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# LOCAL LINEAR QUANTILE REGRESSION WITH DEPENDENT CENSORED DATA

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**Supplementary Material**

This is a supplementary technical note that contains the proofs of Theorem 1 and Theorem 2.

**Proof of Theorem 1.** Let $\beta = (\beta_0, \beta_1)^T$, $H_n = \text{diag}(1, h_1)$ and $\tilde{Z}_t = Z_t - \beta_0 - \beta_1(X_t - x_0)$. Put $\hat{\theta} = a_n^{-1} H_n(\hat{\beta} - \beta)$, or equivalently $\hat{\theta} = \arg\min_\theta L_n(\theta)$, with

$$L_n(\theta) = \sum_{t=1}^{n} [\tilde{Z}_t - a_n \theta^T \tilde{X}_{ht}][\pi - \frac{\delta_t}{\hat{\bar{G}}_{X_t}(Z_t)} I(\tilde{Z}_t < a_n \theta^T \tilde{X}_{ht})] K_1(X_{ht}).$$

The quasi-gradient of $-L_n(\theta)$ is given by

$$\hat{V}_n(\theta) = a_n \sum_{t=1}^{n} [\pi - \frac{\delta_t}{\hat{\bar{G}}_{X_t}(Z_t)} I(\tilde{Z}_t < a_n \theta^T \tilde{X}_{ht})] \tilde{X}_{ht} K_1(X_{ht}).$$

We also define $V_n(\theta)$ to be the same as $\hat{V}_n(\theta)$ but with $G_x$ instead of $\hat{G}_x$. To prove the asymptotic (Bahadur) representation given in Theorem 1, we need the following lemma whose proof is similar to the proof of Lemma A.4 given in Koenker and Zhao (1996).

**Lemma 1** *Let $W_n(\theta)$ be a function such that for any $0 < M < \infty$,*

*(1)* $-\theta^T W_n(\lambda\theta) \geq -\theta^T W_n(\theta)$, $\forall \lambda \geq 1$

*(2)* $\sup_{||\theta|| \leq M} ||W_n(\theta) + D\theta - A_n|| = O_p(v_n)$,

*where $||A_n|| = O_p(1)$, $D$ is a positive definite matrix, and $0 < v_n = O(1)$. If $\theta_n$ is such that $||W_n(\theta_n)|| = o_p(v_n)$, then $||\theta_n|| = O_p(1)$ and $\theta_n = D^{-1} A_n + O_p(v_n) + o_p(1)$.*

We will start by showing the following :

(L1) $||[V_n(\theta) - V_n(0)] - \mathbb{E}[V_n(\theta) - V_n(0)]|| = o_p(1)$, uniformly in $\theta$ over $A_M := \{\theta : ||\theta|| \leq M\}$.

(L2) $||\mathbb{E}[V_n(\theta) - V_n(0)] + D\theta|| = o(1)$, uniformly in $\theta$ over $A_M$, where $D = f(x_0, \beta_0)\Lambda_u$.

(L3) $||V_n(0)|| = O_p(1)$.

(L4) $-\theta^T\hat{V}_n(\lambda\theta) \geq -\theta^T\hat{V}_n(\theta)$, $\forall\lambda \geq 1$.

(L5) $||\hat{V}_n(\hat{\theta})|| = O_p(a_n)$.

(L6) $\sup_{||\theta||\leq M} ||\hat{V}_n(\theta) - V_n(\theta)|| = O_p(a_n^{-1}h_0^2)$.

From now on, $C$ will denote a generic positive constant independent of $n$ and $\theta$ and whose value may change from line to line. Put $a_x^1 = \beta_0 + \beta_1(x - x_0)$, $a_x^2(\theta) = a_n(\theta_0 + \theta_1(x - x_0)/h_1)$.

