Some construction methods for uniform designs with large size

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Outline

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2. Construction Method I

   -Based on integer programming method
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1. Motivation

Uniform design

- An important type of computer experiments and physical experiments
- The main idea of uniform design is to scatter the design points to be uniformly on the experimental domain
- Searching a uniform design is an optimization problem

1. Motivation

Existing construction method

1. Number theoretic method (Fang and Wang, 1994)
   - good lattice point (glp) methods
   - glp method with power generator (pglp method)
2. Stochastic optimization algorithm

   (Winker and Fang, 1997; Fang, Tang and Yin, 2005)
3. Combinatorial construction method

   (Fang, Li and Sudjianto, 2006)

1. Motivation

Shortcoming of Existing construction method

- Not suitable to construct uniform design tables with a large size (a large number of runs or/and a large number of factors)
- Some construction methods have high computational complexity.

1. Motivation

New construction methods should be proposed.

- One construction method is based on the integer programming method
- The other construction method is an extension of glp method

In this talk, the centered L_2-discrepancy (CD) is considered. An analytical expression of squared CD of $\boldsymbol{X} = (X_1, \cdots, X_n) = (x_{ij})$ is given by

$$\mathrm{CD}^2(\boldsymbol{X}) = \left(\frac{13}{12}\right)^s - \frac{2}{n}\sum_{i=1}^{n}\prod_{j=1}^{s}\left(1 + \frac{1}{2}|x_{ij} - 0.5| - \frac{1}{2}|x_{ij} - 0.5|^2\right)$$

$$+\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\prod_{j=1}^{s}\left(1 + \frac{1}{2}|x_{ij} - 0.5| + \frac{1}{2}|x_{kj} - 0.5| - \frac{1}{2}|x_{ij} - x_{kj}|\right).$$

2. Construction Method I

- For a given $\boldsymbol{X} \in \mathcal{U}(n; q_1, \cdots, q_s)$, let $\boldsymbol{y} = \boldsymbol{y}(\boldsymbol{X})$ be a column vector of $n(i_1, ..., i_s)$ arranged lexicographically, where $n(i_1, ..., i_s)$ is the number of runs at the level-combination $(i_1, ..., i_s)$ in the design $\boldsymbol{X}$
- Call the vector $\boldsymbol{y}$ be the frequency vector
- For example, when the design $\boldsymbol{X}$ is as follows,

$$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix},$$

then $\boldsymbol{y} = (0, 0, 1, 1, 0, 1, 0, 2, 0, 1, 0, 0)'$

Lemma

Let design $\boldsymbol{X} \in \mathcal{U}(n; q_1, \cdots, q_s)$ *and* $\boldsymbol{y} = \boldsymbol{y}(\boldsymbol{X})$ *be the frequency vector, we have*

$$CD^2(\boldsymbol{X}) = \left(\frac{13}{12}\right)^s - \frac{2}{n}\boldsymbol{a}'\boldsymbol{y} + \frac{1}{n^2}\boldsymbol{y}'\boldsymbol{A}\boldsymbol{y},$$

where $\boldsymbol{a} = \boldsymbol{a}_1 \otimes \boldsymbol{a}_2 \otimes \cdots \otimes \boldsymbol{a}_s$, $\boldsymbol{A} = A_1 \otimes A_2 \otimes \cdots \otimes A_s$, *and* $\otimes$ *is the Kronecker product,* $\boldsymbol{a}_k = (v_1, \cdots, v_{q_k})'$, $A_k = (v_{ij}^k)$, *for* $i, j = 1, \cdots, q_k$, $k = 1, \cdots, s$

$$\begin{aligned} v_i &= 1 + \frac{1}{2}\left|\frac{2i-1-q_k}{2q_k}\right| - \frac{1}{2}\left|\frac{2i-1-q_k}{2q_k}\right|^2, \\ v_{ij}^k &= 1 + \frac{1}{2}\left|\frac{2i-1-q_k}{2q_k}\right| + \frac{1}{2}\left|\frac{2j-1-q_k}{2q_k}\right| - \frac{1}{2}\left|\frac{i-j}{q_k}\right|. \end{aligned}$$

