Information Theory in Volume Visualization

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Today's Menu

- Entropy
- Joint and Conditional Entropy
- Information Channel
- Mutual Information
- Entropy Rate
- Informational Divergence

Entropy

- Random variable X taking values in an alphabet X

$$\mathrm{X}:\{x_1,x_2,\ldots,x_n\}, p(x)=\Pr\{X=x\}, p(X)=\{p(x), x\in\mathrm{X}\}$$

- **Shannon entropy** $H(X)$: uncertainty, information, homogeneity, uniformity

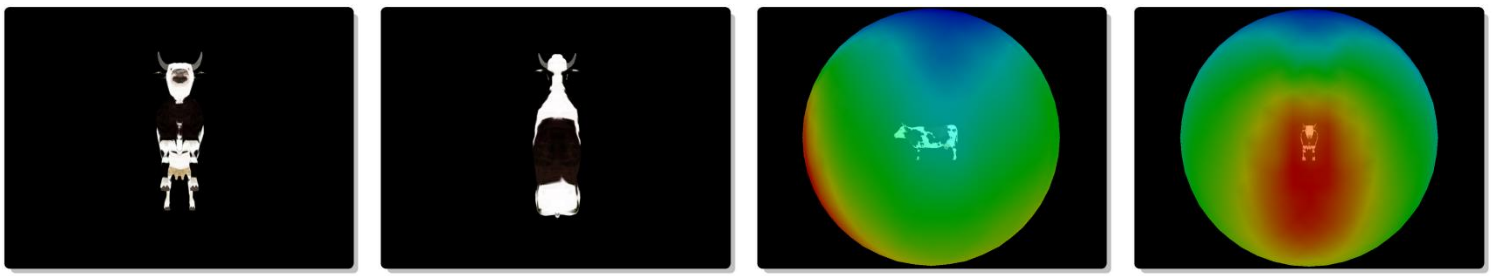
$$H(X)=-\sum_{x\in\mathrm{X}} p(x)\log p(x)\equiv-\sum_{i=1}^{n} p(x_i)\log p(x_i)$$

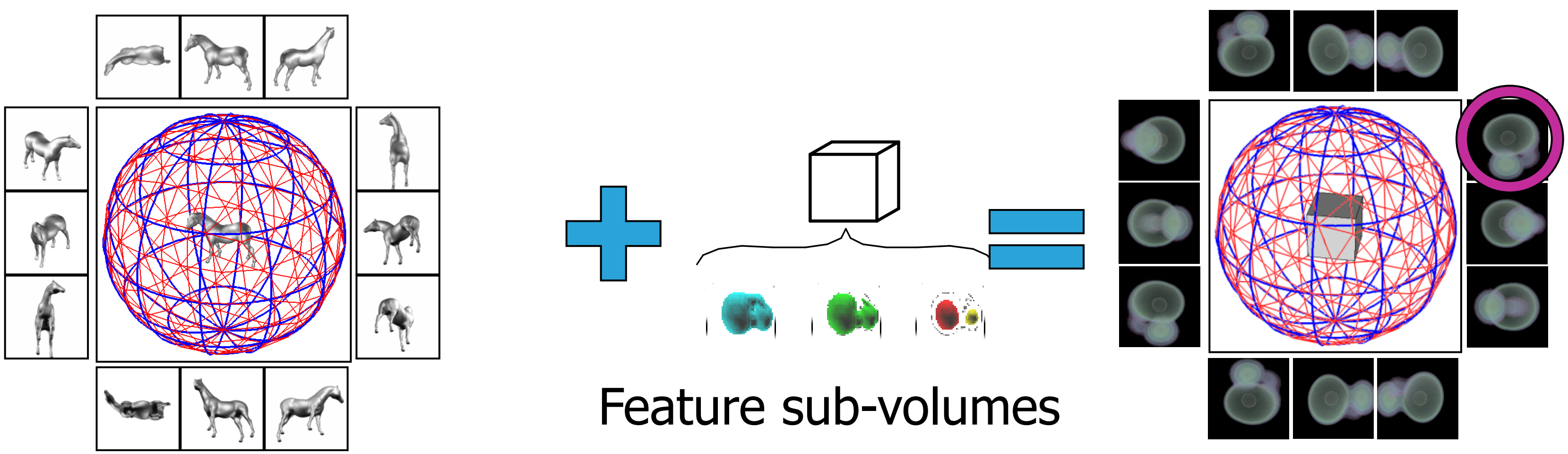
- **Viewpoint entropy** $H(v)$ based on triangle area ratio

$$H(v)=-\sum_{i=0}^{N_f}\frac{a_i}{a_t}\log\frac{a_i}{a_t},$$

View Selection for Set of Iso-Surfaces

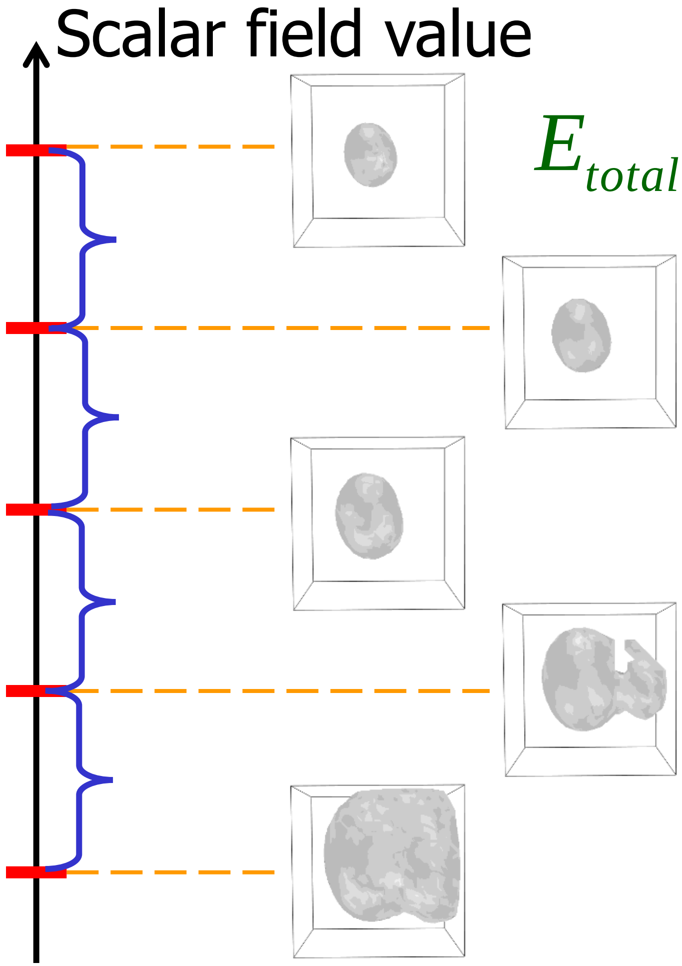
- Decompose volume into feature sub-volumes
- Surface-based method for local optimal viewpoints
- Find global compromise between local optimal viewpoints





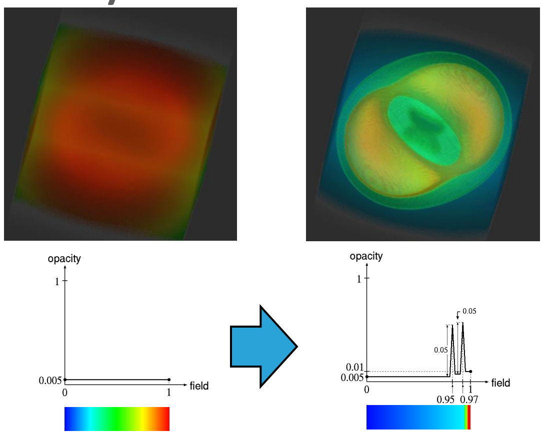
[Takahashi et al. 2005]

View Selection for Set of Iso-Surfaces



$$E_{total} = w_1 E_1 + w_2 E_2 + w_3 E_3 + w_4 E_4 + w_5 E_5$$

- Calculate the weighted sum of the viewpoint entropies of the extracted iso-surfaces
- Assign higher weights to feature iso-surfaces with opacity transfer functions



