

The 2011 Taipei International Statistical Symposium and  
7th Conference of the Asian Regional Section of the IASC

# Exploring Symbolic Data Structure using Matrix Visualization

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Dec. 17, 2011



# Outline

- Matrix Visualization and related techniques
- Symbolic data Matrix Visualization
- Examples
- Summary

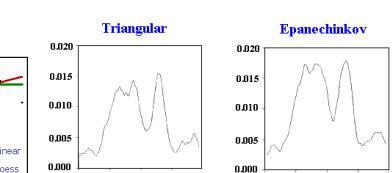
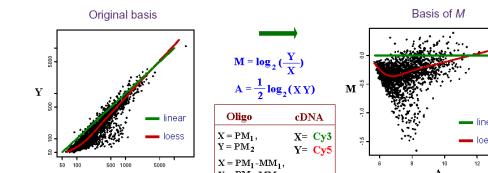
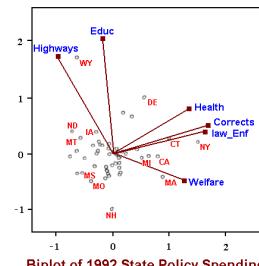
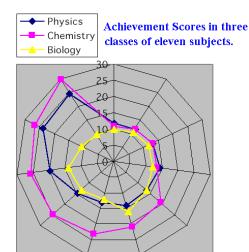
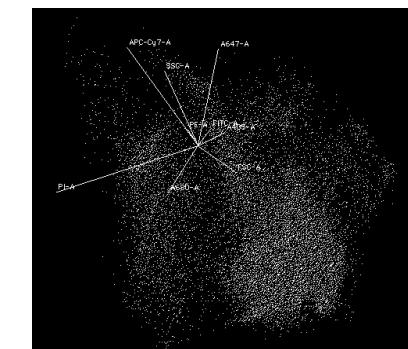
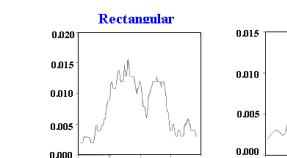
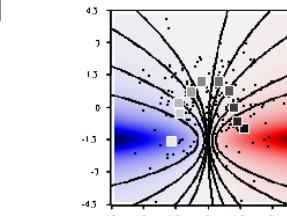
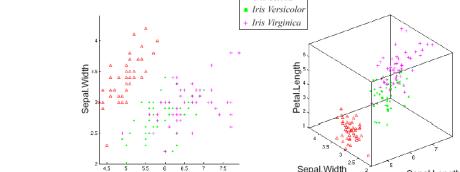
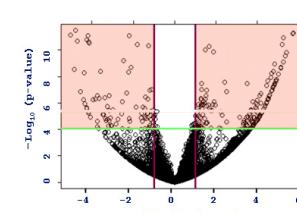
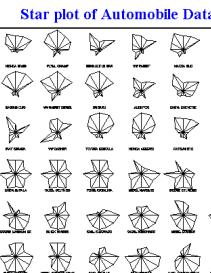
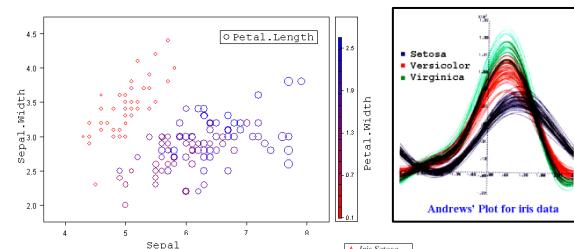
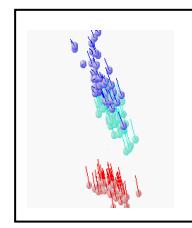
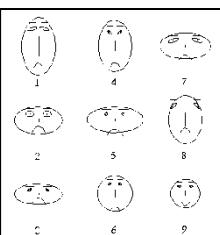
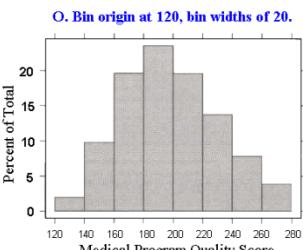
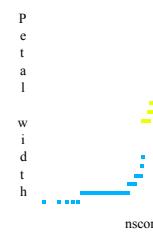
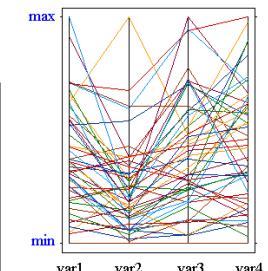
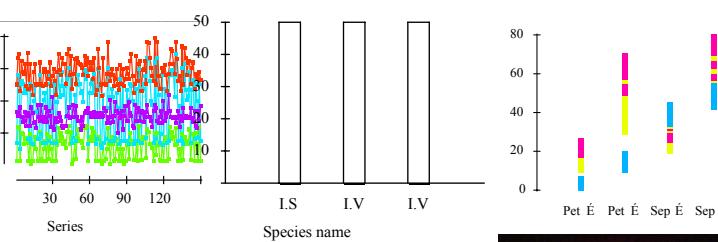
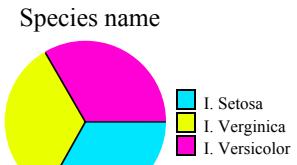
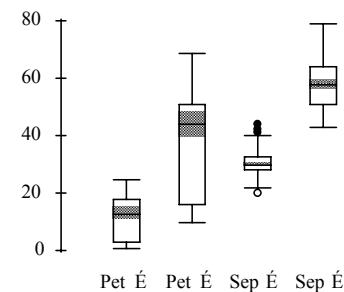
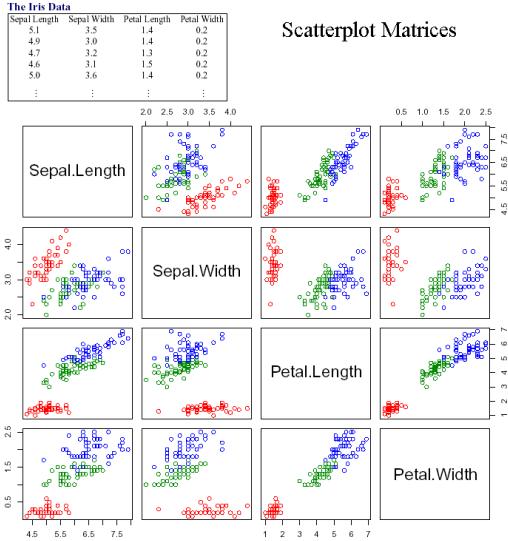
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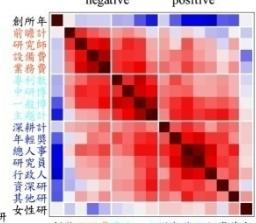


# Graphics/Visualization for high dimensional data?

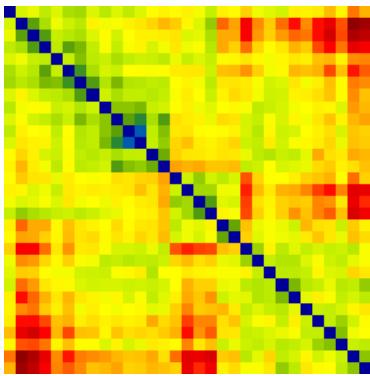
P>5 p>10 p>100 p>10000



A. Correlation Matrix  
-1 negative +1 positive

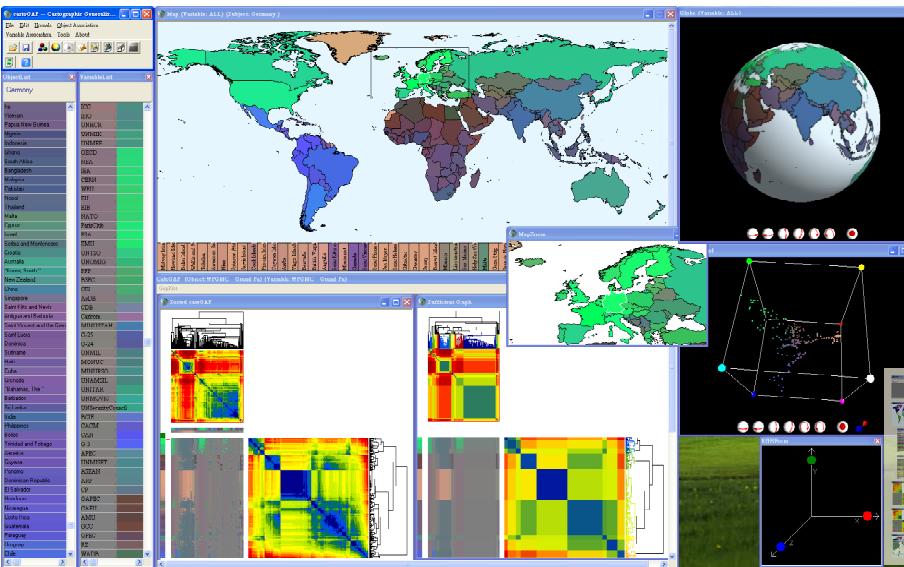


## Continuous GAP

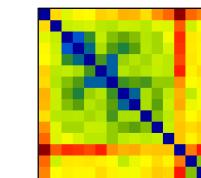
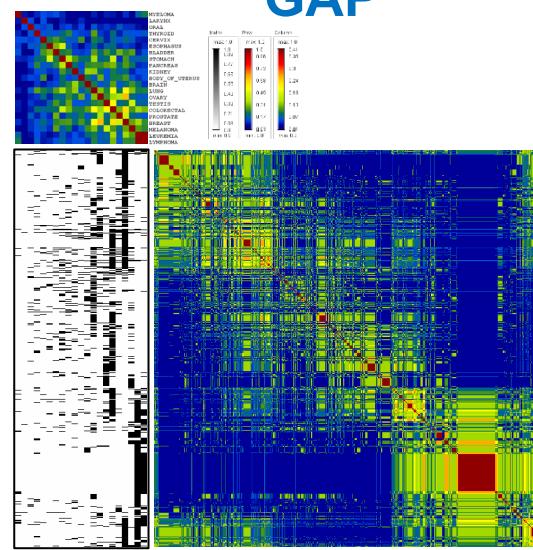