For given $n, q_1, \cdots, q_s$, the problem of constructing uniform design $U(n; q_1, \cdots, q_s)$ can be formulated as the following optimization problem:

$$\text{(OP)} \qquad \min \quad f_0(\boldsymbol{y}) = \left(\frac{13}{12}\right)^s - \frac{2}{n}\boldsymbol{a}'\boldsymbol{y} + \frac{1}{n^2}\boldsymbol{y}'\boldsymbol{A}\boldsymbol{y},$$
$$s.t. \quad \mathbf{1}_m'\boldsymbol{y} = n, \ \boldsymbol{y} \in Z_n^m,$$

where $Z_n^m = Z_n \times \cdots \times Z_n$, $Z_n = \{0, 1, 2, \cdots, n\}$, $m = q_1 \cdots q_s$, $\boldsymbol{y} = (y_1, \cdots, y_m)' \in Z_n^m$ means $y_i \in Z_n$.

2. Construction Method I

If the constraint $\boldsymbol{y} \in Z_n^m$ is relaxed, we get the following semidefinite programming problem

$$\text{(SDP)} \qquad \begin{aligned} \min \quad & f_1(\boldsymbol{y}) = \boldsymbol{y}'\boldsymbol{A}\boldsymbol{y} - 2n\boldsymbol{a}'\boldsymbol{y} \\ s.t. \quad & \mathbf{1}_m'\boldsymbol{y} = n. \end{aligned}$$

The Lagrange dual function is an effective tool for solving the problem (SDP).

2. Construction Method I

Theorem 1 For given $n, q_1, \cdots, q_s$, let $q_1, \cdots, q_r$ $(0 \le r \le s)$ be odd numbers and $q_{r+1}, \cdots, q_s$ be even numbers. Then, the minimizer of problem (SDP) is

$$\begin{aligned}
\boldsymbol{y}^* &= \frac{n}{m}\mathbf{1}_{m_r} \otimes \left(\otimes_{k=r+1}^{s} \begin{pmatrix} \mathbf{1}_{q_k/2-1} \\ 1 - \frac{1}{4(4q_k+1)} \\ 1 - \frac{1}{4(4q_k+1)} \\ \mathbf{1}_{q_k/2-1} \end{pmatrix} \right) \\
&\quad + n\frac{1 - \prod_{r+1}^{s}\left(1 - \frac{1}{2q_k(4q_k+1)}\right)}{2^{s-r}m_r}\mathbf{1}_{m_r} \otimes \left(\otimes_{k=r+1}^{s} \begin{pmatrix} \mathbf{0}_{q_k/2-1} \\ 1 \\ 1 \\ \mathbf{0}_{q_k/2-1} \end{pmatrix} \right),
\end{aligned}$$

where $m_r = q_1 \cdots q_r$, $m = q_1 \cdots q_s$ and $\mathbf{1}_t$ represents the t-column vector with all the elements as one.

2. Construction Method I

Corollary 1   For $q_1, \cdots, q_s$, if every number of level q_i is odd or all equals to 2, then the minimizer of problem (SDP) is

$$\boldsymbol{y}^* = \frac{n}{m}\mathbf{1}_m.$$

Corollary 2 Let the number of levels $q_1, \cdots, q_r$ be odd numbers and $q_{r+1} = \cdots = q_s = 2$ $(0 \leq r \leq s)$, the corresponding full factorial design $\boldsymbol{X} \in \mathcal{U}(n; q_1, \cdots, q_r, 2, \cdots, 2)$ with $n = tm$ runs is the uniform design under CD, and its squared *CD* is given by

$$\mathrm{CD}^2(\boldsymbol{X}) = \left(\frac{13}{12}\right)^s - \frac{2}{m}\left(\frac{35}{16}\right)^{s-r}\prod_{k=1}^{r}\frac{13q_k^2-1}{12q_k} + \frac{1}{m^2}\left(\frac{9}{2}\right)^{s-r}\prod_{k=1}^{r}\frac{13q_k^2-1}{12},$$

where $m = 2^{s-r}q_1\cdots q_r$.