**Proof of (L1)**

For any $\theta$ and $\tilde{\theta}$ such that $||\theta|| \leq M$ and $||\tilde{\theta} - \theta|| \leq \iota$, for some $M$ and $\iota > 0$, define

$$\Delta_n^i(\theta, \tilde{\theta}) = a_n \sum_t \frac{\delta_t}{\bar{G}_{X_t}(Z_t)} Z_t^*(\theta, \tilde{\theta}) X_{ht}^i K_1(X_{ht}),\ i = 0, 1,$$

with $Z_t^*(\theta, \tilde{\theta}) := I(Z_t < a_{X_t}^1 + a_{X_t}^2(\tilde{\theta})) - I(Z_t < a_{X_t}^1 + a_{X_t}^2(\theta))$. When no confusion is possible, we will omit $\theta$ and $\tilde{\theta}$ in all our notations. Clearly $V_n(\theta) - V_n(\tilde{\theta}) = (\Delta_n^0, \Delta_n^1)^T$, and, by stationarity,

$$\begin{aligned}\mathbb{V}ar[\Delta_n^0] &= a_n^2\{n\mathbb{V}ar[\frac{\delta_t}{\bar{G}_{X_t}(Z_t)} Z_t^* K_1(X_{ht})] + 2n\sum_{j=1}^n (1 - j/n)C_j(\theta, \tilde{\theta})\} \\ &\leq a_n^2\{n\mathbb{V}ar[\frac{\delta_t}{\bar{G}_{X_t}(Z_t)} Z_t^* K_1(X_{ht})] + 2n\sum_{j=1}^n |C_j|\} \qquad (0.1)\end{aligned}$$

where $C_j = \mathbb{C}ov\left(Z_1^*[\delta_1/\bar{G}_{X_1}(Z_1)]K_1(X_{h1}), Z_{j+1}^*[\delta_{j+1}/\bar{G}_{X_{j+1}}(Z_{j+1})]K_1(X_{h(j+1)})\right)$.
Using the fact that $|I(y < b) - I(y < a)| \leq I(b - |a - b| \leq y \leq b + |a - b|)$, we can see that

$$|Z_t^*(\theta, \tilde{\theta})| \leq I(a_{X_t}^1 + a_{X_t}^2(\tilde{\theta}) - \mathcal{C}\iota a_n \leq Z_t \leq a_{X_t}^1 + a_{X_t}^2(\tilde{\theta}) + \mathcal{C}\iota a_n) := \tilde{Z}_t^*(\tilde{\theta}), \qquad (0.2)$$

for some $\mathcal{C} > 0$ and for any $t$ such that $|X_t - x_0| \leq h_1$.

By assumptions (A1) and (A2.b), since $a_x^1 + a_x^2(\tilde{\theta}) \xrightarrow[x\to x_0]{} \beta_0$, we can see that, for $n$ sufficiently large and for any $t \in \{t : |X_t - x_0| \leq h_1, Z_t \leq a_{X_t}^1 + a_{X_t}^2 + \mathcal{C}\iota a_n\}$, $G_{X_t}(Z_t) \leq G_{x_0}(\beta_0) + \epsilon < 1$, for some $\epsilon > 0$. So, as $n \to \infty$,

$$\begin{aligned}&\mathbb{V}ar[\frac{\delta_t}{\bar{G}_{X_t}(Z_t)} Z_t^* K_1(X_{ht})] \\ &\leq C\mathbb{E}[\frac{\delta_t}{\bar{G}_{X_t}(Z_t)} \tilde{Z}_t^* K_1^2(X_{ht})] \\ &= C\mathbb{E}\{[F_{X_t}(a_{X_t}^1 + a_{X_t}^2 + \mathcal{C}\iota a_n) - F_{X_t}(a_{X_t}^1 + a_{X_t}^2 - \mathcal{C}\iota a_n)]K_1^2(X_{ht})\} \\ &\leq C\iota a_n h_1, \qquad (0.3)\end{aligned}$$

where in the last inequality we have used the facts that $\mathbb{E}(K_1^2(X_{ht})) = O(h_1)$, and, by Taylor development, $F_x(a_x^1 + a_x^2(\tilde{\theta}) + \mathcal{C}\iota a_n) - F_x(a_x^1 + a_x^2(\tilde{\theta}) - \mathcal{C}\iota a_n) \leq C\iota a_n$.