B. Data Rank  
□ 人社科組  
■ 數理科學組  
■ 生命科學組  
small large 31

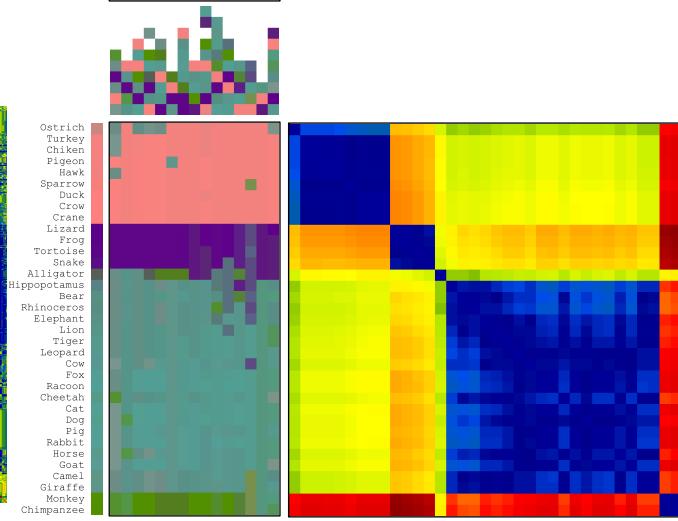
## Cartography GAP



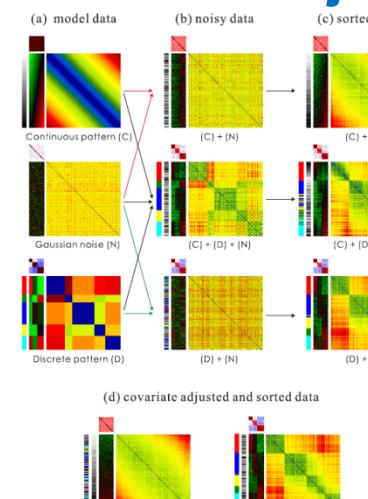
## Binary GAP



## Categorical GAP

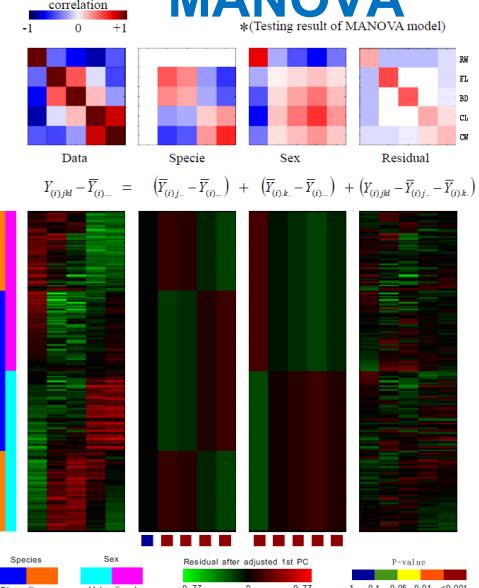


## GAP with Covariate-Adjust

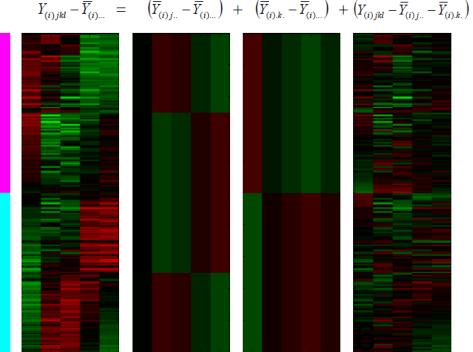


Correlation Euclidean Distance Data Value  
-1 1 min. max. -2.8 0 2.8

## GAP for MANOVA

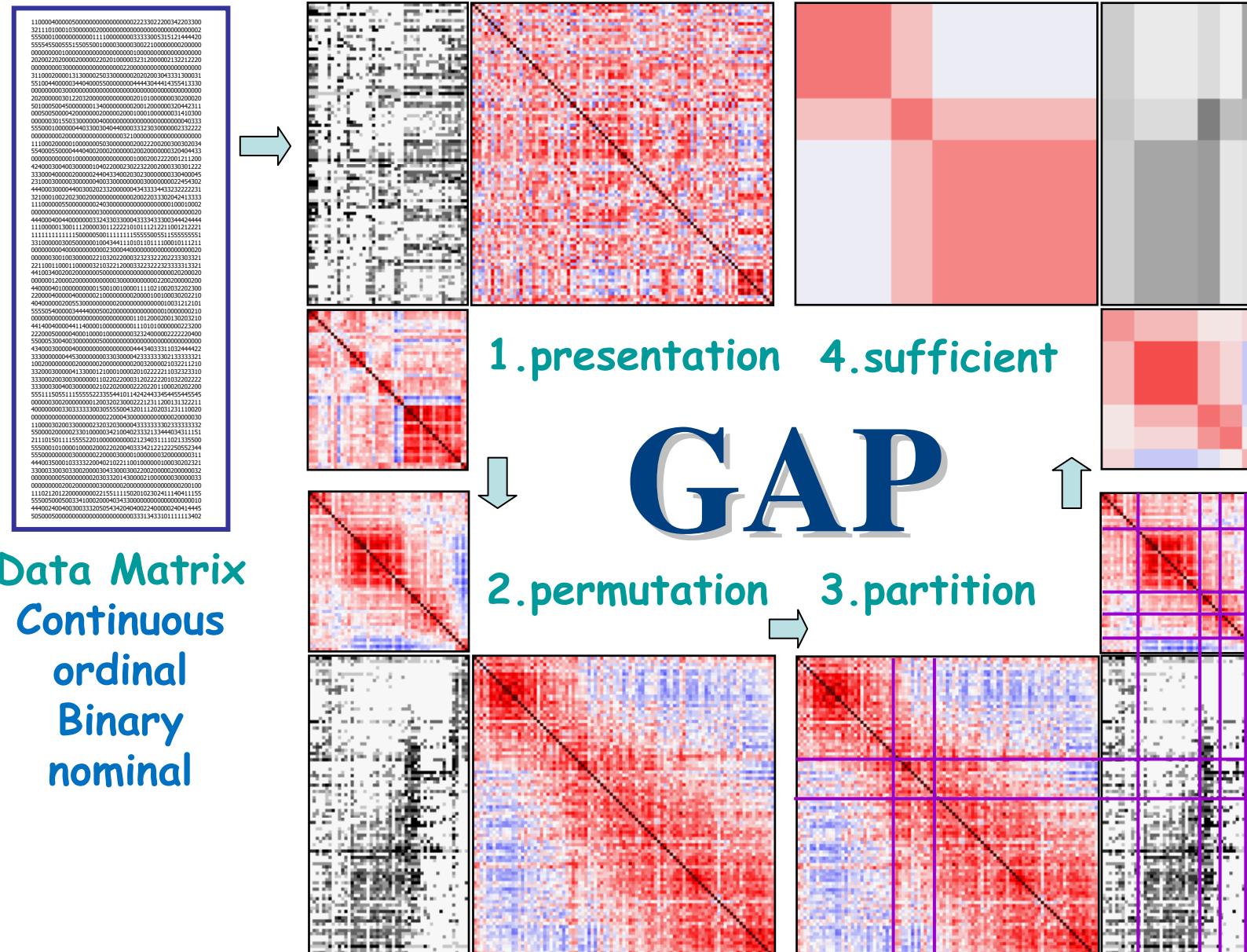


$$Y_{(ij)l} - \bar{Y}_{(i)l} = (\bar{Y}_{(i)j} - \bar{Y}_{(i)l}) + (\bar{Y}_{(i)k} - \bar{Y}_{(i)l}) + (Y_{(i)l} - \bar{Y}_{(i)j} - \bar{Y}_{(i)k})$$



P-value  
Blue Orange Male Female  
-0.77 0 0.77 1 0.1 0.05 0.01 <0.001

# Generalized Association Plots (GAP, Chen 2002) for MV



# Some essential elements in a GAP MV procedure

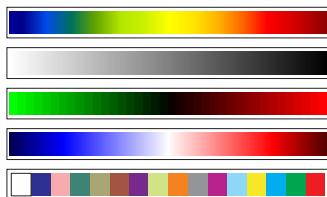
## 3. Proximity (Variable $p * p$ )