2. Construction Method I

When the number of runs $n < m = q_1 \cdots q_s$, under some algebra operation, the construction problem (OP) is equivalent to the following unconstrained quadratic zero-one programming problem:

$$\text{(BQP)} \qquad \begin{array}{ll} \min & f(\boldsymbol{y}) = \boldsymbol{y}'\boldsymbol{Q}\boldsymbol{y} \\ s.t. & \boldsymbol{y} \in \{0,1\}^m, \end{array}$$

where

$$\boldsymbol{Q} = (\boldsymbol{B}_0 + K\mathbf{1}_m\mathbf{1}_m' - 2Km\boldsymbol{I}_m)/K = (q_{ij})$$

is a symmetric matrix and $\boldsymbol{I}_m$ is the identity matrix, and $\boldsymbol{B}_0 = \boldsymbol{A} - \mathrm{diag}(2n\boldsymbol{a}') = (b_{ij})$, $K = 2\sum_{i=1}^m \sum_{j=1}^m |b_{ij}| + 1$.

Construction Method

SA-based integer programming method (SA-IPM), which combines

- Simulated annealing algorithm
- 1-*opt* local search with best improvement move strategy

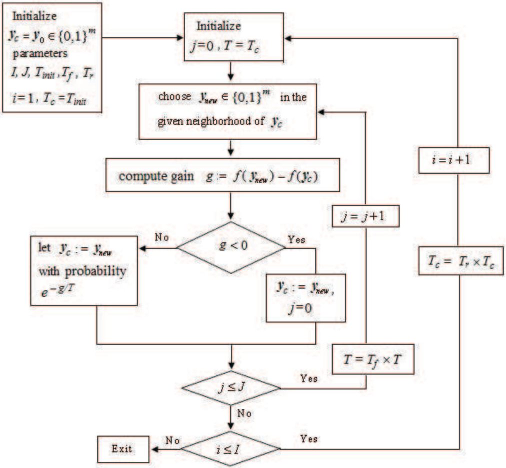

Figure 1: The framework of SA-based integer programming method

Symmetrical design

Table 1: Comparison between SA-IPM and other construction methods.

No.	n	s	q	m	CD_{SA}	time (seconds)	designs on web	glp or pglp method	time (seconds)
1	12	3	3	27	0.033758	4.1	0	−0.2666	0.2
2	24	3	3	27	0.032779	3.8	0.1486	−0.7209	0.4
3	51	5	3	243	0.063902	34.8	0.1132	−0.0977	31.7
4	120	5	3	243	0.062996	35.9	–	−0.1670	∗
5	240	5	3	243	0.062706	35.2	–	−5.9826	∗
6	51	6	3	729	0.085128	154.2	0.2868	−0.0711	353.2
7	201	6	3	729	0.081655	158.0	–	−0.0718	∗
8	702	6	3	729	0.081159	340.1	–	−1.6562	∗
9	52	3	4	64	0.018787	12.6	0.0262	−0.0067	1.2

The notation "–" in 8th column means the corresponding design does not exist on web yet. The notation "∗" in last column means the design is constructed by pglp method.

2. Construction Method I – Simulation results

Table 2: Comparison between SA-IPM and other methods (Cont.)

No.	n	s	q	m	CD_{SA}	time (seconds)	designs on web	glp or pglp method	time (seconds)
10	48	4	4	256	0.027896	56.7	0.0835	−0.0096	9.9
11	120	4	4	256	0.027226	59.1	–	−0.0844	960.3
12	240	4	4	256	0.027118	58.7	–	−1.2218	39044.2
13	48	5	4	1024	0.039058	411.3	0.1289	−0.0391	720.5
14	500	5	4	1024	0.036959	408.1	–	−0.3713	$*$
15	1000	5	4	1024	0.036954	420.2	–	−5.7425	$*$
16	250	4	5	625	0.016930	249.0	–	−0.0322	$*$
17	600	4	5	625	0.016879	253.2	–	−0.4072	$*$
18	800	5	5	3125	0.022852	1353.5	–	−0.0011	$*$
19	2000	5	5	3125	0.022820	1372.0	–	−0.0010	$*$
20	204	4	6	1296	0.011999	327.5	–	−0.1313	8007.6
21	402	4	6	1296	0.011920	329.2	–	−0.0286	$*$
22	804	4	6	1296	0.011893	334.5	–	−0.1304	$*$

Asymmetrical design

Table 3: Comparison between SA-IPM and other construction methods.