Using Cauchy-Schwartz inequality, (0.3) implies that

$$|C_j| \leq \mathbb{V}ar[Z_j^*[\delta_j/\bar{G}_{X_j}(Z_j)]K_1(X_{hj})] = o(h_1). \tag{0.4a}$$

Remark also that, by Assumption (A3.c), we have, for any $j \geq j_*$

$$\begin{aligned} |C_j| &\leq \mathbb{E}|Z_1^* Z_{j+1}^* [\delta_1/\bar{G}_{X_1}(Z_1)][\delta_{j+1}/\bar{G}_{X_{j+1}}(Z_{j+1})]K_1(X_{h1})K_1(X_{h(j+1)})| \\ &\qquad + [\mathbb{E}|Z_j^*[\delta_j/\bar{G}_{X_j}(Z_j)]K_1(X_{hj})|]^2 \\ &\leq C\mathbb{E}[K_1(X_{h1})K_1(X_{h(j+1)})] + C[\mathbb{E}|K_1(X_{h1})|]^2 \\ &\leq C u_0^2 M_* h_1^2 + C[\mathbb{E}(K_1(X_{ht}))]^2 \\ &= O(h_1^2). \end{aligned} \tag{0.4b}$$

By applying Billingsley's inequality, see e.g. Corollary 1.1 in Bosq (1998), as $n \to \infty$,

$$|C_j| \leq C j^{-\nu}. \tag{0.4c}$$

Let $0 < k_n \to \infty$. From (0.4) it follows that $\sum_{j=1}^{n} |C_j| \leq \sum_{j=1}^{j_*} |C_j| + \sum_{j=j_*+1}^{k_n} |C_j| + \sum_{j \geq k_n+1} |C_j| = o(h_1) + O(k_n h_1^2) + O(k_n^{1-\nu})$. This together with (0.1) and (0.3) leads to $\mathbb{V}ar[\Delta_n^0] = o(1) + O(k_n h_1) + O(h_1^{-1} k_n^{1-\nu})$, which converges to 0 whenever $k_n = h_1^{-s}$, with $(\nu-1)^{-1} < s < 1$. We deduce that $\Delta_n^0 - \mathbb{E}[\Delta_n^0] = o_p(1)$. The same procedure can also be applied to show that $\Delta_n^1 - \mathbb{E}[\Delta_n^1] = o_p(1)$, hence, we conclude that

$$||[V_n(\theta) - V_n(\tilde{\theta})] - \mathbb{E}[V_n(\theta) - V_n(\tilde{\theta})]|| = o_p(1). \tag{0.5a}$$

On the other hand, using (0.2), as $n \to \infty$,

$$||\mathbb{E}[V_n(\theta) - V_n(\tilde{\theta})]|| \leq C a_n \mathbb{E}[\sum_{t=1}^{n} \frac{\delta_t}{\bar{G}_{X_t}(Z_t)} \tilde{Z}_t^*(\tilde{\theta}) K_1(X_{ht})] \tag{0.5b}$$

$$||V_n(\theta) - V_n(\tilde{\theta})|| \leq C a_n \sum_{t=1}^{n} \frac{\delta_t}{\bar{G}_{X_t}(Z_t)} \tilde{Z}_t^*(\tilde{\theta}) K_1(X_{ht}). \tag{0.5c}$$

Note that the right part of the inequalities (7.5b) and (7.5c) do not depend on $\theta$. Moreover, following the same treatment as we have done above, see (0.3),

$$\mathbb{E}[a_n \sum_t \frac{\delta_t}{\bar{G}_{X_t}(Z_t)} \tilde{Z}_t^* K_1(X_{ht})] \leq C\iota$$

Therefore, by letting $\iota \to 0$, we get

$$||\mathbb{E}[V_n(\theta) - V_n(\tilde{\theta})]|| = o_p(1) \qquad ||V_n(\theta) - V_n(\tilde{\theta})|| = o_p(1). \tag{0.5d}$$

The desired uniform consistency given in (L1) follows from (0.5) by using a chaining argument as in Hallin et al. (2005).

**Proof of (L2)**

First note that, by definition of $V_n(\theta)$,

$$\mathbb{E}[V_n(\theta) - V_n(0)] = na_n\mathbb{E}[\tilde{b}(\theta, X_t)\tilde{X}_{ht}K_1(X_{ht})],$$

where $\tilde{b}(\theta, x) = F_x(a_x^1) - F_x(a_x^1 + a_x^2(\theta))$.