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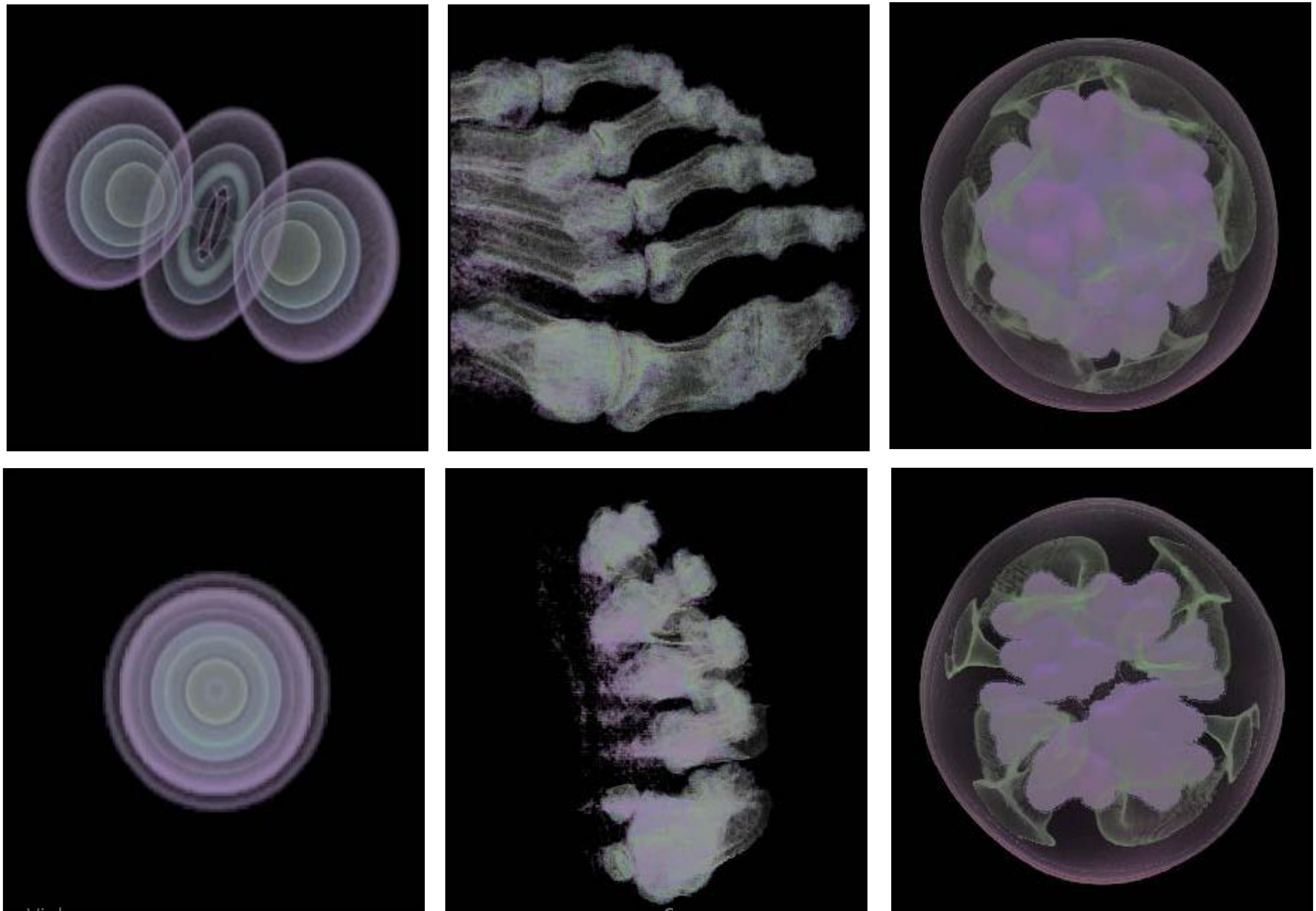
View Selection for Set of Iso-Surfaces



[Takahashi et al. 2005]

Derived Entropy Measures

- Discrete random variable Y in an alphabet Y

$$\mathrm{Y}:\{y_1, y_2, \ldots, y_n\}, p(y) = \Pr\{Y = y\}$$

- **Joint entropy** $H(X,Y)$

$$H(X,Y) = -\sum_{x \in \mathrm{X}} \sum_{y \in \mathrm{Y}} p(x,y) \log p(x,y)$$

- **Conditional entropy** $H(Y|X)$

$$H(Y \mid X) = \sum_{x \in \mathrm{X}} p(x) H(Y \mid x) = -\sum_{x \in \mathrm{X}} p(x) \sum_{y \in \mathrm{Y}} p(y \mid x) \log p(y \mid x)$$

$$= -\sum_{x \in \mathrm{X}} \sum_{y \in \mathrm{Y}} p(x,y) \log p(y \mid x)$$

$$p(x,y) = p(x) p(y \mid x) = p(y) p(x \mid y)$$

View Selection for Volume Data

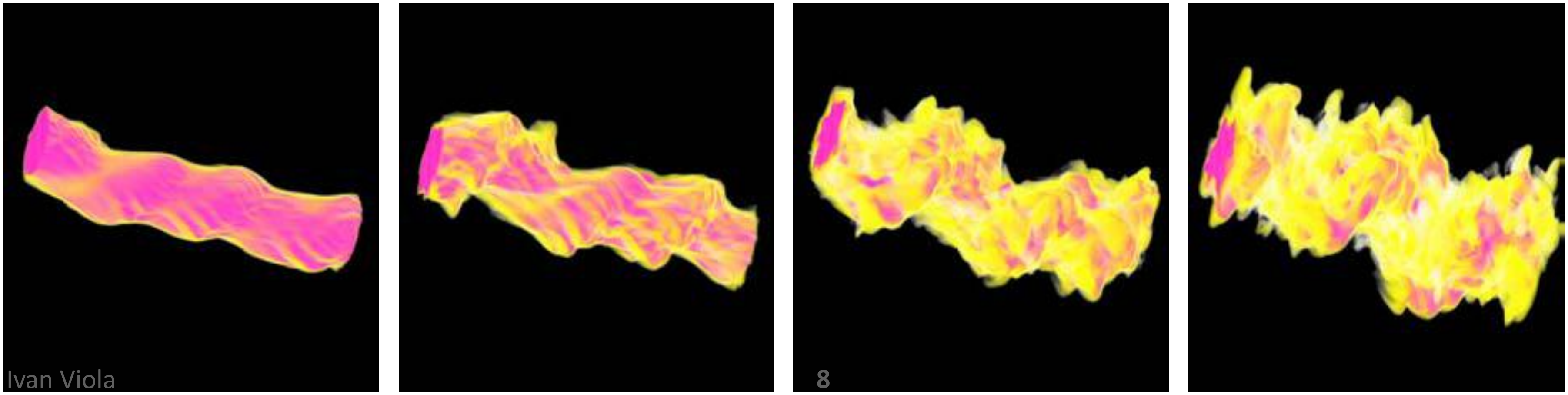
- 3D or 4D scalar fields (over time)
- Probability function q_j

$$q_j \equiv q_j(V) = \frac{1}{\sigma} \cdot \frac{v_j(V)}{W_j} \qquad \text{where, } \sigma = \sum_{j=0}^{J-1} \frac{v_j(V)}{W_j}$$

- Importance W_j : voxel opacity and information content
- View selection for time-series uses Conditional entropy

$$H(X) = H(X_1) + H(X_2 \mid X_1) + \ldots + H(X_n \mid X_{n-1}); H(X_2 \mid X_1) = -\sum_{x_1 \in X_1} \sum_{x_2 \in X_2} p(x_1, x_2) \log p(x_2 \mid x_1)$$

[Bordoloi and Shen 2005]



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Information Channel

- Communication or information channel $X \to Y$

$$\underset{p(X)}{\boldsymbol{X}} \xrightarrow{p(Y|X)} \underset{p(Y)}{\boldsymbol{Y}}$$

$$p(X): \begin{bmatrix} p(x_1) \\ p(x_2) \\ \ldots \\ p(x_n) \end{bmatrix} \qquad p(Y|X): \begin{bmatrix} p(y_1|x_1) & p(y_2|x_1) & \ldots & p(y_m|x_1) \\ p(y_1|x_2) & p(y_2|x_2) & \ldots & p(y_m|x_2) \\ \ldots & \ldots & \ldots & \ldots \\ p(y_1|x_n) & p(y_2|x_n) & \ldots & p(y_m|x_n) \end{bmatrix} \; p(Y|x)$$

$$p(Y): \begin{bmatrix} p(y_1) & p(y_2) & \ldots & p(y_m) \end{bmatrix}$$

$$\sum_{x \in X} p(y|x) = 1$$

$$p(y) = \sum_{x \in X} p(x) p(y|x)$$

Mutual Information

- Mutual information *I(X,Y)*: shared information, correlation, dependence, information transfer

$$I(X,Y) = H(Y) - H(Y \mid X) = \sum_{x \in X} \sum_{y \in Y} p(x,y) \log \frac{p(x,y)}{p(x)\,p(y)}$$