No.	Design	m	CD_{SA}	time (seconds)	RGM	glp or pglp method	time (seconds)
1	$U(12;3^2,4)$	36	0.029032	4.5	−0.2700	−0.0042	0.1
2	$U(24;3^2,4)$	36	0.027874	4.7	−0.4627	−0.0316	0.2
3	$U(24;3,4^2)$	48	0.023415	5.8	−0.4121	−0.0118	0.3
4	$U(24;3,6^2)$	108	0.017500	10.7	−0.3126	−0.0010	0.4
5	$U(102;3,6^2)$	108	0.016315	9.3	−0.1342	−0.2555	11.6
6	$U(40;4,5^2)$	100	0.014327	10.5	−0.2752	0.0450	1.2
7	$U(80;4,5^2)$	100	0.013979	10.5	−0.4295	−0.0253	2.8
8	$U(150;5,6^2)$	180	0.009388	18.5	−0.3432	−0.0181	37.8
9	$U(48;3^2,4^2)$	144	0.037073	31.6	−0.3710	−0.0163	8.7
10	$U(96;3^2,4^2)$	144	0.036675	31.6	−0.3693	−0.0257	382.6
11	$U(24;3^2,6^2)$	324	0.032136	69.5	−0.4260	−0.0548	9.0

The notation “$*$” in last column means the design is constructed by pglp method.

2. Construction Method I – Simulation results

Table 4: Comparison between SA-IPM and other methods (Cont.)

No.	Design	m	CD_{SA}	time (seconds)	RGM	glp or pglp method	time (seconds)
12	$U(72; 3^2, 6^2)$	324	0.029690	68.2	−0.5975	−0.0357	90.5
13	$U(200; 4^2, 5^2)$	400	0.021773	103.9	−0.6490	−0.0440	3623.9
14	$U(210; 5^2, 6^2)$	900	0.014451	280.0	−0.6484	−0.0022	$*$
15	$U(600; 5^2, 6^2)$	900	0.014345	285.3	−0.7724	−0.0236	$*$
16	$U(48; 3^3, 4^2)$	432	0.053476	198.0	−0.6807	−0.0154	118.2
17	$U(300; 3^3, 4^2)$	432	0.052011	183.1	−1.0314	−0.2637	$*$
18	$U(100; 4^2, 5^3)$	2000	0.028665	642.5	−1.0257	−0.0046	$*$
19	$U(1000; 4^2, 5^3)$	2000	0.028016	677.8	−1.1440	−0.2302	$*$
20	$U(1500; 4^2, 5^3)$	2000	0.028006	668.4	−1.0908	−0.7864	$*$
21	$U(48; 3^3, 4^3)$	1728	0.066923	676.9	−1.0404	−0.0031	5066.9
22	$U(960; 3^3, 4^3)$	1728	0.064053	697.4	−1.3524	−0.5872	$*$
23	$U(180; 3^4, 6^2)$	2916	0.061626	1679.3	−1.2624	−0.0146	$*$
24	$U(1800; 3^4, 6^2)$	2916	0.061250	1626.1	−1.3199	−0.2406	$*$

3. Construction Method II – Mixture Method

For given $(n, s, q_1, \cdots, q_s)$, we mixture two nearly uniform designs to construct UDs. The idea is as follows.

1. The *pqlp* method is employed to generate two nearly uniform designs with n and p points, respectively. ($p \gg n$)
2. we mixture $n + p$ points in the two designs and choose the 'best' n points from the $n + p$ points by some stochastic optimization method.

