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Title	Bayesian analysis of spatially-dependent functional responses with spatially-dependent multi-dimensional functional predictors
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the expansion coefficients associated with $\alpha_i(\mathbf{s})$, $\xi_{jk}(\mathbf{s})$, and $\theta_i(\mathbf{s})$, respectively. As with the $\boldsymbol{\delta}(d)$ coefficients, if one is interested in considering the spatial predictors as functionals, then it would be appropriate to expand the pth element of $\mathbf{z}(\mathbf{s})$ as $z_p(\mathbf{s}) = \sum_{\ell=1}^{\infty} w_\ell(\mathbf{s}) q_{p\ell}$. Substituting the above expansions into (2.3), we obtain the representation

$$\sum_{i=1}^{\infty}\sum_{\ell=1}^{\infty} w_\ell(\mathbf{s})\psi_i(d)a_{i\ell} = \sum_{j=1}^{J}\sum_{k=1}^{\infty}\sum_{i=1}^{\infty}\sum_{\ell=1}^{\infty} w_\ell(\mathbf{s})\psi_i(d) f_{jk\ell} b_{jki} + \mathbf{z}'(\mathbf{s})\boldsymbol{\delta}(d) + \sum_{i=1}^{\infty}\sum_{\ell=1}^{\infty} w_\ell(\mathbf{s})\psi_i(d) g_{i\ell}. \quad (2.4)$$

From a functional data analysis perspective, using the spatia[illegible] to a[illegible]modate spatial structure is quite reasonable. However, one could alternatively co[illegible] a more traditional spatial co-kriging approach (see the references in Section 1). We [illegible]e the basis expansion formulation here because our application in Sect[illegible]ned with fairly smooth functional spatial surfaces, which, along with the potential for future "big data" applications, is facilitated by the use of rank[illegible]ced spatial models (e.g., see the review in Wikle (2010)).

Similar to applications in traditional functional data [illegible]sis, one can consider finite approximations to the infinite summations i[illegible]), e.g.

$$\sum_{i=1}^{n_i}\sum_{\ell=1}^{n_\ell} w_\ell(\mathbf{s})\psi_i(d)a_{i\ell} = \sum_{j=1}^{J}\sum_{k=1}^{n_k}\sum_{i=1}^{n_i}\sum_{\ell=1}^{n_\ell} w_\ell(\mathbf{s})\psi_i(d) f_{jk\ell} b_{jki} + \mathbf{z}'(\mathbf{s})\boldsymbol{\delta}(d) + \sum_{i=1}^{n_i}\sum_{\ell=1}^{n_\ell} w_\ell(\mathbf{s})\psi_i(d) g_{i\ell}. \quad (2.5)$$

In practice, th[illegible] n_i, and n_ℓ are typically problem specific and can be [illegible] based on p[illegible]ent[illegible]ariance explained, cross-validation, and/or sensitivity analysis. [illegible] hierarchical Bayesian implementation (see Section 2.2), one can account for pote[illegible]l truncation and observation error correspond to the response and co[illegible]s. Given these truncations, we denote the basis vectors associated wi[illegible] and spatial location by $\boldsymbol{\psi}(d) \equiv [\psi_1(d), \ldots, \psi_{n_i}(d)]'$ and $\mathbf{w}(\mathbf{s}) \equiv$