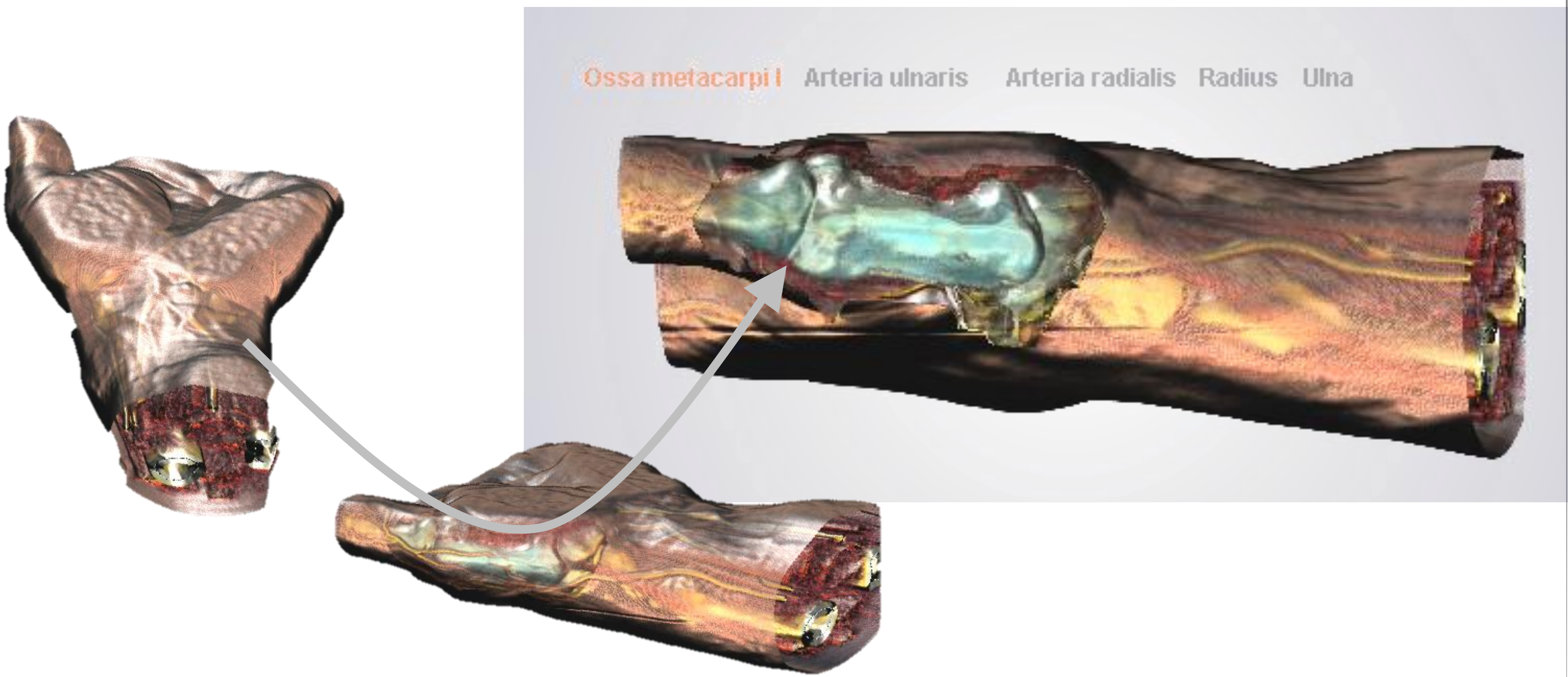
$$= \sum_{x \in X} p(x) \sum_{y \in Y} p(y \mid x) \log \frac{p(y \mid x)}{p(y)}$$

- Viewpoint Mutual Information *I(v,O)*: dependence of viewpoint *v* on set of objects *O*

$$I(V,O) = \sum_{x \in X} \sum_{y \in Y} p(v,o) \log \frac{p(v,o)}{p(v)\,p(o)} = \sum_{v \in V} p(v) \sum_{o \in O} p(o \mid v) \log \frac{p(o \mid v)}{p(o)} = \sum_{v \in V} p(v) I(v,O)$$

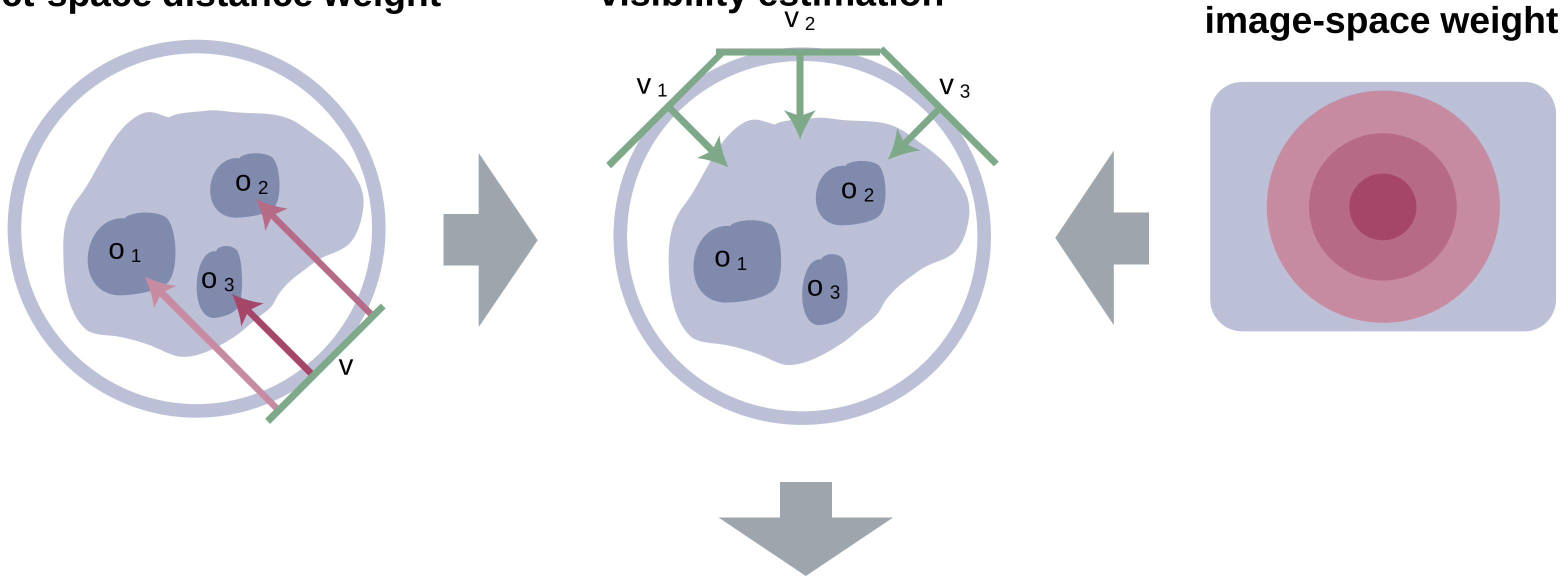
View Selection for Volumetric Objects

- Characteristic view
- Emphasis of focus object
- Guided navigation between characteristic views

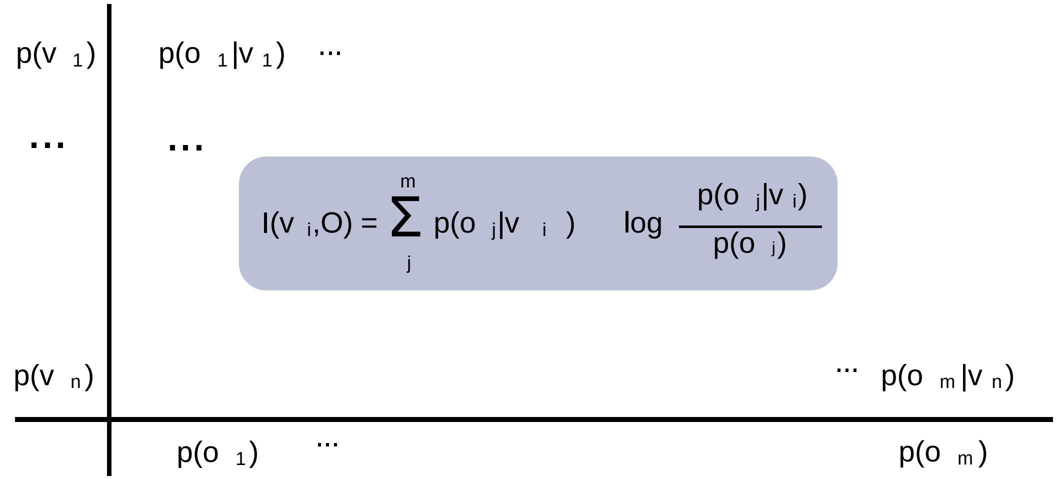


[Viola et al. 2006]