The goodness of the mixture method

1. The resulted designs have lower discrepancy
2. The computational complexity can be decreased significantly comparing with existing methods

3. Construction Method II

Table 5: The pseudo-code for the new algorithm

The MM Algorithm: searching a uniform design under CD

Generate $\mathcal{P}_0 \in \mathcal{U}(n; q_1, \cdots, q_s)$ and $\xi_0 \in \mathcal{U}(p; q_1, \cdots, q_s)$,
  denote $\mathcal{P}_c := \mathcal{P}_0$, $\xi_c := \xi_0$;
Initialize $I, J, \tau_1, \cdots, \tau_I$;
for i=1:I
  for j=1:J
    Generate $\mathcal{P}_{\text{new}} \in \mathcal{N}(\mathcal{P}_c)$ and corresponding design ξ_{new};
    Calculate $\triangle\text{CD} = \text{CD}^2(\mathcal{P}_{\text{new}}) - \text{CD}^2(\mathcal{P}_c)$;
    if $\triangle\text{CD} < \tau_i$, then $\mathcal{P}_c := \mathcal{P}_{\text{new}}$, $\xi_c := \xi_{\text{new}}$;
    otherwise $\mathcal{P}_c = \mathcal{P}_c$, $\xi_c = \xi_c$;
  end
  Change the threshold $\tau_i := \tau_{i+1}$;
end

Comparison with existing designs

Table 6: Designs generated by MM and existing designs.

Design	n	s	q	Mean of RPM	Std of RPM	Existing design	MM
1	15	24	3	8.0414	0.5878	6.1981(-3.14)	3.9038(-7.04)
2	15	22	3	5.3145	0.3572	4.1379(-3.29)	2.8674(-6.85)
3	15	20	3	3.4936	0.2138	2.7296(-3.57)	2.1366(-6.35)
4	9	20	3	5.3760	0.4679	4.3353(-2.22)	2.6129(-5.91)
5	15	18	3	2.2802	0.1269	1.7755(-3.98)	1.5452(-5.80)
6	12	16	3	1.7410	0.1033	1.3704(-3.59)	1.2128(-5.11)
7	24	15	4	0.9654	0.0321	0.7074(-8.03)	0.6832(-8.79)
8	12	15	4	1.6923	0.0771	1.3778(-4.08)	1.0806(-8.05)
9	8	15	4	2.4517	0.1332	2.1016(-2.63)	1.3790(-8.03)
10	15	15	3	1.1828	0.0580	0.9086(-4.73)	0.8952(-4.96)
11	9	15	3	1.7357	0.1204	1.4030(-2.76)	1.1805(-4.61)

3. Construction Method II – Simulation Results

Table 7: Designs generated by MM and existing designs (Cont.)

Design	n	s	q	Mean of RPM	Std of RPM	Existing design	MM
12	60	20	6	1.3213	0.0347	0.9312(-11.25)	0.8604(-13.30)
13	60	25	6	4.2126	0.1244	3.1277(-8.73)	2.2785(-15.55)
14	102	25	6	2.6508	0.0586	1.9298(-12.30)	1.6282(-17.45)
15	60	29	6	10.4771	0.3495	8.0190(-7.03)	4.5032(-17.09)
16	102	29	6	6.4478	0.1631	4.8205(-9.97)	3.3380(-19.06)
17	70	25	7	3.2238	0.1043	2.3096(-8.77)	1.9159(-12.54)
18	70	29	7	7.9226	0.2886	5.7927(-7.38)	3.7370(-14.50)
19	105	29	7	5.4253	0.1591	3.9241(-9.44)	2.9659(-15.46)
20	80	25	8	3.0815	0.0845	2.2062(-10.36)	1.7424(-15.85)
21	112	29	8	5.6299	0.1461	4.1067(-10.43)	2.9069(-18.64)
22	110	25	11	2.1104	0.0544	1.4583(-11.99)	1.3011(-14.88)
23	154	29	11	3.8375	0.0939	2.7079(-12.03)	2.2306(-17.11)

Symmetrical designs

Table 8: The CD^2 of designs generated by different methods, where n or $n+1$ is a prime number.