Continuous  
Ordinal  
Binary  
Nominal

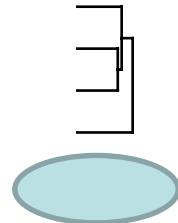
## 1. Data Matrix ( $n * p$ )

(w/ Color coding)

Continuous  
Ordinal  
Binary  
Nominal



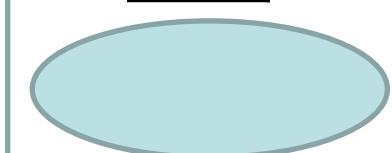
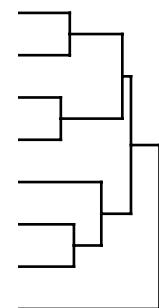
## 4. Permutation (variable)



## 2. Proximity Matrix for Subject ( $n * n$ )

Continuous  
Ordinal  
Binary  
Nominal

## 4. Permutation (subject)



Methodology article

Open Access

## Methods for simultaneously identifying coherent local clusters with smooth global patterns in gene expression profiles

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<sup>4</sup>Department of Mathematics, Tamkang University, Tamsui 25137, Taiwan and <sup>5</sup>Institute of Statistical Science, Academia Sinica, Taipei, 11529, Taiwan

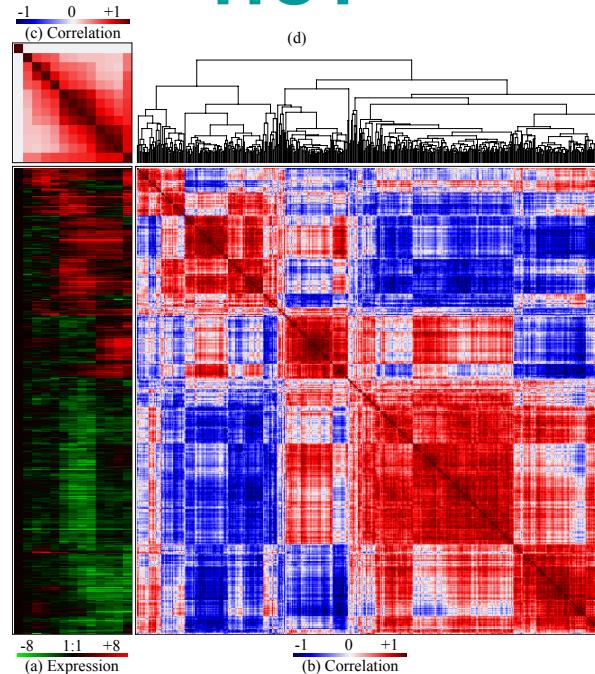
Local  
HCT

+

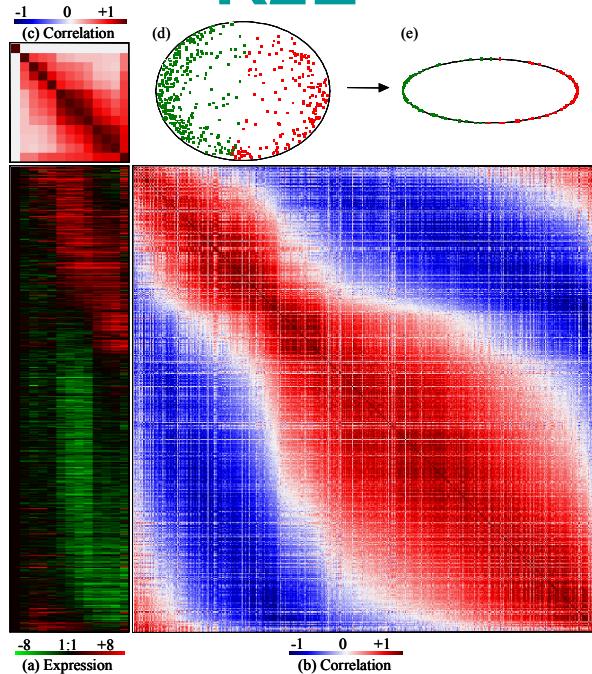
Global  
R2E

=

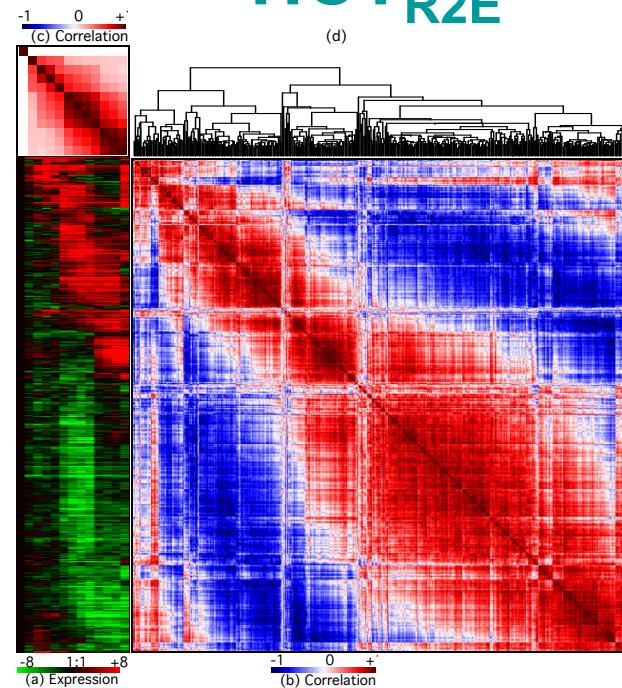
Local/Global  
HCT<sub>R2E</sub>



Hierarchical Tree Seriation



GAP Elliptical (R2E) Seriation



Tree guided by (R2E)

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# Symbolic data

- Classical Data : Individuals: single value  
Single player  
age = 25, eye color = blue
- Symbolic Data : Symbolic units (groups/classes)  
Team  
interval : age range = [20, 36]  
multiple values: eye color = {blue,brown,black}

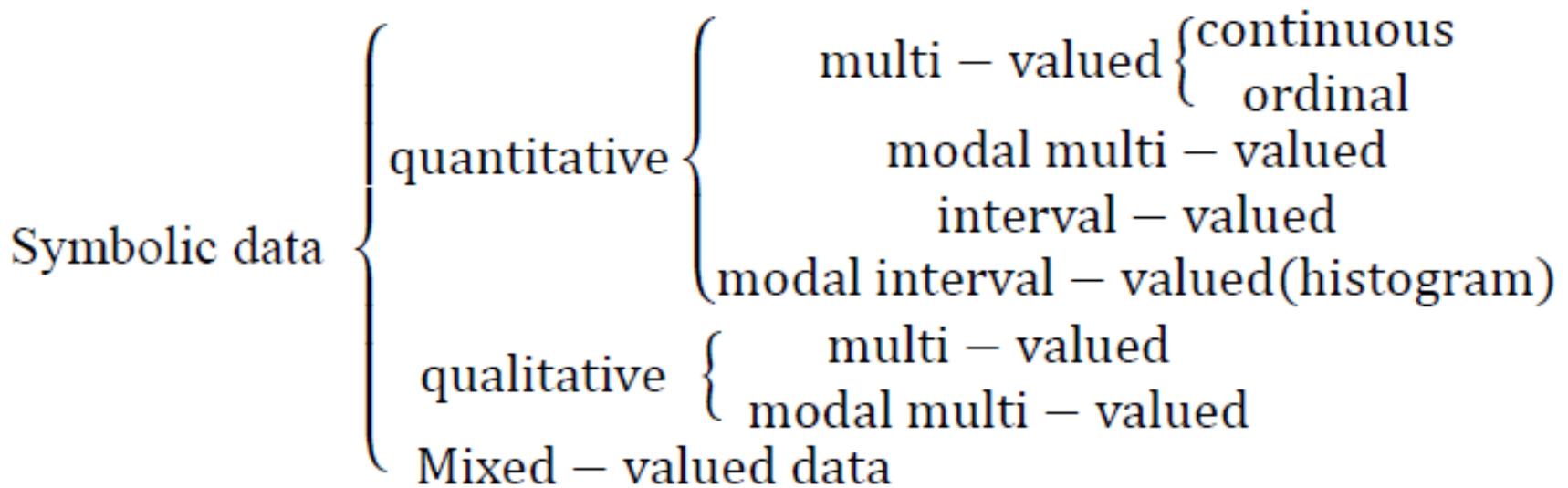
(Billard and Diday (2006))

# Symbolic data analysis

## When?

- When we are interested the **higher level units** (groups/classes ).
- When the **initial data are composed** by Symbolic data tables

# Symbolic data types



# Symbolic data types

Multi-valued symbolic random variable Y is  
one or more values {12,23,20}

Interval-valued symbolic random variable Y  
is one that takes values  
in an interval [17, 25]