Viewpoint Estimation



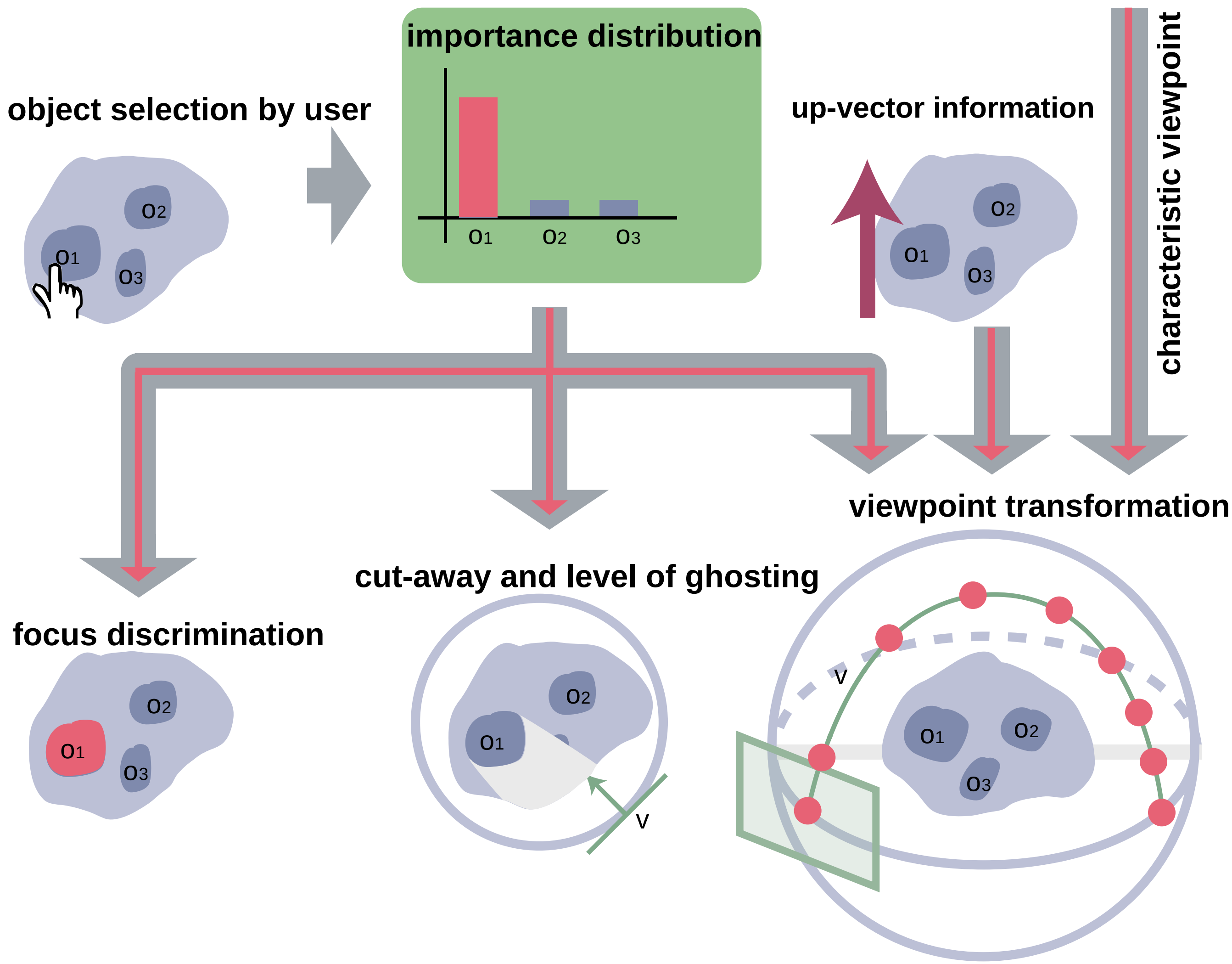
information-theoretic framework for optimal viewpoint estimation



$$I(v_i,O) = \sum_{j}^{m} p(o_j|v_i) \log \frac{p(o_j|v_i)}{p(o_j)}$$

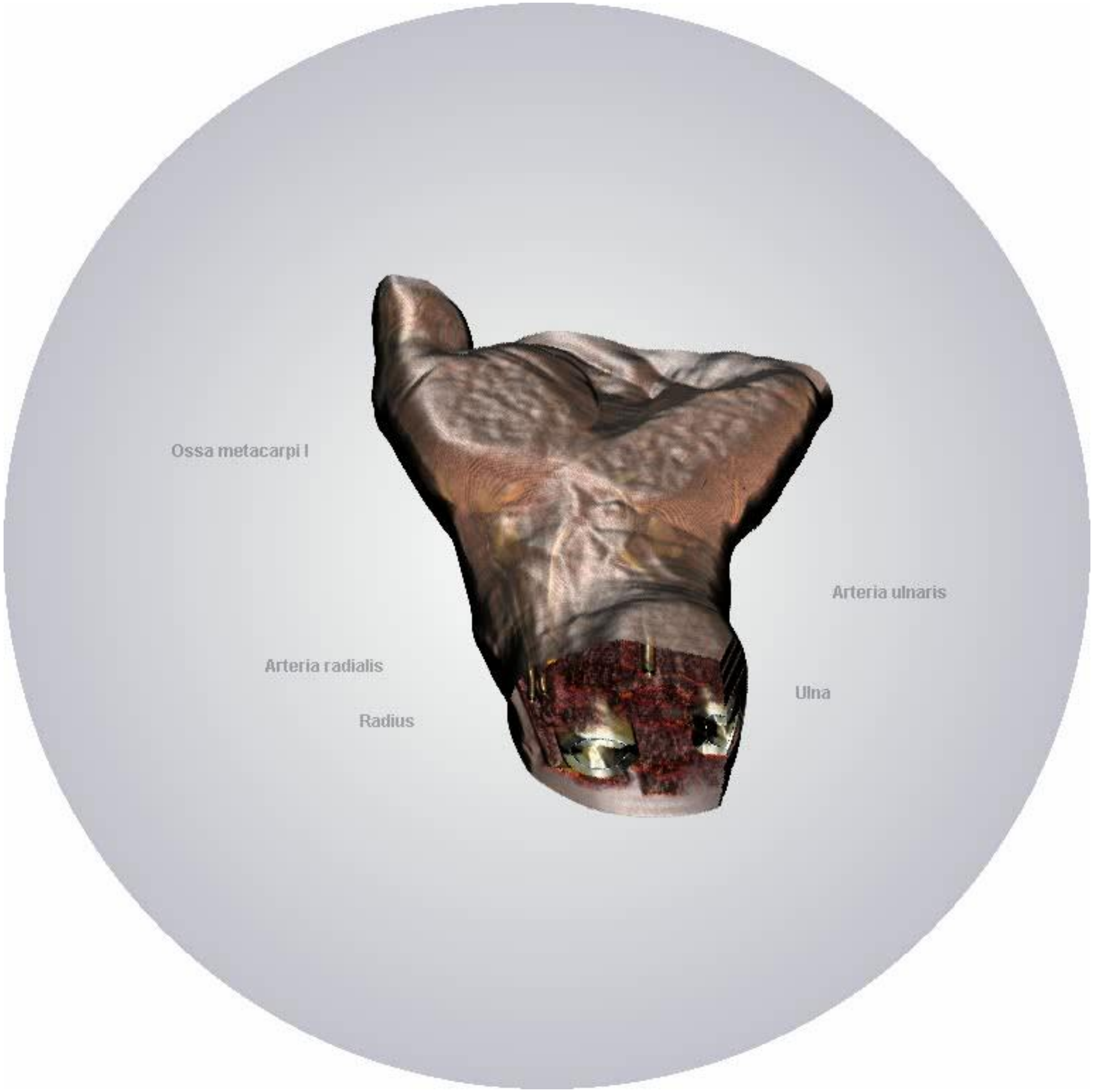
[Viola et al. 2006]

Focus of Attention



[Viola et al. 2006]

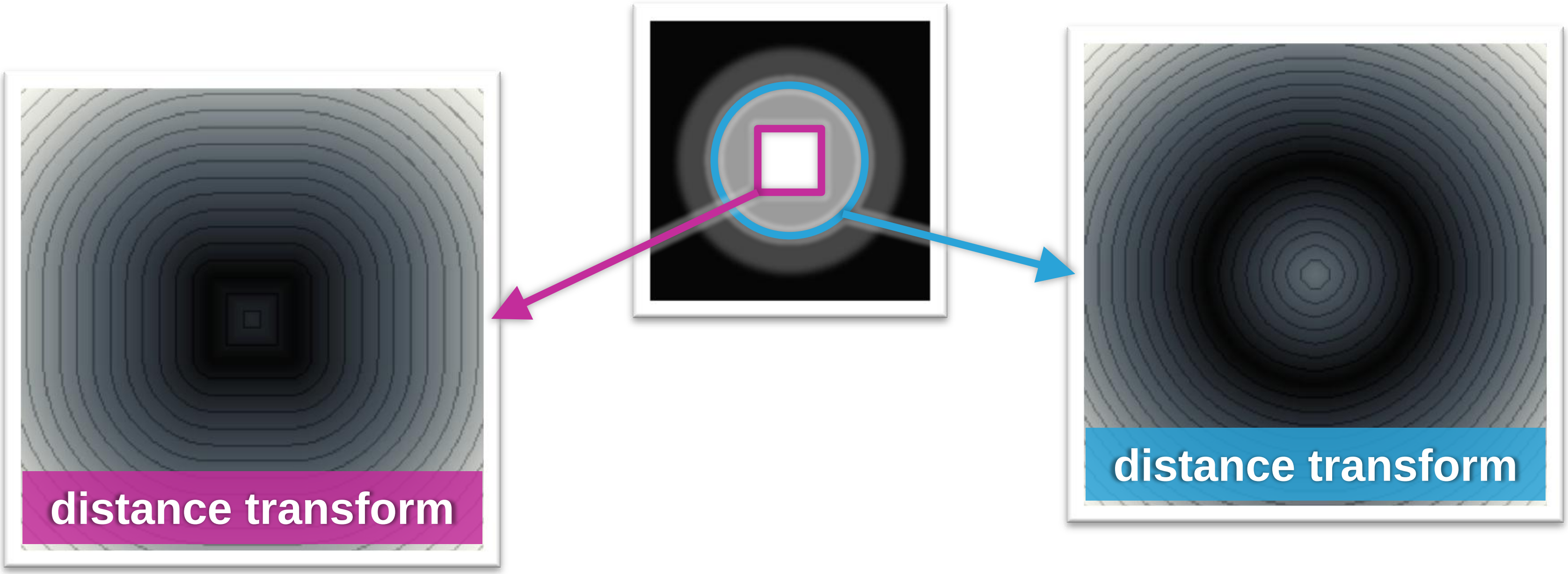
View Selection for Volumetric Objects



[Viola et al. 2006]