By Taylor development, we have that, for some $0 < \eta < 1$, $\tilde{b}(\theta, x) = -a_x^2(\theta)f_x(a_x^1 + \eta a_x^2(\theta))$. This implies that

$$\mathbb{E}[V_n(\theta) - V_n(0)] = -h_1^{-1}\mathbb{E}[\tilde{X}_{ht}\tilde{X}_{ht}^T f_{X_t}(a_{X_t}^1 + \eta a_{X_t}^2(\theta))K_1(X_{ht})]\theta.$$

To complete the proof observe that, by Assumption (A2.a) and (A2.b),

$$\sup_{||\theta||\leq M, |x-x_0|\leq h_1} |f_x(a_x^1 + \eta a_x^2(\theta)) - f_{x_0}(\beta_0)| \to 0, \text{ and}$$

$$\frac{1}{h_1}\int\left(\frac{x-x_0}{h_1}\right)^i K_1\left(\frac{x-x_0}{h_1}\right) f_0(x)dx \to f_0(x_0)u_i, \;\; i = 0, 1, 2.$$

**Proof of (L3)**

Remark that $V_n(0) = (V_n^0(0), V_n^1(0))^T$, with

$$V_n^i(0) = a_n\sum_t[\pi - I(Z_t < a_{X_t}^1)\delta_t/\bar{G}_{X_t}(Z_t)]X_{ht}^i K_1(X_{ht}), \; i = 0, 1.$$

We have that

$$\mathbb{E}[V_n^0(0)] = na_n\int(\pi - F_x(a_x^1))K_1(\frac{x-x_0}{h_1})f_0(x)dx.$$

By first and second order Taylor development of $t \to F_x(t)$ and $x \to Q_\pi(x)$, respectively, we can see that, for some $0 < \eta_1, \eta_2 < 1$,

$$\begin{aligned}\pi - F_x(a_x^1) &= F_x(Q_\pi(x)) - F_x(a_x^1)\\ &= [Q_\pi(x) - a_x^1]f_x(a_x^1 + \eta_1(Q_\pi(x) - a_x^1))\\ &= 2^{-1}(x-x_0)^2\ddot{Q}_\pi(x + \eta_2(x-x_0))f_x(a_x^1 + \eta_1(Q_\pi(x) - a_x^1)).\end{aligned}$$

By assumption (A2.a) and (A2.b) and the fact that $nh_1^5 = O(1)$ (see assumption (i) in the statement of the theorem), we deduce that $\mathbb{E}[V_n^0(0)] = na_nh_1^3(u_2/2)\ddot{Q}_\pi(x_0)f(x_0, \beta_0) + o(1) = O(na_nh_1^3) + o(1) = O(1)$. Now, we need to show that $\mathbb{V}(V_n^0(0)) = O(1)$. This can be done by first noticing that $\mathbb{V}ar\{[\pi - I(Z_t < a_{X_t}^1)\delta_t/\bar{G}_{X_t}(Z_t)]K_1(X_{ht})\} \leq C\mathbb{E}[K_1^2(X_{ht})] = O(h_1)$ and then by following a similar treatment as we have done above for $\mathbb{V}ar(\Delta_n^0)$ in the proof of (L1). So, this shows that $V_n^0(0) = O_p(1)$. Similarly one can also verify that $V_n^1(0) = O_p(1)$. From this we conclude that $V_n(0) = O_p(1)$.

**Proof of (L4)**

Using the fact that $\delta_t/\bar{\hat{G}}_{X_t}(Z_t)$ and $K_1(X_{ht})$ are nonnegative quantities, it is easy to check that , for a given $\theta$, $\lambda \to -\theta\hat{V}_n(\lambda\theta)$ is a nondecreasing function which implies the desired result.

**Proof of (L5)**
(L5) is a direct application of the following result :

**Lemma 2** *For any random vectors* $\mathbf{X}_t \in \mathbb{R}^p$ *and* $(A_t, B_t, C_t)^T \in \mathbb{R}^3$, $t = 1\ldots, n$, *let* $\theta_n = \arg\min_{\theta\in\mathbb{R}^p} \sum_t [A_t - \theta^T\mathbf{X}_t][\pi - B_t I(A_t < \theta^T\mathbf{X}_t)]C_t$. *If* $B_t$ *and* $C_t \geq 0$, $\mathbf{X}_t$ *is continuous and* $||\theta_n|| < \infty$ *then, with probability one*

$$||\sum_t \mathbf{X}_t[\pi - B_t I(A_t < \theta_n^T\mathbf{X}_t)]C_t|| \leq p \max_t ||B_t C_t \mathbf{X}_t||$$

The proof of this lemma follows along the same lines as in the proof of Lemma A.2 in Ruppert and Carroll (1980).