Modal multi-valued {single, 3/8, married, 5/8}

$$Y(u) = \{\eta_k, \pi_k; k = 1, 2, \dots, s_u\}$$

Modal interval-valued (Histogram) {[12,40), 1/7, [40, 60), 2/7, [60, 80], 4/7}

$$Y(u) = \{[a_{uk}, b_{uk}), p_{uk}; k = 1, 2, \dots, s_u\}$$

# Visualization Tools for Symbolic Data (Analysis)

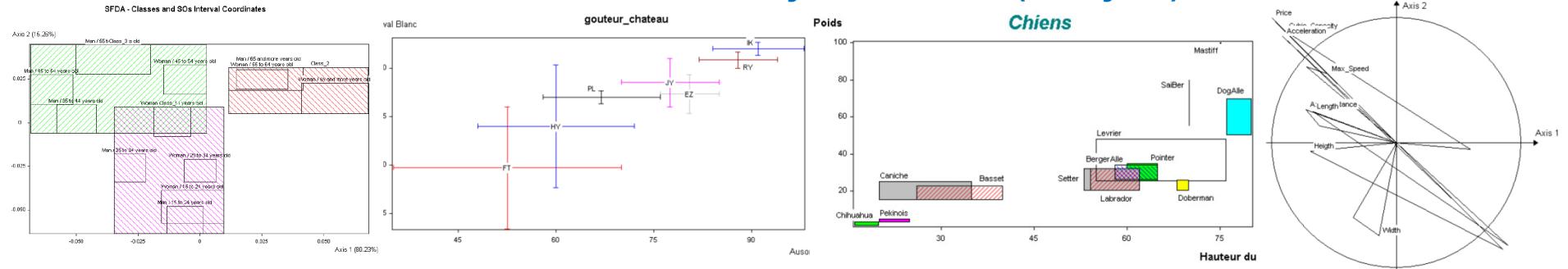
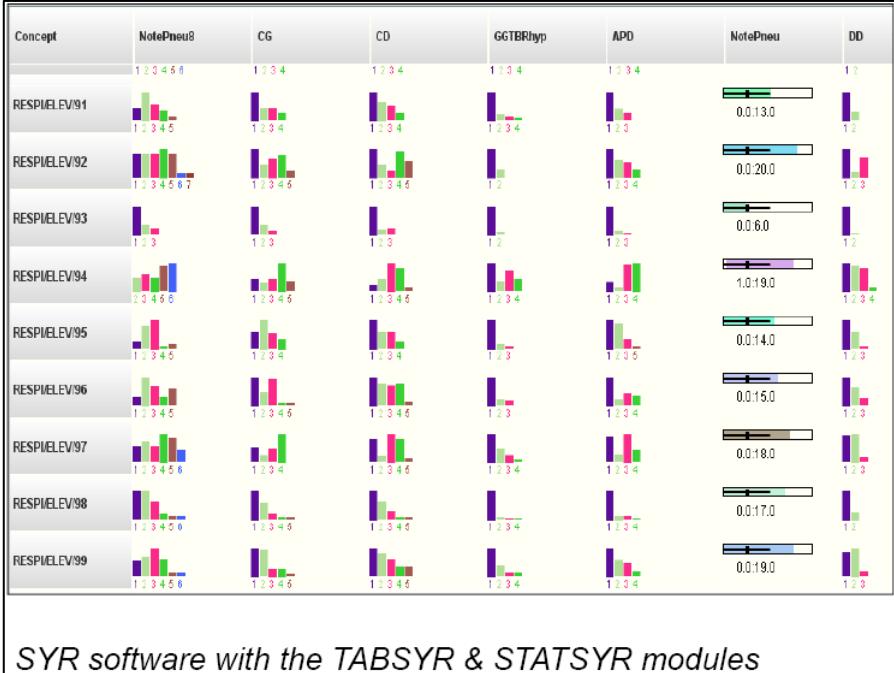


Figure 4: Graphical output of SFDA



SYR software with the TABSYR & STATSYR modules

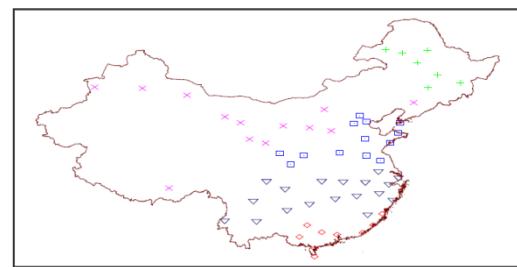


Fig. 1: Visualization on the Map of China of the 5 Classes of the partition of the 60 meteorological stations

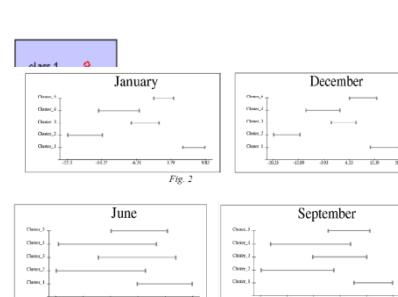


Figure 51: Example of 3D Zoom Star

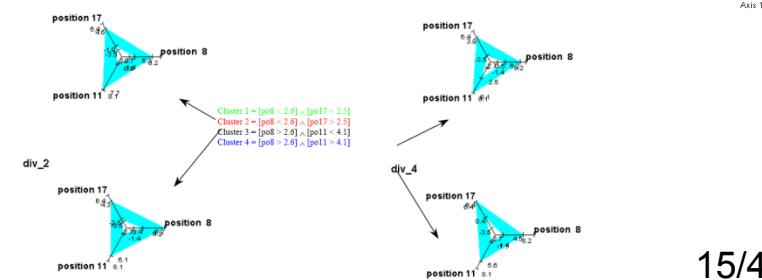


Figure 9: Stars of the symbolic interpretation of each cluster

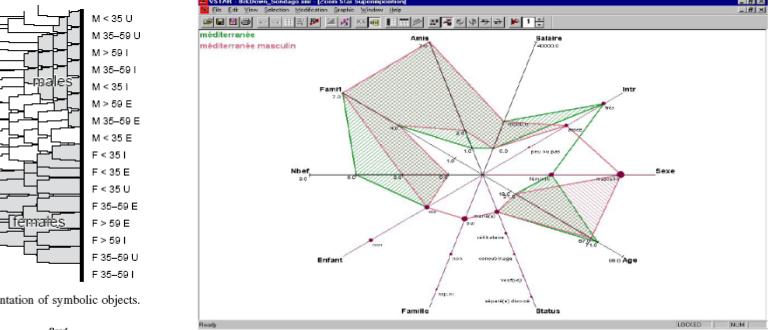


Figure 23.5 Pyramidal representation of symbolic objects.

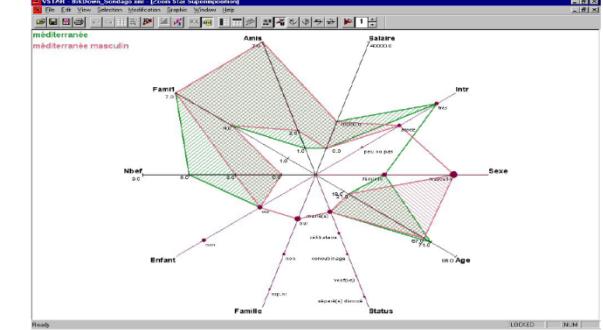
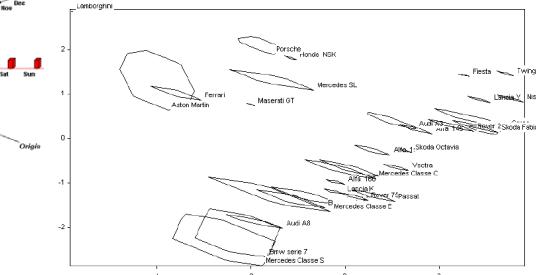


Figure 56: Breakdown - Superimposition of the new and the old objects  
SPCA - SOs Convex Hulls



# Essential elements in a GAP MV procedure?

Classical

2. Subject  
Proximity

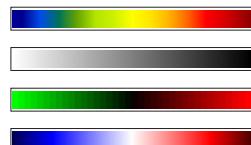
Euclidean Distance  
Manhattan Distance  
Correlation ...

3. Variable  
Proximity

Correlation  
Covariance  
polychoric  
Correlation ...

Symbolic

1. Data  
Matrix



3. Variable  
Proximity

?

1. Data  
Matrix

?

2. Subject  
Proximity

?

# 1. Computation of proximity matrix for variables

Notations :

$\omega_u$  : The symbolic units (observations),  $u = 1, 2, \dots, m$ .

$d_u = (\xi_{u1}, \xi_{u2}, \dots, \xi_{up})$  : The symbolic description of an observation  $\omega_u \in E$ ,  $u = 1, 2, \dots, m$ .