Iso-Surface Similarity Maps

- Compare iso-surfaces through evaluating mutual information of their distance volume
  - *X* and *Y* are **independent**: $I(X,Y) = 0$
  - *X* and *Y* are **identical**: $I(X,Y) = H(X) = H(Y)$

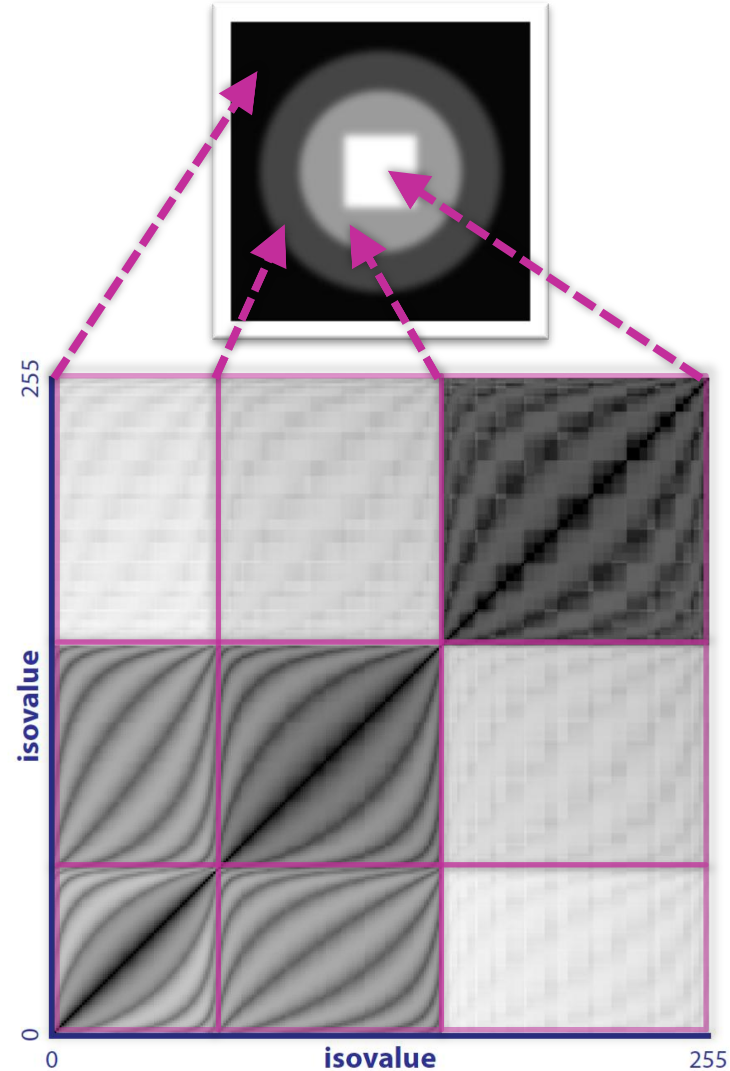


[Bruckner and Möller 2010]

Iso-Surface Similarity Maps

- Normalized measure

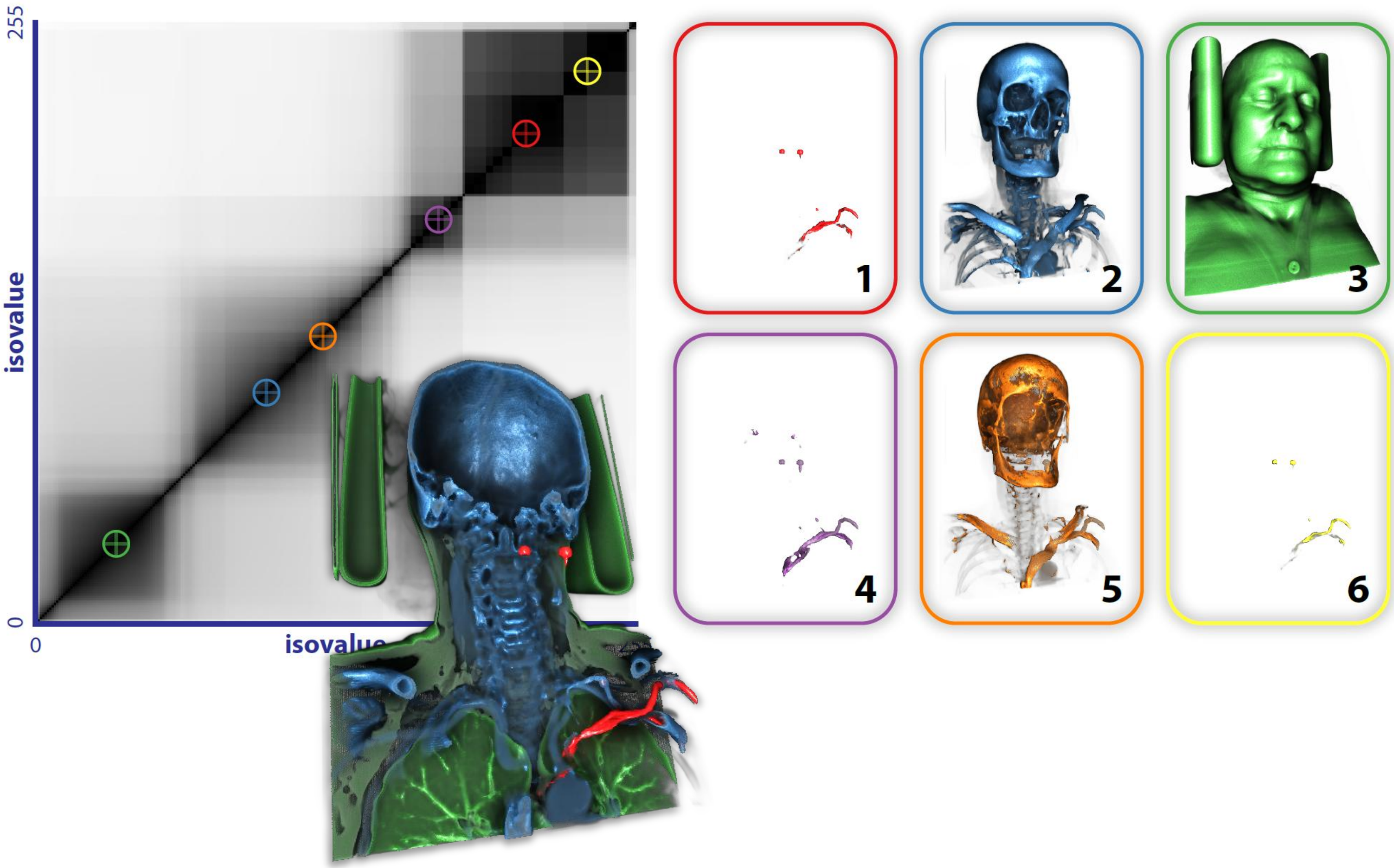
$$\hat{I}(X,Y) = \frac{2I(X,Y)}{H(X)+H(Y)}$$



[Bruckner and Möller 2010]

Iso-Surface Similarity Maps

- Selection of characteristic iso-surfaces



[Bruckner and Möller 2010]

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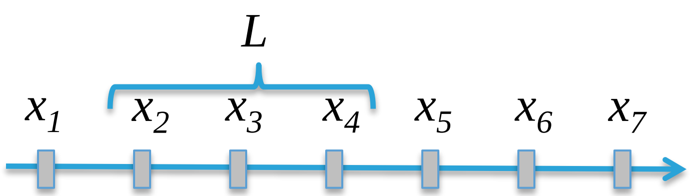
Entropy Rate



