

# Orthogonalized Moment Aberration for Multi-Stratum Factorial Designs

**Ming-Chung Chang**

*Institute of Statistical Science, Academia Sinica*

## ABSTRACT

Multi-stratum factorial designs, such as block designs and row–column designs, are widely used for screening treatment factors in experiments involving complex structures of experimental units due to multiple sources of error. In this study, we propose a unified model-free approach, termed orthogonalized moment aberration, to compare the similarities between level combinations of treatment factors assigned to heterogeneous experimental units. The proposed approach, which uses kernel functions to evaluate the rows of design matrices rather than the columns, can assess a wide variety of mixed-level regular/nonregular factorial designs with an extensive class of heterogeneous experimental unit structures called partially-relaxed orthogonal block structures. This approach is flexible in that it can be adapted to various scenarios by choosing different kernel functions, with certain choices yielding well-known minimum aberration criteria proposed in the literature. Although model-free, the proposed method is justified by using linear mixed- effect models and Gaussian process models. Theoretical results and numerical examples presented in this article collectively demonstrate that the proposed approach can generate multi-stratum factorial designs with high D-efficiencies within a Bayesian framework.

**Keywords:** Block design; Minimum aberration; Minimum moment aberration; Orthogonal array; Split-plot design

# Efficient and Robust Block Designs for Order-of-Addition Experiments

**Chang-Yun Lin**

*Department of Applied Mathematics and Institute of Statistics, National Chung Hsing University,  
Taichung, Taiwan, 40227*

## ABSTRACT

Designs for Order-of-Addition (OofA) experiments have received growing attention due to their impact on responses based on the sequence of component addition. In certain cases, these experiments involve heterogeneous groups of units, which necessitates the use of blocking to manage variation effects. Despite this, the exploration of block OofA designs remains limited in the literature. As experiments become increasingly complex, addressing this gap is essential to ensure that the designs accurately reflect the effects of the addition sequence and effectively handle the associated variability.

Motivated by this, this paper seeks to address the gap by expanding the indicator function framework for block OofA designs. We propose the use of the word length pattern as a criterion for selecting robust block OofA designs. To improve search efficiency and reduce computational demands, we develop algorithms that employ orthogonal Latin squares for design construction and selection, minimizing the need for exhaustive searches.

Our analysis, supported by correlation plots, reveals that the algorithms effectively manage confounding and aliasing between effects. Additionally, simulation studies indicate that designs based on our proposed criterion and algorithms achieve power and type I error rates comparable to those of full block OofA designs. This approach offers a practical and efficient method for constructing block OofA designs and may provide valuable insights for future research and applications.

**Keywords:** Component orthogonal array; indicator function; position-based; orthogonal Latin squares; word length pattern

# Results on Large Strong Orthogonal Arrays of Strength Three

**Chenlu Shi<sup>1</sup>**, Ye Tian<sup>2</sup>, and Hongquan Xu<sup>3,\*</sup>

*<sup>1</sup>Department of Mathematical Sciences, New Jersey Institute of Technology*

*<sup>2</sup>School of Mathematical Sciences, Beijing University of Posts and Telecommunications*

*<sup>3</sup>Department of Statistics and Data Science, University of California, Los Angeles*

## ABSTRACT

Strong orthogonal arrays are widely recognized as effective space-filling designs for computer experiments. Among them, those of strength three are particularly useful, since strong orthogonal arrays of strength four or higher may be too expensive for some investigations. Strong orthogonal arrays of strength three that possess some of the space-filling properties of strength four are more desirable. Such arrays with small numbers of factors have been thoroughly investigated, whereas those of large sizes remain relatively unexplored. In this paper, we develop a characterization and construction method for large-sized arrays with better space-filling properties. We further present theoretical and computational results that facilitate the implementation of our construction method. Additionally, we use a simulation study to illustrate the usefulness of arrays produced by our method for developing statistical surrogate models.

**Keywords:** Computer experiment, Latin hypercube, space-filling design, strong orthogonal array

# A Stratified $L_2$ -Discrepancy with Application to Space-Filling Designs

**Hongquan Xu**\*

*Department of Statistics and Data Science, University of California, Los Angeles*

## ABSTRACT

Space-filling designs are widely used in computer experiments. We propose a stratified  $L_2$ -discrepancy to evaluate the uniformity of a design when the design domain is stratified into various subregions. Weights are used to adjust preferences for the uniformity over subregions in each stratification. The stratified  $L_2$ -discrepancy is easy to compute, satisfies a Koksma–Hlawka type inequality, and overcomes the curse of dimensionality that exists for other discrepancies. It is applicable to a broad class of designs, and covers several minimum aberration-type criteria as special cases. Strong orthogonal arrays of maximum strength are shown to have low stratified  $L_2$ -discrepancies, and thus are suitable for computer experiments. In addition, we develop a lower bound for the stratified  $L_2$ -discrepancy and provide a construction method for designs that achieve the lower bound. We further introduce a general version of the stratified  $L_2$ -discrepancy for evaluating designs with flexible stratification properties.

**Keywords:** Computer experiment; curse of dimensionality; generalised minimum aberration; space-filling hierarchy principle; strong orthogonal array.