Consider two variables there are some association measures for different symbolic data type (Billard, and Diday 2006)

**Multi-valued data :**

For quantitative multi-valued variables  $Z_1$  and  $Z_2$ , the **empirical covariance function**

$Cov(Z_1, Z_2)$  is given by

$$Cov(Z_1, Z_2) = \frac{1}{m} [\sum_{\xi_1 \in Z_1} \sum_{\xi_2 \in Z_2} (\xi_1 \times \xi_2) O_{Z_1, Z_2}(\xi_1, \xi_2)] - \bar{Z}_1 \bar{Z}_2$$

**Interval data :**

For interval-valued variables  $Z_1$  and  $Z_2$ , the **empirical covariance function**  $Cov(Z_1, Z_2)$  is given by

$$Cov(Z_1, Z_2) = \frac{1}{3m} \sum_{u \in E} G_1 G_2 [Q_1 Q_2]^{1/2}$$

Where, for  $j = 1, 2$ ,  $Q_j = (a_{uj} - \bar{Z}_j)^2 + (a_{uj} - \bar{Z}_j)(b_{uj} - \bar{Z}_j) + (b_{uj} - \bar{Z}_j)^2$ ,  $G_j = \begin{cases} 1, & \text{if } \bar{Z}_{uj} \leq \bar{Z}_j \\ -1, & \text{if } \bar{Z}_{uj} > \bar{Z}_j \end{cases}$

**Correlation coefficient**  $r(Z_1, Z_2) = Cov(Z_1, Z_2) / S_{Z_1} S_{Z_2}$

Where  $\bar{Z}_{uj} = (b_{uj} + a_{uj})/2$ ,  $\bar{Z}_j = \frac{1}{2m} \sum_{u \in E} (b_{uj} + a_{uj})$ ,  $S_{Z_j} = \frac{1}{3m} \sum_{u \in E} (b_{uj}^2 + b_{uj}a_{uj} + a_{uj}^2) - \frac{1}{4m^2} [\sum_{u \in E} (b_{uj} + a_{uj})]^2$

**Histogram data :**

For modal interval-valued (i.e., histogram-valued) variables  $Z_1$  and  $Z_2$ , the **empirical covariance function** is given by

$$Cov(Z_1, Z_2) = \frac{1}{3m} \sum_{u \in E} \sum_{k_1=1}^{S_{u1}} \sum_{k_2=1}^{S_{u2}} p_{u1k_1} p_{u2k_2} G_1 G_2 [Q_1 Q_2]^{1/2}$$

Where, for  $j = 1, 2$ ,  $Q_j = (a_{ujk_j} - \bar{Z}_j)^2 + (a_{ujk_j} - \bar{Z}_j)(b_{ujk_j} - \bar{Z}_j) + (b_{ujk_j} - \bar{Z}_j)^2$ ,  $G_j = \begin{cases} 1, & \text{if } \bar{Z}_{uj} \leq \bar{Z}_j \\ -1, & \text{if } \bar{Z}_{uj} > \bar{Z}_j \end{cases}$

**Correlation coefficient**  $r(Z_1, Z_2) = Cov(Z_1, Z_2) / S_{Z_1} S_{Z_2}$

Where

$$\bar{Z}_{uj} = \frac{1}{2} \sum_{k_j=1}^{S_{uj}} p_{ujk_j} (b_{uj} + a_{uj}), \quad \bar{Z}_j = \frac{1}{2m} \sum_{u \in E} \left[ \sum_{k_j=1}^{S_{uj}} (b_{ujk_j} + a_{ujk_j}) p_{ujk_j} \right], \quad S_{Z_j} = \frac{1}{3m} \sum_{u \in E} \sum_{k_j=1}^{S_{uj}} (b_{ujk_j}^2 + b_{ujk_j}a_{ujk_j} + a_{ujk_j}^2) p_{ujk_j} - \frac{1}{4m^2} \left[ \sum_{u \in E} \sum_{k_j=1}^{S_{uj}} (b_{ujk_j} + a_{ujk_j}) p_{ujk_j} \right]^2$$

## 2. Computation of proximity matrix for symbolic units

Consider two symbolic units there are some proximity measures (Diday and Noirhomme 2008).

### Dissimilarity measures for Boolean symbolic descriptions

Gowda and Diday's dissimilarity measure (1991)

Ichino and Yaguchi's first formulation of a dissimilarity measure (1994)

Ichino and Yaguchi's normalized dissimilarity measure (1994)

Ichino and Yaguchi's normalized and weighted dissimilarity measure (1994)

de Carvalho's normalized dissimilarity measure for constrained Boolean descriptions (1998)

de Carvalho's dissimilarity measure (1994)

de Carvalho's extension of Ichino and Yaguchi's dissimilarity (1994)

de Carvalho's first dissimilarity measure based on description potential (1998)

de Carvalho's second dissimilarity measure based on description potential (1998)

de Carvalho's normalized dissimilarity measure based on description potential (1998)

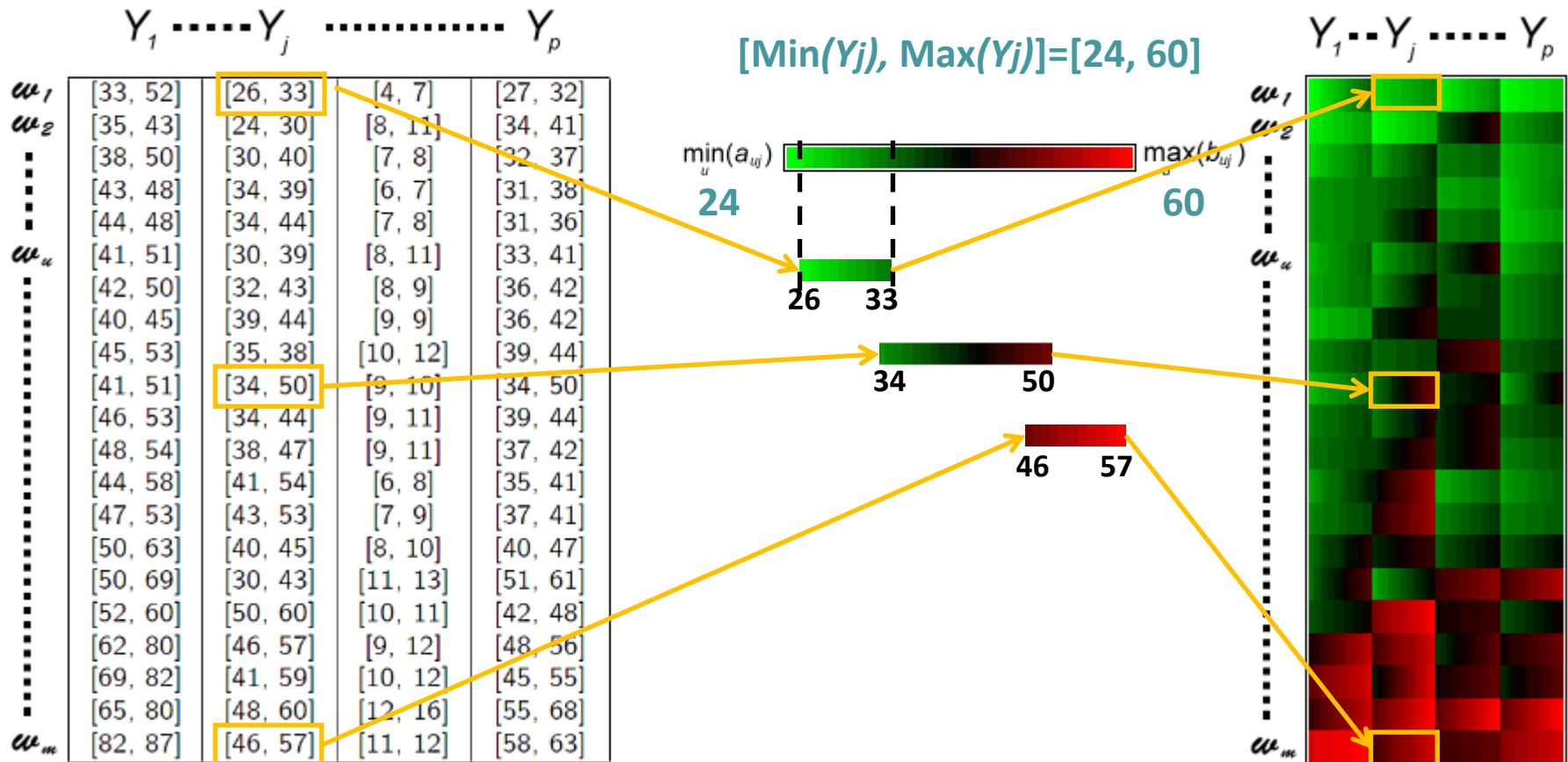
### The Hausdorff distance, The $L_1$ distance, The $L_2$ distance.

### Dissimilarity measures for probabilistic symbolic descriptions

The Kullback–Leibler (KL) divergence, The  $\chi^2$  divergence, The Hellinger coefficient, Renyi's divergence, The variation distance, The Kullback divergence