No.	n	s	q	Mean of RPM	Std of RPM	*pglp* method	Cutting method	MM
1	149	7	5	0.0443	0.00078	0.0393 (-6.42)	0.0398 (-5.74)	0.0389 (-6.95)
2	150	10	5	0.0892	0.0015	0.0789 (-6.87)	0.0785 (-7.13)	0.0756 (-9.07)
3	180	10	4	0.1341	0.0014	0.1205 (-9.71)	0.1237 (-7.43)	0.1186 (-11.07)
4	180	25	5	1.5739	0.0245	1.3968 (-7.29)	1.4758 (-4.00)	1.2135 (-14.71)
5	180	40	3	23.5270	0.6503	64.2879 (62.68)	21.5729 (-3.00)	15.0197 (-13.08)
6	311	10	3	0.1914	0.00077	0.1860 (-7.11)	0.1865 (-6.50)	0.1847 (-8.81)
7	311	25	4	1.7853	0.0132	1.6068 (-13.52)	1.8459 (4.58)	1.3750 (-31.08)
8	311	40	5	20.2028	0.4506	25.2497 (11.20)	20.2994 (0.21)	11.0574 (-20.30)
9	401	10	5	0.0757	0.00055	0.0701 (-10.21)	0.0707 (-9.11)	0.0697 (-10.93)
10	401	25	3	1.7374	0.0071	1.6921 (-6.38)	1.7438 (0.90)	1.6256 (-15.75)
11	401	40	4	24.9007	0.3047	25.8474 (3.11)	27.2040 (7.56)	12.9535 (-39.21)

Asymmetrical designs

Table 9: The CD^2 by the different methods.

No.	Design	Mean of RPM	Std of RPM	*pglp* method	MM
1	$U(20; 4^5 5^5)$	0.2592	0.0123	0.3119(4.28)	0.2027(-4.59)
2	$U(60; 4^5 5^5)$	0.1441	0.0040	0.1175(-6.72)	0.1144(-7.51)
3	$U(160; 4^5 5^5)$	0.1092	0.0015	0.0966(-8.52)	0.0950(-9.60)
4	$U(240; 4^5 5^5)$	0.1023	0.0010	0.0928(-9.60)	0.0917(-10.71)
5	$U(320; 4^5 5^5)$	0.0989	0.0008	0.0913(-10.03)	0.0904(-11.17)
6	$U(400; 4^5 5^5)$	0.0968	0.0006	0.0908(-10.02)	0.0897(-11.85)
7	$U(29; 3^{10} 4^{10})$	2.4242	0.0880	2.4098(-0.16)	1.5838(-9.55)
8	$U(89; 3^{10} 4^{10})$	1.2129	0.0202	1.0925(-5.95)	0.9700(-12.01)
9	$U(173; 3^{10} 4^{10})$	0.9348	0.0090	0.8657(-7.65)	0.7986(-15.07)
10	$U(241; 3^{10} 4^{10})$	0.8561	0.0064	0.8098(-7.29)	0.7469(-17.19)
11	$U(317; 3^{10} 4^{10})$	0.8022	0.0046	0.7544(-10.30)	0.7173(-18.30)
12	$U(401; 3^{10} 4^{10})$	0.7689	0.0036	0.7376(-8.68)	0.6985(-19.53)

Flow rate of water

$$y = \frac{2\pi T_u[H_u - H_l]}{\log\left(\frac{r}{r_w}\right)\left[1 + \frac{2LT_u}{\log(r/r_w)r_w^2K_w} + \frac{T_u}{T_l}\right]}. \tag{1}$$

where the 8 input variables are as follows:

r_w (m) : radius of borehole, $r_w \in [0.05, 0.15]$,

r (m) : radius of influence, $r \in [100, 50000]$,

T_u (m^2/year) : transmissivity of upper aquifer, $T_u \in [63070, 115600]$,

T_l (m^2/year) : transmissivity of lower aquifer, $T_u \in [63.1, 116]$,

H_u (m) : potentiometric head of upper aquifer, $H_u \in [990, 1110]$

H_l (m) : potentiometric head of lower aquifer, $H_l \in [700, 820]$,

L (m) : length of borehole, $L \in [1120, 1680]$,

K_w (m/year) : hydraulic conductivity of borehole, $K_w \in [9855, 12045]$.