- Shannon entropy

$$H(X) = -\sum_{x \in \mathrm{X}} p(x) \log p(x)$$

- Joint entropy of L vector

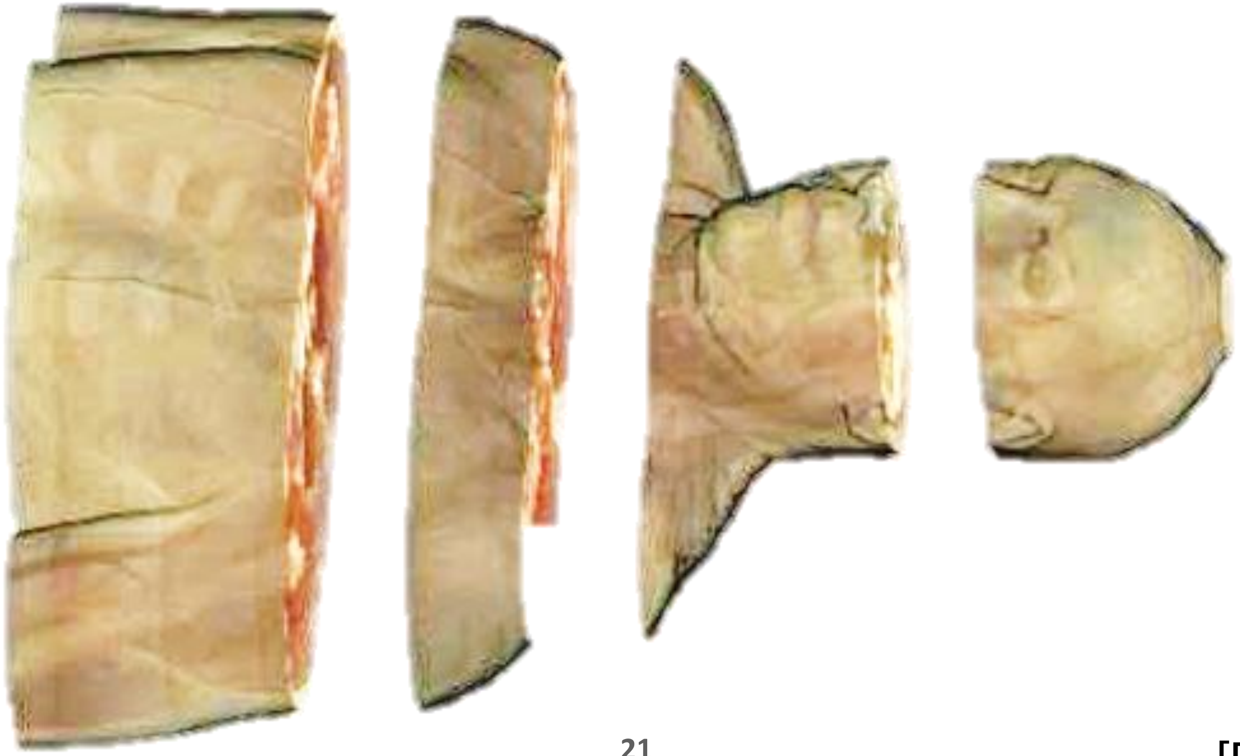
$$H(X^L) = -\sum_{x^L \in \mathrm{X}^L} p(x^L) \log p(x^L)$$

- **Entropy rate**
  represents the average information content per symbol in a stochastic process

$$h = \lim_{L \to \infty} \frac{H(X^L)}{L} = \lim_{L \to \infty} (H(X^L) - H(X^{L-1}))$$

Similarity-Based Exploded Views

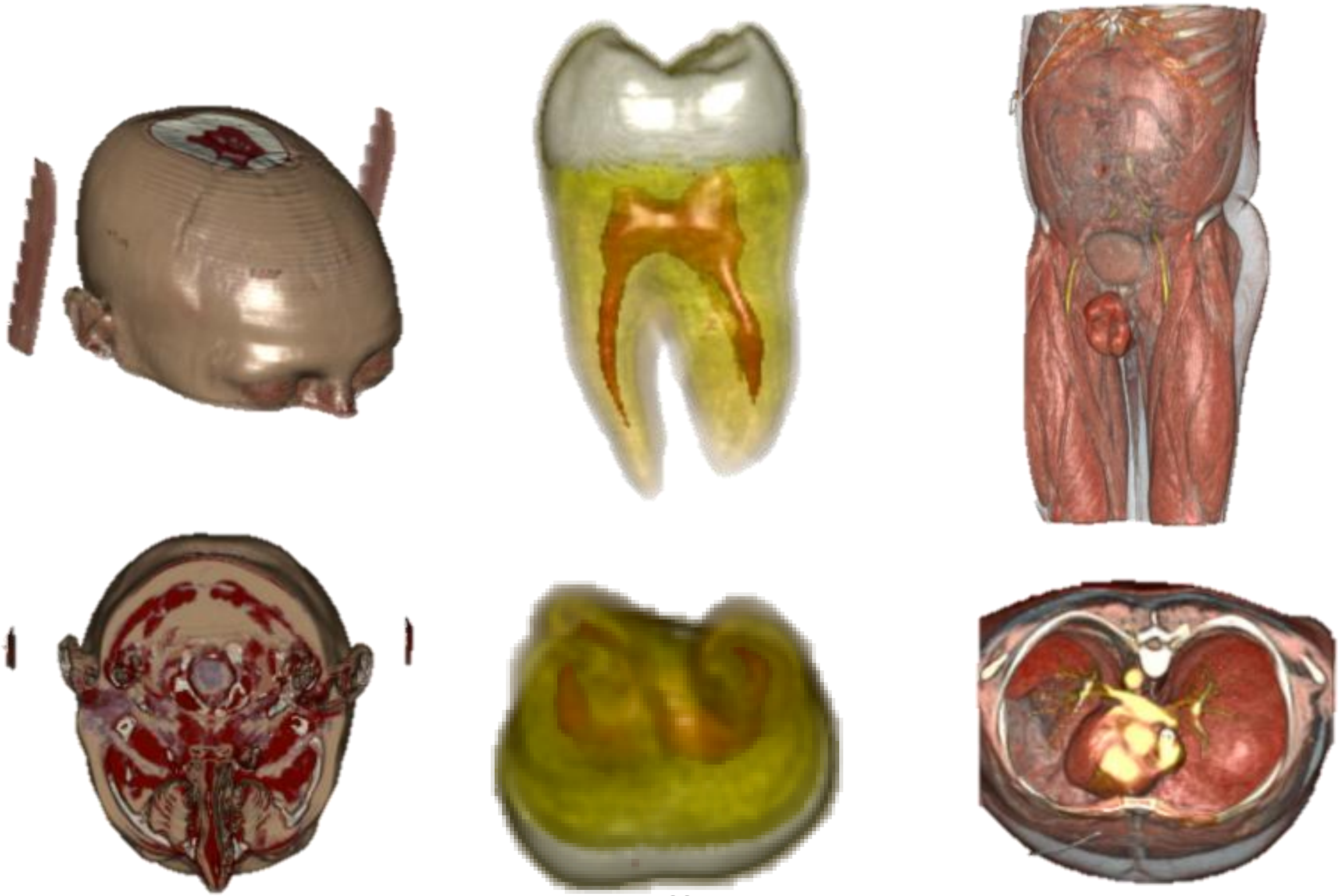
- A two step process is proposed to automatically obtain the partitioning planes:
  - Explosion axis: selection of the most structured view
  - Partitioning of the data: slices are grouped according to the maximization of a similarity criterion



[Ruiz et al. 2008]

Similarity-Based Exploded Views

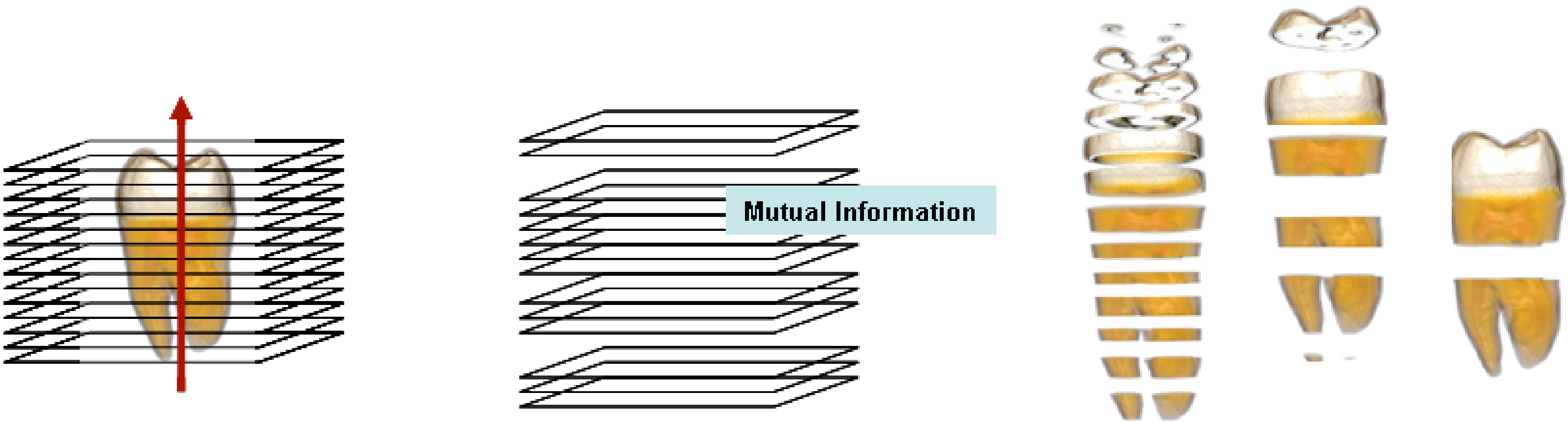
- **Structured View** measured through Entropy Rate
  captures the randomness or unpredictability of a system