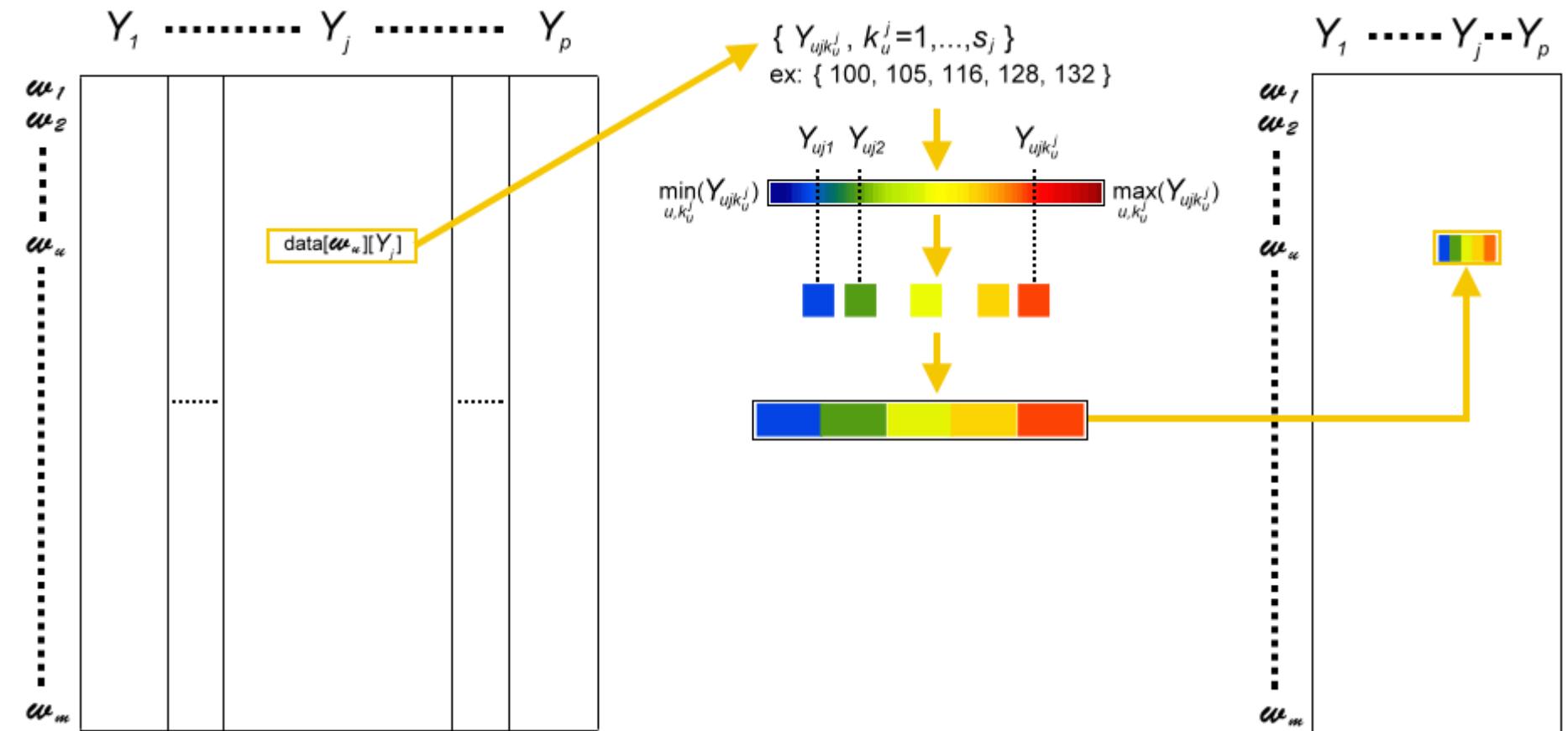
### 3. Color coding for symbolic data

Interval data



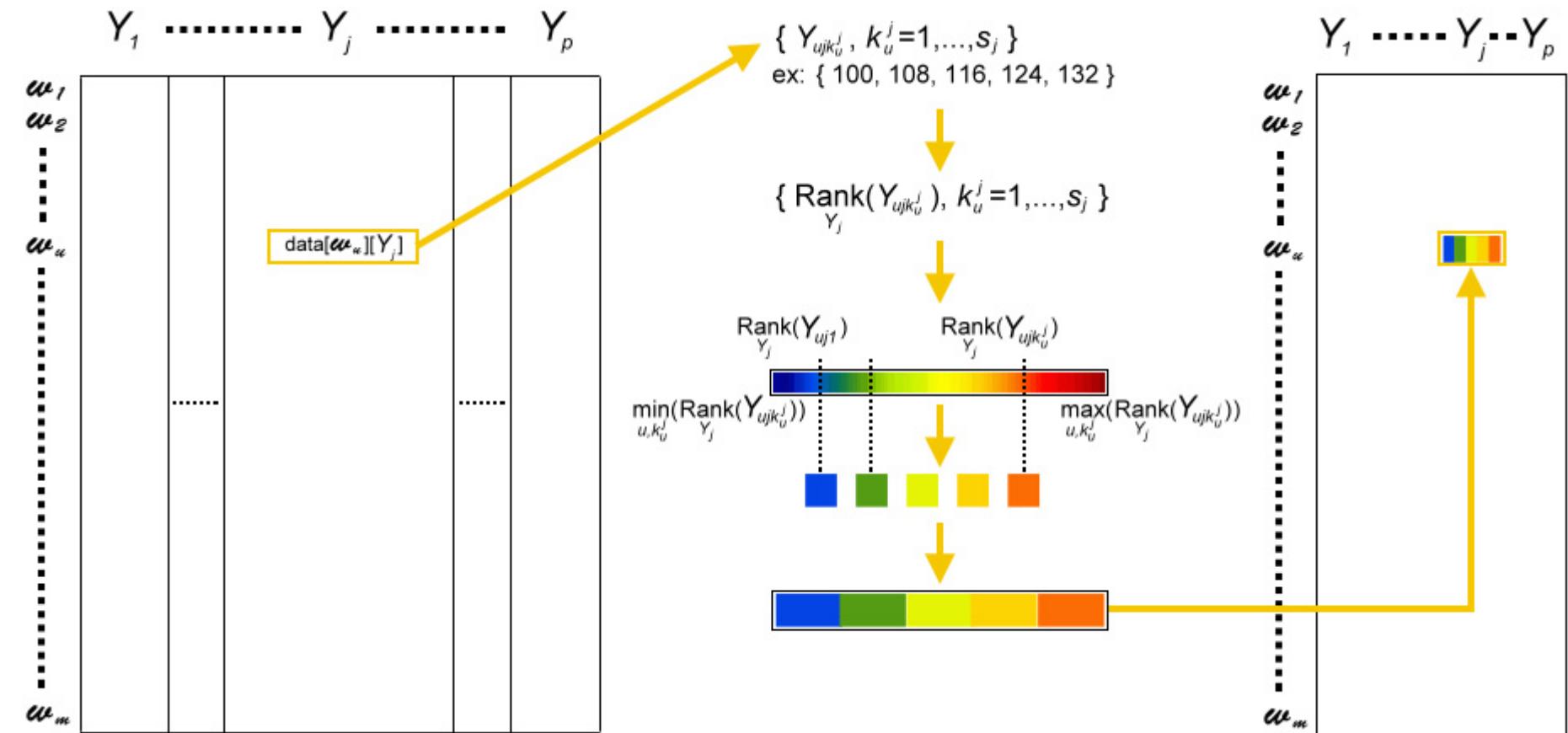
### 3. Color coding for symbolic data

Continuous multi-valued



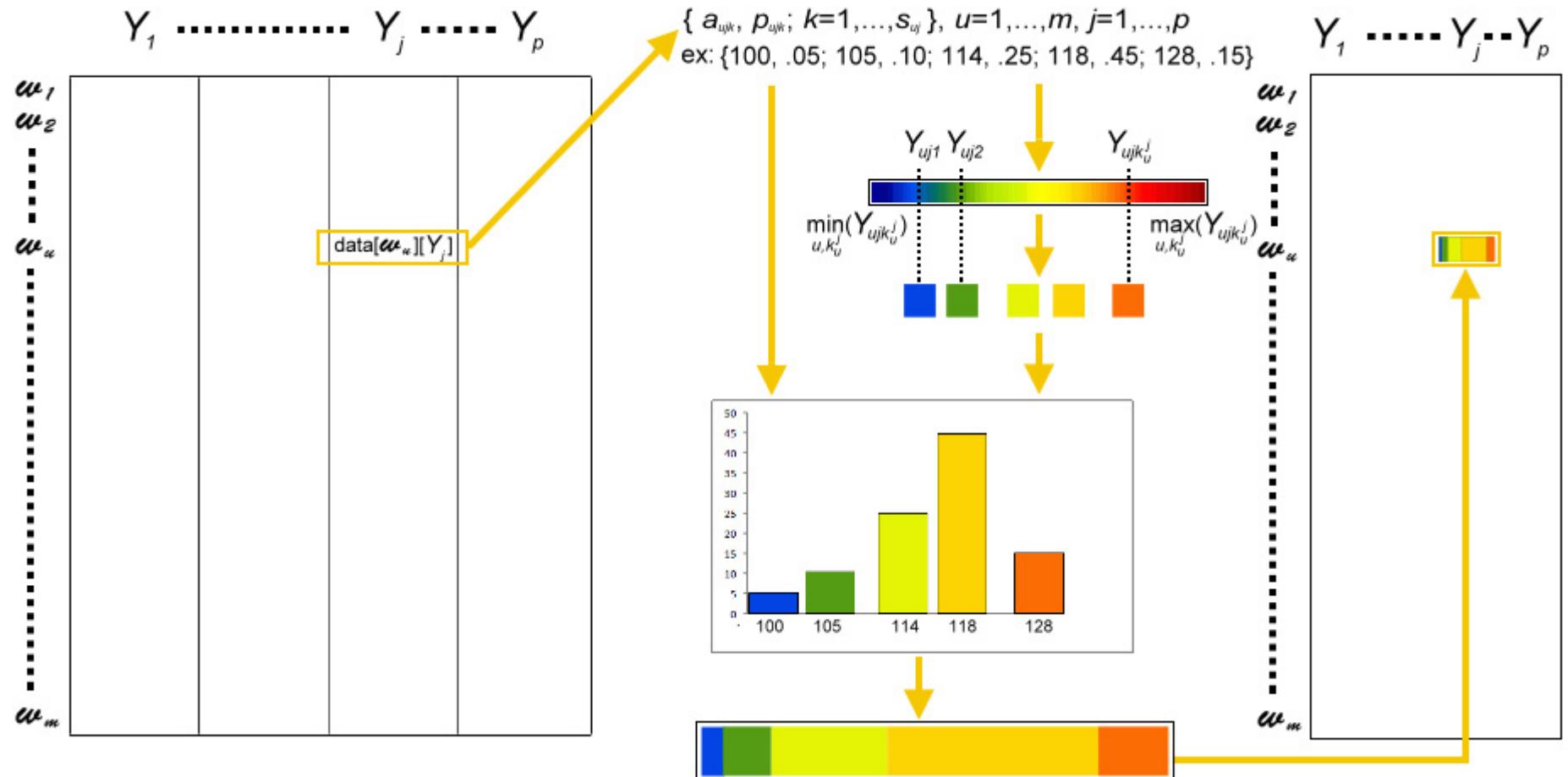
### 3. Color coding for symbolic data

#### Ordinal multi-valued



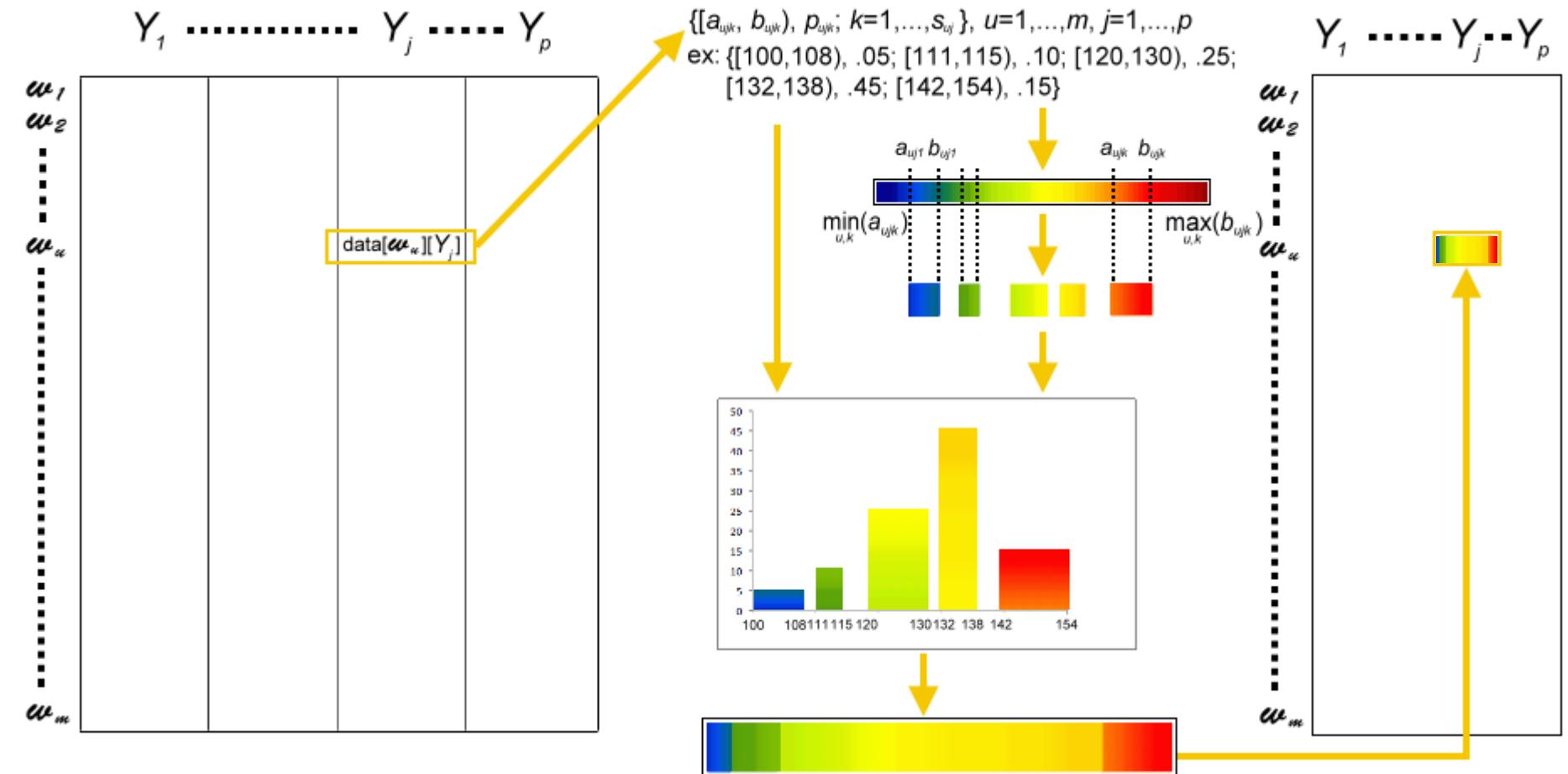