**Proof of (L6)**
Since $K_1$ is nonnegative,

$$a_n||\hat{V}_n(\theta) - V_n(\theta)|| \leq [\frac{1}{nh_1}\sum_{i=1}^n K_1\left(\frac{X_i - x_0}{h_1}\right)] \sup_{t\in A_n(\theta)} \frac{|\hat{G}_{X_t}(Z_t) - G_{X_t}(Z_t)|}{\bar{\hat{G}}_{X_t}(Z_t)\bar{G}_{X_t}(Z_t)},$$

where $A_n(\theta) = \{t : |X_t - x_0| \leq h_1 \text{ and } Z_t < \beta_0 + \beta_1(X_t - x_0) + a_n\theta^T\tilde{X}_{ht}\}$.

For $n$ sufficiently large and for any $\theta$ such that $||\theta|| \leq M$, using the fact that $K_1$ has a compact support, assumption (A1) and (A2.b), one can find a neighborhood $\tilde{J} \subset J$ of $x_0$ and an $\tilde{\epsilon} > 0$ such that, if $t \in A_n(\theta)$ then $X_t \in \tilde{J}$, $Z_t \leq \beta_0 + \tilde{\epsilon} < \mathcal{T}_{x_0}$ and $G_{X_t}(Z_t) \leq G_{x_0}(\beta_0) + \tilde{\epsilon} < 1$. On the other hand, by Theorem 3.1(II) in El Ghouch and Van Keilegom (2008), we have that

$$\sup_{x\in\tilde{J}} \sup_{s\in[0,\beta_0+\tilde{\epsilon}]} |\hat{G}_x(s) - G_x(s)| = O_p(h_0^2),$$

whenever Assumptions (A3) and (A4) and Assumptions (ii) and (iv) given in the statement of the theorem are fulfilled. To conclude the proof, one can easily check that, by Assumptions (A2.a) and (A3.c),

$$\frac{1}{nh_1}\sum_{i=1}^n K_1\left(\frac{X_i - x_0}{h_1}\right) = O_p(1).$$

Now that we have shown (L1)-(L6), we continue with the proof of Theorem 1. (L1), (L2) and (L6) imply that

$$\begin{aligned}||\hat{V}_n(\theta) + D\theta - V_n(0)|| &\leq ||\hat{V}_n(\theta) - V_n(\theta)|| + ||(V_n(\theta) - V_n(0)) - \mathbb{E}(V_n(\theta) - V_n(0))|| \\ &\quad + ||\mathbb{E}(V_n(\theta) - V_n(0)) + D\theta|| \\ &= o_p(1) + O_p(a_n^{-1}h_0^2), \qquad (0.6)\end{aligned}$$

uniformly over $\{\theta : ||\theta|| \leq M\}$. This together with (L3), (L4), (L5), (iii) (see statement of the theorem) and Lemma 1, implies that $||\hat{\theta}|| = O_p(1)$, which, by (0.6), leads to

$$\begin{aligned}\hat{\theta} &= D^{-1}V_n(0) + O_p(a_n^{-1}h_0^2) + o_p(1)\\ &= \Lambda_u^{-1}/f(x_0,\beta_0)[a_n \sum_t e_t \tilde{X}_{ht}K_1(X_{ht}) + B_n] + O_p(a_n^{-1}h_0^2) + o_p(1),\end{aligned}$$

where $e_t$ is defined in the theorem and $B_n = (B_n^0, B_n^1)^T$, with

$$B_n^i = a_n \sum_{i=1}^{n} [I(Z_t < Q_\pi(X_t)) - I(Z_t < a_{X_t}^1)] \frac{\delta_t}{\bar{G}_{X_t}(Z_t)} X_{ht}^i K_1(X_{ht}), \text{ for } i = 0, 1.$$