[Ruiz et al. 2008]

Similarity-Based Exploded Views

- **Bottom-up Grouping**: group the most similar slices or slabs through normalized mutual information
  degree of similarity or shared information between two slices or slabs



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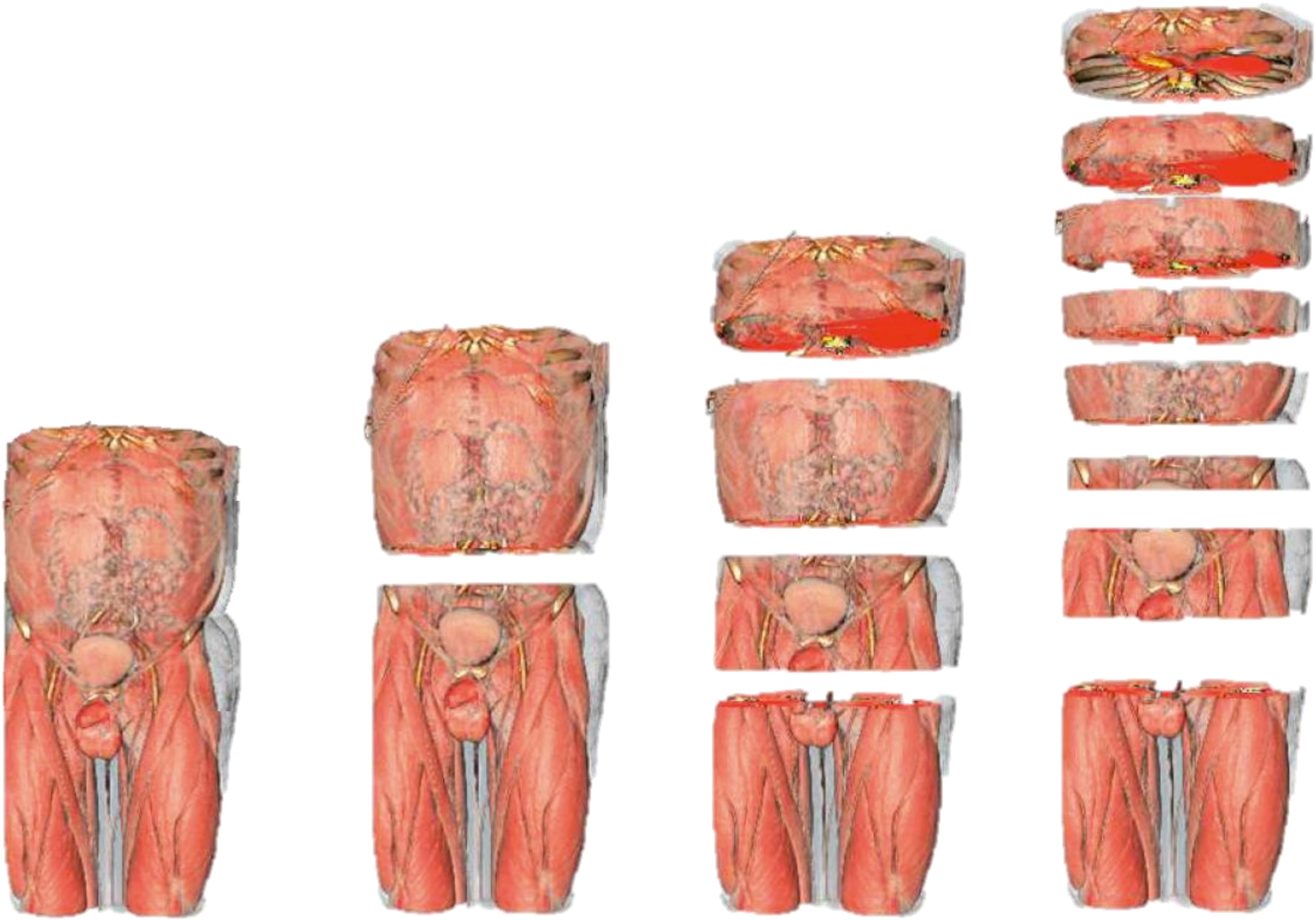
Similarity-Based Exploded Views



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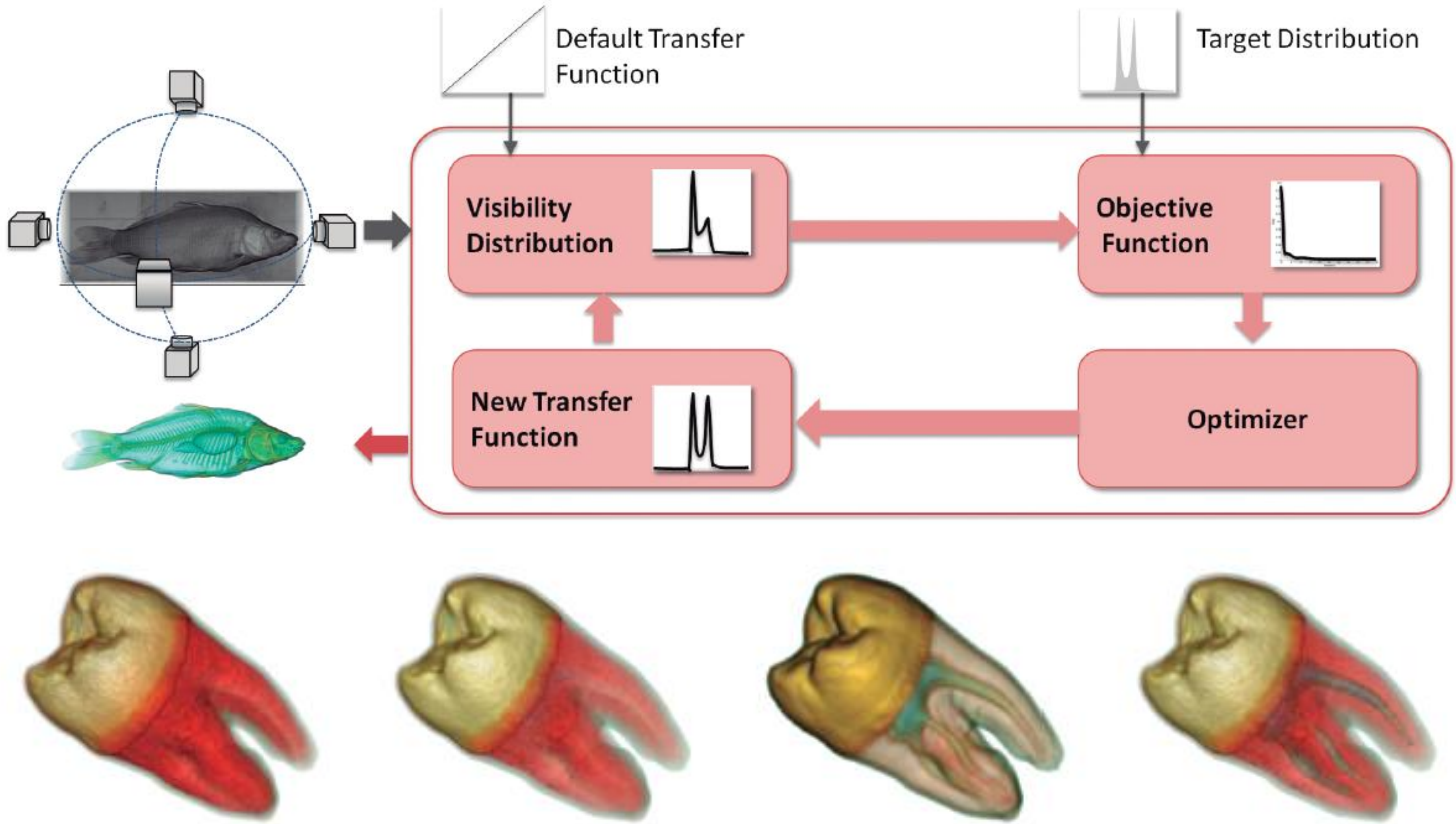
Informational Divergence

- **Informational divergence** (relative entropy or Kullback-Leibler distance)
  $D_{KL}(p,q)$: how much p is different from q (on a common alphabet X)

$$D_{KL}(p,q) = \sum_{x \in \mathrm{X}} p(x) \log \frac{p(x)}{q(x)}$$

  - Convention: $0 \log 0/q = 0$ and $p \log p/0 = \infty$
  - $D_{KL}(p,q) >= 0$
  - Not a true metric or *distance*
  - Mutual information is a special case
    $I(X,Y) = D_{KL}(p(X,Y), p(X)p(Y))$

