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Detecting sparse cone alternatives for Gaussian random fields, with an application to fMRI

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Abstract: Our problem is to find a good approximation to the P-value of the maximum of a random field of test statistics for a cone alternative at each point in a sample of Gaussian random fields. These test statistics have been proposed in the neuroscience literature for the analysis of fMRI data allowing for unknown delay in the hemodynamic response. However the null distribution of the maximum of this 3D random field of test statistics, and hence the threshold used to detect brain activation, was unsolved. To find a solution, we approximate the P-value by the expected Euler characteristic (EC) of the excursion set of the test statistic random field. Our main result is the required EC density, derived using the Gaussian Kinematic Formula.

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