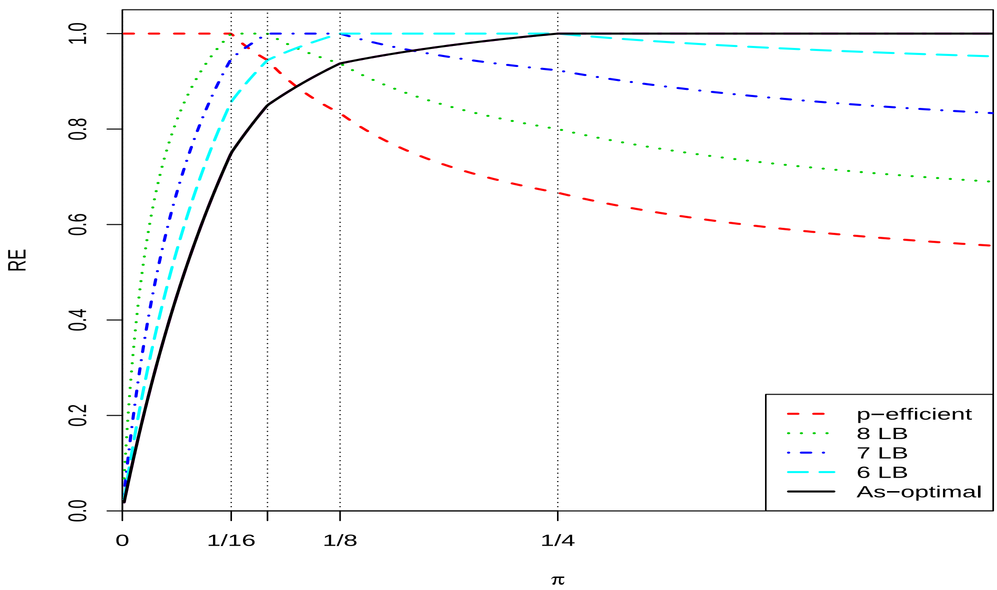

Figure 1: Q_B-efficiency under different π for the 10-run two-level designs.

The secondary criterion: A_s-criterion for the full main-effects model

Here we consider designs with 10 runs having 6 level-balanced factors and 3 non-level-balanced factors. We consider the following two designs:

$$
D_{10}^{(1)} = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 \\
1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1
\end{bmatrix}, \quad
D_{10}^{(2)} = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \\
1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 \\
1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1
\end{bmatrix}
$$

These designs are obtained by replacing the 0 diagonal elements of the conference matrix of order 10 with different 10×1 vectors. The first design is obtained by replacing it by the vector $(1,1,1,1,-1,-1,-1,-1,-1,-1)$, and the second design is replaced by the vector $(1,1,1,-1,-1,1,-1,-1,-1,-1)$. The information matrices for these designs are, respectively,

$$
M^{(1)} = \begin{bmatrix}
10 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 10 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 2 & 10 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 2 & 2 & 10 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 10 & -2 & -2 & 2 & 2 & -2 \\
0 & 0 & 0 & 0 & -2 & 10 & -2 & -2 & 2 & 2 \\
0 & 0 & 0 & 0 & -2 & -2 & 10 & 2 & -2 & 2 \\
0 & 0 & 0 & 0 & 2 & -2 & 2 & 10 & -2 & -2 \\
0 & 0 & 0 & 0 & 2 & 2 & -2 & -2 & 10 & -2 \\
0 & 0 & 0 & 0 & -2 & 2 & 2 & -2 & -2 & 10
\end{bmatrix}, \quad
M^{(2)} = \begin{bmatrix}
10 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 10 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 2 & 10 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & -2 & -2 & 10 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 10 & 2 & 2 & -2 & 2 & 2 \\
0 & 0 & 0 & 0 & 2 & 10 & -2 & 2 & 2 & -2 \\
0 & 0 & 0 & 0 & 2 & -2 & 10 & 2 & -2 & 2 \\
0 & 0 & 0 & 0 & -2 & 2 & 2 & 10 & -2 & -2 \\
0 & 0 & 0 & 0 & 2 & 2 & -2 & -2 & 10 & -2 \\
0 & 0 & 0 & 0 & 2 & -2 & 2 & -2 & -2 & 10
\end{bmatrix}.
$$