Transfer Functions for Scalar Fields



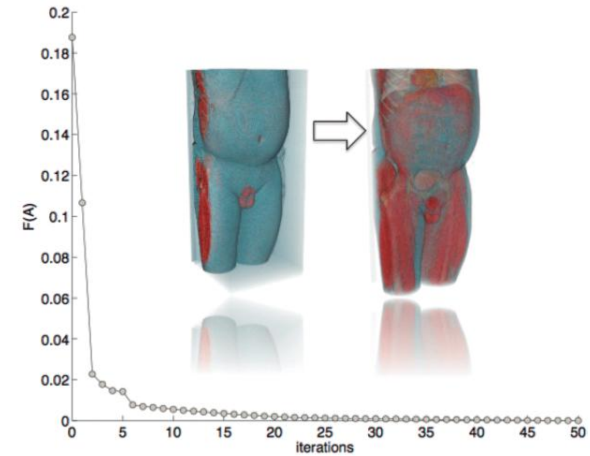
[Ruiz et al. 2011]

Transfer Functions for Scalar Fields

- Target Function: Intuitive specification of visual prominence for density values
- Minimize informational divergence ($F(A)$) between the average projected visibility distribution from viewpoints and a target distribution
- Optimizer: Steepest Gradient Descent

$$A^t = A^{t-1} - s^{t-1} \nabla F(A)$$

$$\nabla F(A) = \left(\frac{\partial F(A)}{\partial \alpha_0}; \frac{\partial F(A)}{\partial \alpha_1}; \ldots; \frac{\partial F(A)}{\partial \alpha_{n-1}} \right)$$



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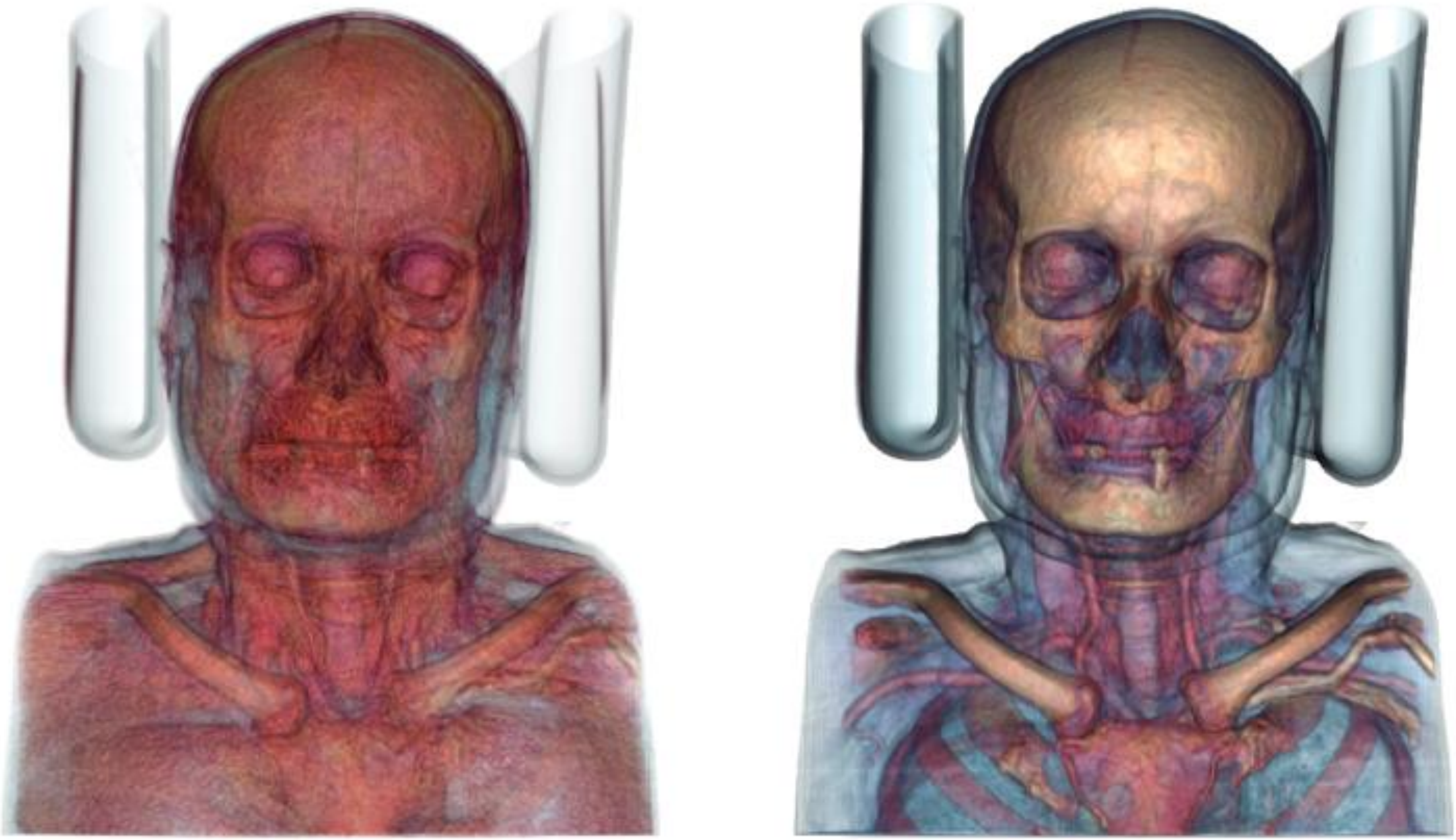
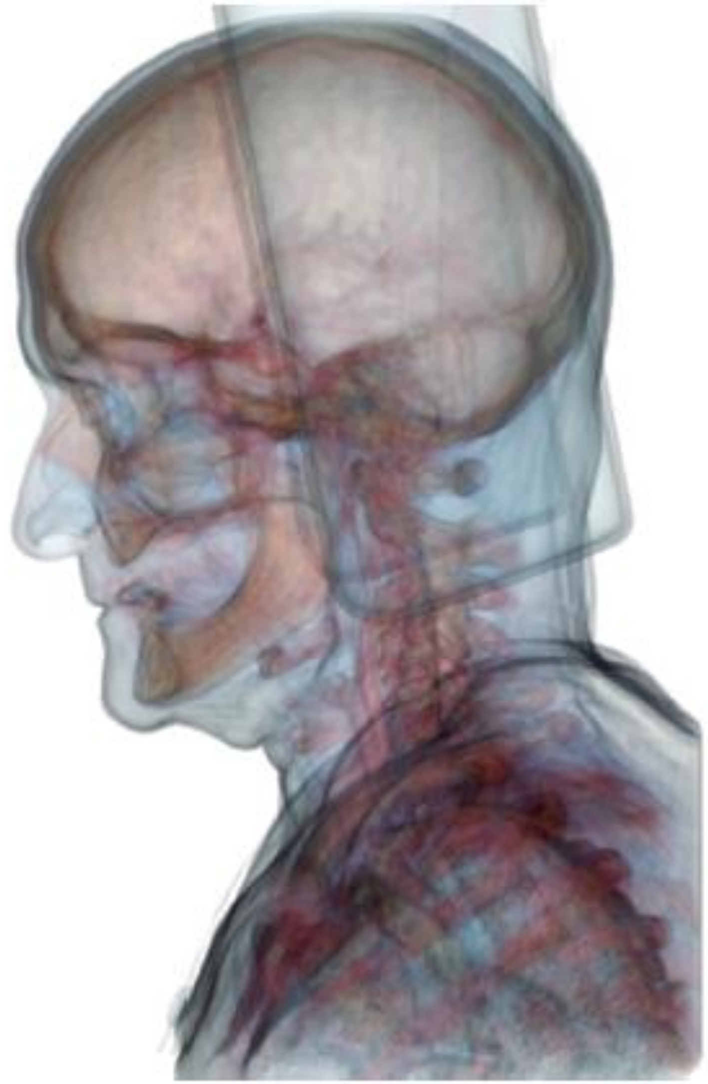
Transfer Functions for Scalar Fields



[Ruiz et al. 2011]

Transfer Functions for Scalar Fields



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References

- [Takahashi et al. 2005] S. Takahashi , I. Fujishiro, Y. Takeshima, T. Nishita: **A Feature-Driven Approach to Locating Optimal Viewpoints for Volume Visualization**, In *Proceedings of IEEE Visualization*, 2005
- [Bordoloi and Shen 2005] U. Bordoloi and H.W. Shen: **View Selection for Volume Rendering**, In *Proceedings of IEEE Visualization*, 2005
- [Viola et al. 2006] I. Viola, M. Feixas, M. Sbert, M.E. Gröller: **Importance-Driven Focus of Attention**, In *IEEE TVCG (Proceedings of IEEE Visualization)*, 2006
- [Bruckner and Möller 2010] S. Bruckner and T. Möller: **Isosurface Similarity Maps**, In *EG CGF (Proceedings of EuroVis)*, 2010
- [Ruiz et al. 2008] M. Ruiz, I. Viola, I. Boada, S. Bruckner, M. Feixas, M. Sbert: **Similarity-based Exploded Views**, In *Springer LNCS (Proceedings of Smart Graphics)*, 2008
- [Ruiz et al. 2011] M. Ruiz, A. Bardera, I. Boada, I. Viola, M. Feixas, M. Sbert: **Automatic Transfer Functions based on Informational Divergence**, *IEEE TVCG (Proceedings of IEEE Visualization)*, 2011

Thank you!

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