To get exactly the asymptotic expression given in Theorem 1, and so to conclude the proof, we still have to show that $B_n = (a_n^{-1}h_1^2/2)(u_2,u_3)^T\ddot{Q}_\pi(x_0)f(x_0,\beta_0) + o_p(a_n^{-1}h_1^2) + o_p(1)$. This can be done by checking that, for $i = 0, 1$,

$$\begin{aligned}\mathbb{E}(B_n^i) &= na_n\mathbb{E}\{[F_{X_t}(Q_\pi(X_t)) - F_{X_t}(a_{X_t}^1)]X_{ht}^i K_1(X_{ht})\}\\ &= a_n^{-1}h_1^2/2[\ddot{Q}_\pi(x_0)f(x_0,\beta_0)u_{i+2} + o(1)],\end{aligned} \quad (0.7a)$$

and

$$\mathbb{V}ar(B_n^i) = o(1). \quad (0.7b)$$

(0.7a) and (0.7b) can be proved by following the same treatment as we have done above for $\mathbb{E}[V_n^i(0)]$, see the proof of (L3), and for $\mathbb{V}ar[\Delta_n^i]$, see the proof of (L1), respectively.

**Proof of Theorem 2.** In order to establish the asymptotic normality, it suffices, by Theorem 1, to show that

$$A_n := a_n \sum_{t=1}^{n} e_t \tilde{X}_{ht} K_1(X_{ht}) \xrightarrow{\mathcal{L}} \mathcal{N}\left(0, f_0(x_0)\zeta(x_0,\beta_0)\Omega_v\right).$$

By the Cramer-Wold device, this is equivalent to showing that for any linear combination $c^T A_n \xrightarrow{\mathcal{L}} \mathcal{N}\big(0, f_0(x_0)\zeta(x_0,\beta_0)c^T\Omega_v c\big)$. First note that $\mathbb{E}(e_t|X_t) = \pi - F_{X_t}(Q_\pi(X_t)) = 0$, and

$$\mathbb{E}(e_t^2|X_t) = \mathbb{E}[\frac{\delta_t}{\bar{G}_{X_t}^2(Z_t)} I(Z_t < Q_\pi(X_t))|X_t] - \pi^2 = \zeta_\pi(X_t, Q_\pi(X_t)).$$

On the other hand, by assumption (A2.a) and (A5), we have that

$$\frac{1}{h_1}\int \zeta_\pi(x,Q_\pi(x))\left(\frac{x-x_0}{h_1}\right)^i K_1\left(\frac{x-x_0}{h_1}\right) f_0(x)dx \to \zeta_\pi(x_0,\beta_0)f_0(x_0)v_i,\ i=0,1,2.$$

This implies that $\mathbb{V}ar[e_t c^T\tilde{X}_{ht}K_1(X_{ht})] = h_1[f_0(x_0)\zeta_\pi(x_0,\beta_0)c^T\Omega_v c + o(1)]$. Now, by stationarity, $\mathbb{V}ar[c^T A_n] = 1/(nh_1)\left\{n\mathbb{V}ar[e_t c^T\tilde{X}_{ht}K_1(X_{ht})] + 2n\sum_{j=1}^n(1-j/n)C_j^+\right\}$, where $C_j^+ = \mathbb{C}ov(e_1 c^T \tilde{X}_{h1} K_1(X_{h1}), e_{j+1} c^T \tilde{X}_{h(j+1)} K_1(X_{h(j+1)}))$. Using Assumption (A3.c) (with $j_* = 1$), we can easily see that $C_j^+ \leq C\mathbb{E}[K_1(X_{h1})K_1(X_{h(j+1)})] = O(h_1^2)$, for any $j \geq 1$. So, by an appropriate choice of $k_n \to 0$ and using Billingsley's inequality, $\sum_{j=1}^{n} |C_j^+| \leq \sum_{j=1}^{k_n} |C_j| + \sum_{j \geq k_n+1} |C_j^+| = O(k_n h_1^2) + O(k_n^{1-\nu}) = o(h_1)$. Thus, we have shown that $\mathbb{E}[c^T A_n] = 0$, and $\mathbb{V}ar[c^T A_n] \to f_0(x_0)\zeta(x_0, \beta_0) c^T \Omega_v c$.

It remains to prove that $c^T A_n$ is asymptotically normal. This can be done by using the well known small-blocks and large-blocks technique and then by verifying the standard Lindeberg-Feller conditions exactly as was done, for example, in Masry and Fan (1997). Details are omitted.

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