By Theorem 2, both these designs are Q_B-optimal when the probability of a factor being active is within $(1/8, 1/4)$. But the values of the A_s criterion function for the saturated main effects model for these design are different, being 1.0714 and 1.0923, respectively. This demonstrates that it is easy to obtain the Q_B-optimal designs from the conference matrices, but some secondary criterion will be useful to identify better designs.

Lists of conference matrices

$N = 10$

$$
\begin{bmatrix}
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & 0 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \\
1 & 1 & 1 & 0 & -1 & 1 & -1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & 0 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & 0 & 1 & 1 & -1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & 0 & -1 & 1 & -1 \\
1 & -1 & -1 & 1 & -1 & 1 & -1 & 0 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 0 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 0
\end{bmatrix}
$$

$N = 14$

$$
\begin{bmatrix}
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 \\
1 & 1 & 0 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 \\
1 & -1 & 1 & 0 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & 1 & -1 & 1 & 0 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \\
1 & 1 & 1 & -1 & 1 & 0 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & 1 & -1 & 1 & 0 & 1 & -1 & 1 & 1 & -1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & 1 & 0 & 1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & 0 & 1 & -1 & 1 & 1 & -1 \\
1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & 0 & 1 & -1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & 0 & 1 & -1 & 1 \\
1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & 0 & 1 & -1 \\
1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & 0 & 1 \\
1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & 0
\end{bmatrix}
$$

$N = 18$

$$\begin{bmatrix}
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & 0 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 \\
1 & -1 & 1 & 1 & 0 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 \\
1 & 1 & -1 & 1 & 1 & 0 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & 1 & 0 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & 0 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 \\
1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 0 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 0 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 \\
1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 0 & 1 & 1 & -1 & 1 & -1 & -1 & -1 \\
1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 0 & 1 & 1 & -1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 0 & 1 & 1 & -1 & 1 & -1 \\
1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 0 & 1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 0 & 1 & 1 & -1 \\
1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 0 & 1 & 1 \\
1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 0
\end{bmatrix}$$

$N = 26(1)$

$$\begin{bmatrix}
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 \\
1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & 1 & 1 & 0 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & 0 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 \\
1 & 1 & 1 & -1 & -1 & 0 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 \\
1 & -1 & 1 & 1 & -1 & 1 & 0 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & -1 & 0 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 \\
1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 0 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 0 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & 0 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 \\
1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 0 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 \\
1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & 0 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 0 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 0 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 \\
1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 \\
1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 0 & -1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 0 & -1 & 1 & -1 & 1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 0 & 1 & 1 & 1 & 1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 0 & -1 & 1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 0 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 0 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & 0 & -1 \\
1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 0
\end{bmatrix}$$

$N = 26(2)$

$$
\begin{bmatrix}
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & 0 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 0 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & -1 & 0 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & 1 & 1 & 0 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & 1 & 1 & -1 & -1 & 0 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 \\
1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 0 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 \\
1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 0 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 0 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 0 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 0 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 \\
1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 0 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 \\
1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 0 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & -1 & 0 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & 0 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 0 & -1 & -1 & 1 & -1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 0 & -1 & 1 & -1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 0 & 1 & 1 & 1 & 1 & -1 \\
1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & -1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 0 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 0 & -1 & -1 \\
1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & 0 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 0
\end{bmatrix}
$$

$N = 30$

$$
\begin{bmatrix}
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 \\
1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 \\
1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 \\
1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 \\
1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 \\
1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 \\
1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 \\
1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 \\
1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 \\
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1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 \\
1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 \\
1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 0
\end{bmatrix}
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