### 3. Color coding for symbolic data

#### Modal multi-valued



### 3. Color coding for symbolic data

#### Modal interval-valued (histogram)



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# Symbolic Data Analysis Tutorial COMPSTAT - August 2010:

## Principal Component Analysis (PCA)

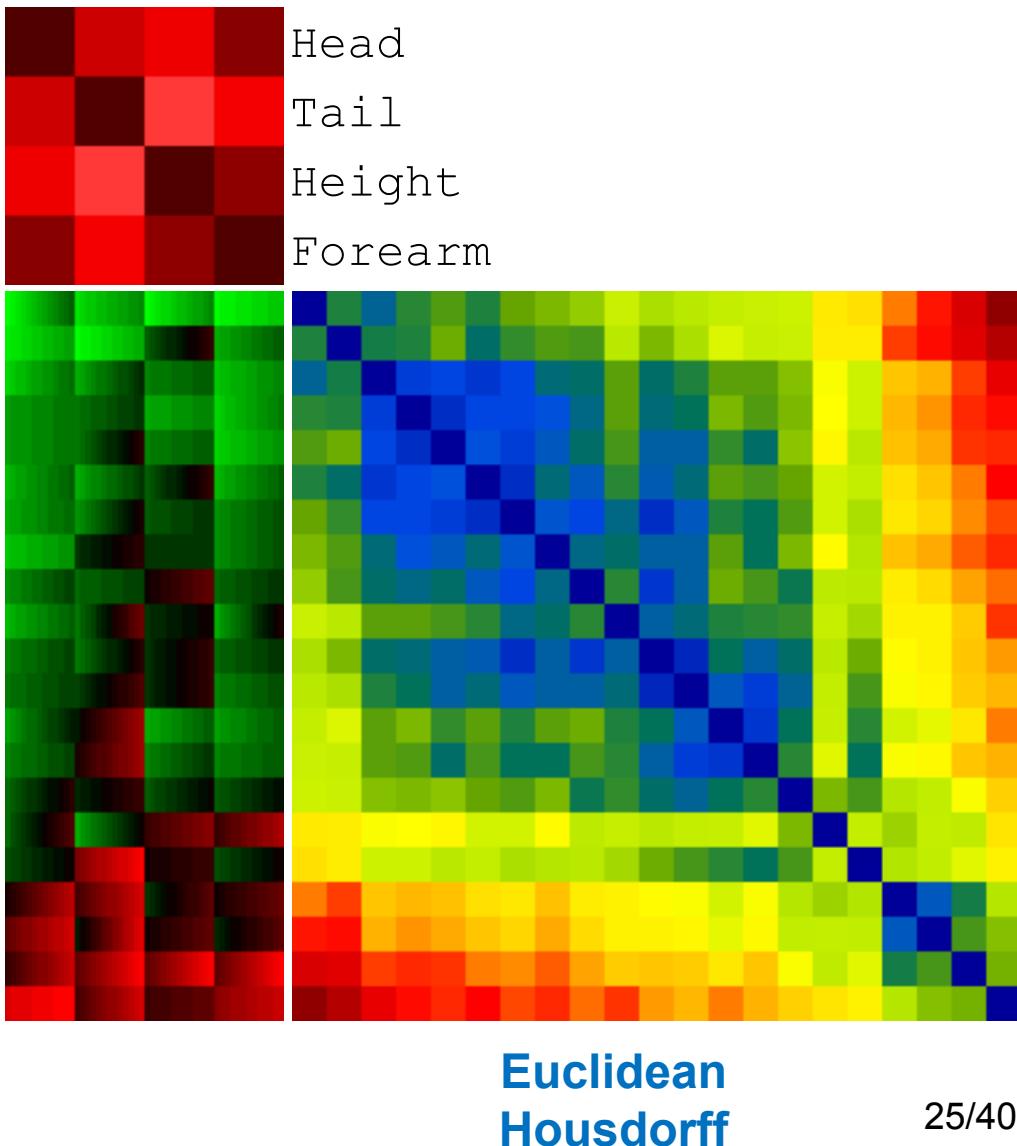
### Lynne Billard

Interval

Bats data:  
(21 Species, 4 variables)