Table 10: A uniform design $U_{32}(16 \times 4^3 \times 8^4)$

No.	design	No.	design	No.	design
1	1 3 4 3 6 4 6 2	12	9 4 1 2 5 7 1 3	23	2 1 2 2 3 2 1 8
2	9 4 2 4 3 6 3 1	13	11 1 4 4 8 7 1 5	24	5 4 2 4 4 2 5 4
3	15 4 3 2 8 7 6 5	14	8 2 3 4 5 5 3 8	25	10 2 1 1 4 2 6 7
4	11 3 2 4 2 6 7 7	15	6 3 2 1 8 2 4 7	26	14 1 1 3 4 4 7 3
5	12 3 2 4 6 1 2 2	16	7 3 3 1 2 7 6 2	27	6 1 3 3 5 1 8 5
6	15 2 3 1 7 3 3 3	17	16 1 3 3 1 3 4 1	28	16 3 2 3 5 7 4 7
7	2 2 3 1 3 5 5 5	18	10 4 4 3 7 3 2 8	29	8 2 4 2 7 8 4 1
8	6 2 1 2 5 3 4 2	19	13 2 4 4 4 1 7 4	30	2 3 1 3 1 5 1 5
9	3 4 4 2 4 8 7 6	20	4 4 1 2 6 5 8 1	31	5 2 4 3 1 6 5 3
10	14 1 1 2 3 6 5 8	21	2 1 2 1 6 8 2 4	32	6 2 1 4 7 5 6 6
11	11 4 3 2 2 3 8 4	22	13 3 4 1 1 3 2 6		

3. Construction Method II – A case study

The approximate model is

$$
\begin{aligned}
\widehat{\log(y)} \quad = \quad & 8.594347 + (5.019538 \times 10^{-3})x_5 - (7.131500 \times 10^{-3})x_6 \\
& -(1.400019 \times 10^{-3})x_7 + (1.528778 \times 10^{-4})x_8 \\
& -(5.205876 \times 10^{-6})x_5^2 - (6.11484001 \times 10^{-6})x_6^2 \\
& +(2.625370 \times 10^{-7})x_7^2 - (2.79650 \times 10^{-9})x_8^2 \\
& -(1.88986016 \times 10^{-6})x_1 x_3 + (3.257402 \times 10^{-3})x_1 x_4 \\
& -(2.613142 \times 10^{-4})x_1 x_5 - (3.08229 \times 10^{-9})x_3 x_4 \\
& +(3.0133 \times 10^{-10})x_3 x_7 + (1.243613499 \times 10^{-5})x_5 x_6 \\
& -(1.0516982 \times 10^{-7})x_6 x_7 + 2.005427 \log(x_1),
\end{aligned}
$$

The predicted MSE of approximate model is 0.112238, which is less than that of all the papers considering this problem before .

4. Conclusion

- The SA-IPM can obtain designs with lower computational complexity and lower WD value comparing with many traditional construction methods, especially, SA-IPM can be used to construct uniform designs with large number of runs. (m is smaller than 2500)

- The mixture method can be used to construct UDs with large number of runs and/or large number of factors.

- Simulation results and case study shows the new methods are useful.

Reference

1. Fang, K.T., Li, R.Z. and Sudjianto, A., 2006. Design and modeling for computer experiments, Chapman and Hall/CRC, New York.
2. Fang, K.T. and Ma, C.X., 2001. Othogonal and Uniform experimental design. Science Press, Beijing.
3. Fang, K.T., Tang, Y. and Yin, J.X., 2005. Lower Bounds for Wrap-around L_2-discrepancy and Constructions of Symmetrical Uniform Designs, Journal of Complexity, 21 , 757-771.
4. Fang, K.T. and Wang, Y (1994), *Number-theoretic Methods in Statistics*, Chapman and Hall, London.
5. Hickernell, F.J., 1998a. A generalized discrepancy and quadrature error bound. *Math. Comp.* 67, 299-322.
6. Winker, P. and Fang, K.T., 1997. Application of threshold-accepting to the evaluation of the discrepancy of a set of points, *SIAM J. Numer. Anal.*, 34, 2028-2042.

Final word

Thank you!

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