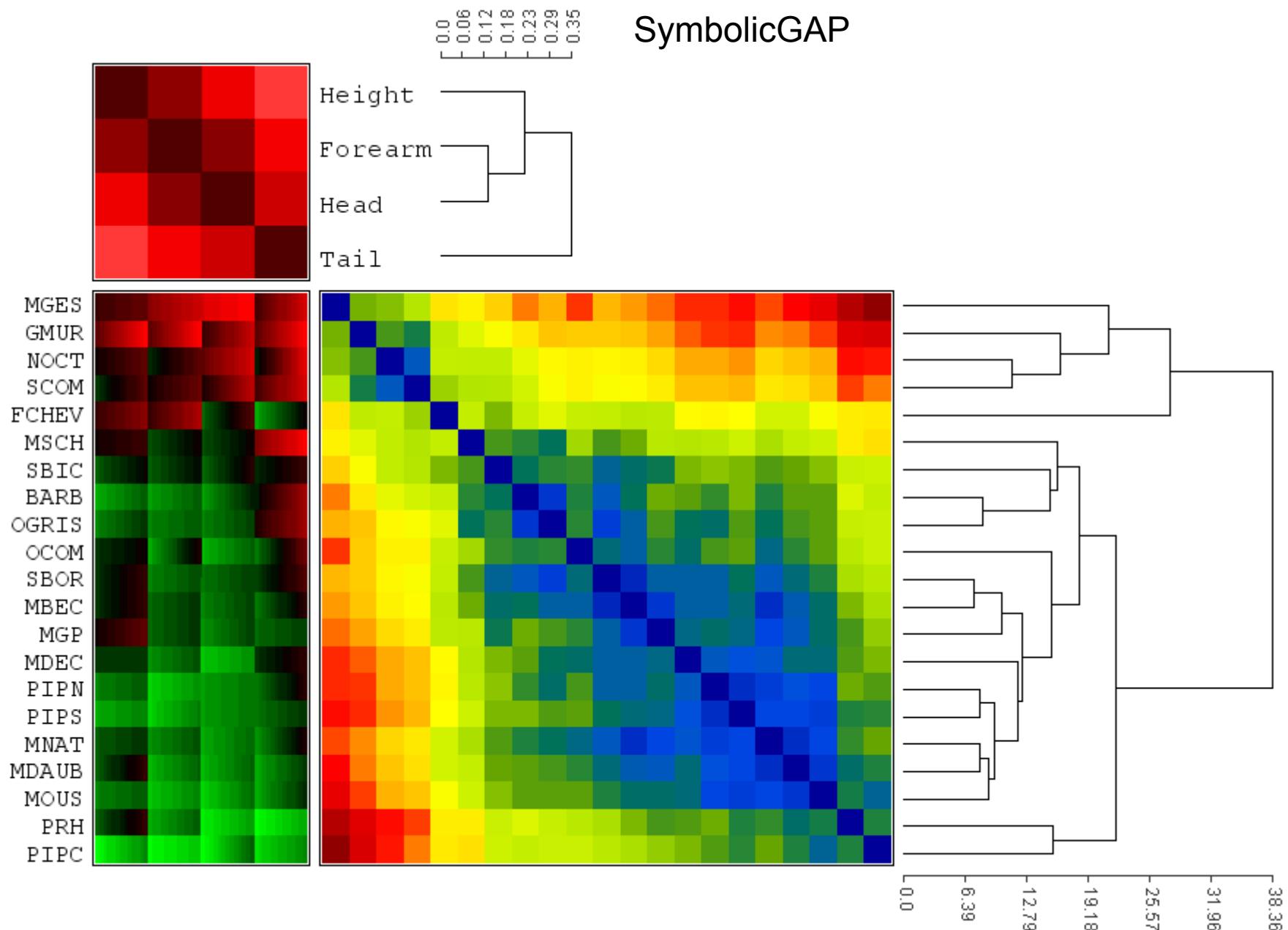
Empirical  
Correlation

Species	Head	Tail	Height	Forearm
PIPC	[33,52]	[26,33]	[4,7]	[27,32]
PRH	[35,43]	[24,30]	[8,11]	[34,41]
MOUS	[38,50]	[30,40]	[7,8]	[32,37]
PIPS	[43,48]	[34,39]	[6,7]	[31,38]
PIPN	[44,48]	[34,44]	[7,8]	[31,36]
MDAUB	[41,51]	[30,39]	[8,11]	[33,41]
MNAT	[42,50]	[32,43]	[8,9]	[36,42]
MDEC	[40,45]	[39,44]	[9,9]	[36,42]
MGP	[45,53]	[35,38]	[10,12]	[39,44]
OCOM	[41,51]	[34,50]	[9,10]	[34,50]
MBEC	[46,53]	[34,44]	[9,11]	[39,44]
SBOR	[48,54]	[38,47]	[9,11]	[37,42]
BARB	[44,58]	[41,54]	[6,8]	[35,41]
OGRIS	[47,53]	[43,53]	[7,9]	[37,41]
SBIC	[50,63]	[40,45]	[8,10]	[40,47]
FCHEV	[50,69]	[30,43]	[11,13]	[51,61]
MSCH	[52,60]	[50,60]	[10,11]	[42,48]
SCOM	[62,80]	[46,57]	[9,12]	[48,56]
NOCT	[69,82]	[41,59]	[10,12]	[45,55]
GMUR	[65,80]	[48,60]	[12,16]	[55,68]
MGES	[82,87]	[46,57]	[11,12]	[58,63]



Euclidean  
Housdorff

# SymbolicGAP



## 2 real examples of MV for SDA:

### A. 民力2010 Database (**interval**) (with Junji Nakano, ISM)

(DVD national manpower-ROM“published by the Asahi Shimbun)  
A collection of **regional data**.

### B. Cross Cultural Comparison of Rural Education Practice in China, Taiwan, and the US (**modal multi-valued**)

# Example: 民力2010 Database (Collabor. with Junji Nakano, ISM)

民力 マーケティング必携  
の地域データベース

トップページ ヘルプ お問い合わせ

DVD-ROM  
**民力**  
マーケティング・データベース

1989-2010 2010

エリア・都市圏・市区町村別に調べる

知りたいエリアを地図から先に指定する場合に使用します。この入口からは、エリア別の民力データを調べることができます。

+ ブロック選択画面へ

都道府県別に調べる

都道府県のデータ資料を20分野にジャンル分けしています。書籍版『民力』の「都道府県別資料集」に該当します。

+ ジャンル選択画面へ

都道府県別民力指数を調べる

都道府県の民力指数を調べる場合に利用します。

+ 都道府県選択画面へ

「民力DVD-ROM2010」について  
最初にお読みください

解説を読む

書籍版『民力』の特集記事、各種の解説記事を探すための入口です。

詳しくはこちら

**民力 WEB版のご案内**

DVD-ROM版にはないさまざまな機能をご利用いただけます。

詳しくはこちら

資料出所先一覧

調べる + エリア・都市圏・市区町村別 | + 都道府県別 | + 都道府県別民力指数

+ 解説を読む

## エリア・都市圏・市区町村別に調べる

### Step.1 ブロックを選択

見たいエリアが含まれるブロック名を地図上(文字の上)でクリックするか、右側のブロック名一覧の該当ボタンをクリックしてください。

#### エリア選択

北海道

甲信越・北陸・東海

名古屋狭域エリア

中国・四国

近畿

大阪狭域エリア

九州・沖縄

東北

関東

東京狭域エリア

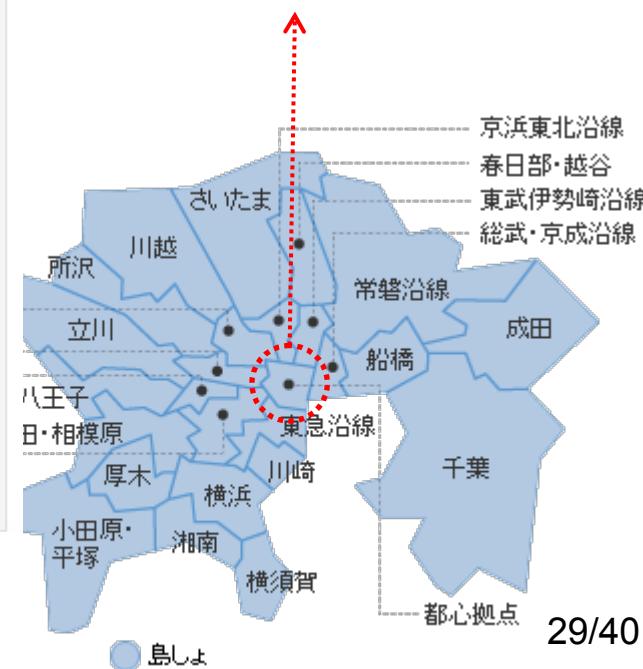
- + 北海道
- + 東北
- + 関東
- + 甲信越・北陸・東海
- + 近畿
- + 中国・四国
- + 九州・沖縄
- + 東京狭域エリア
- + 名古屋狭域エリア
- + 大阪狭域エリア

**東京狭域エリア**

前のページへ戻る

千代田都市圏	千代田区
中央都市圏	中央区
港都市圏	港区
新宿都市圏	新宿区
文京都市圏	文京区
台東都市圏	台東区
渋谷都市圏	渋谷区
豊島都市圏	豊島区

## 都心拠点地域



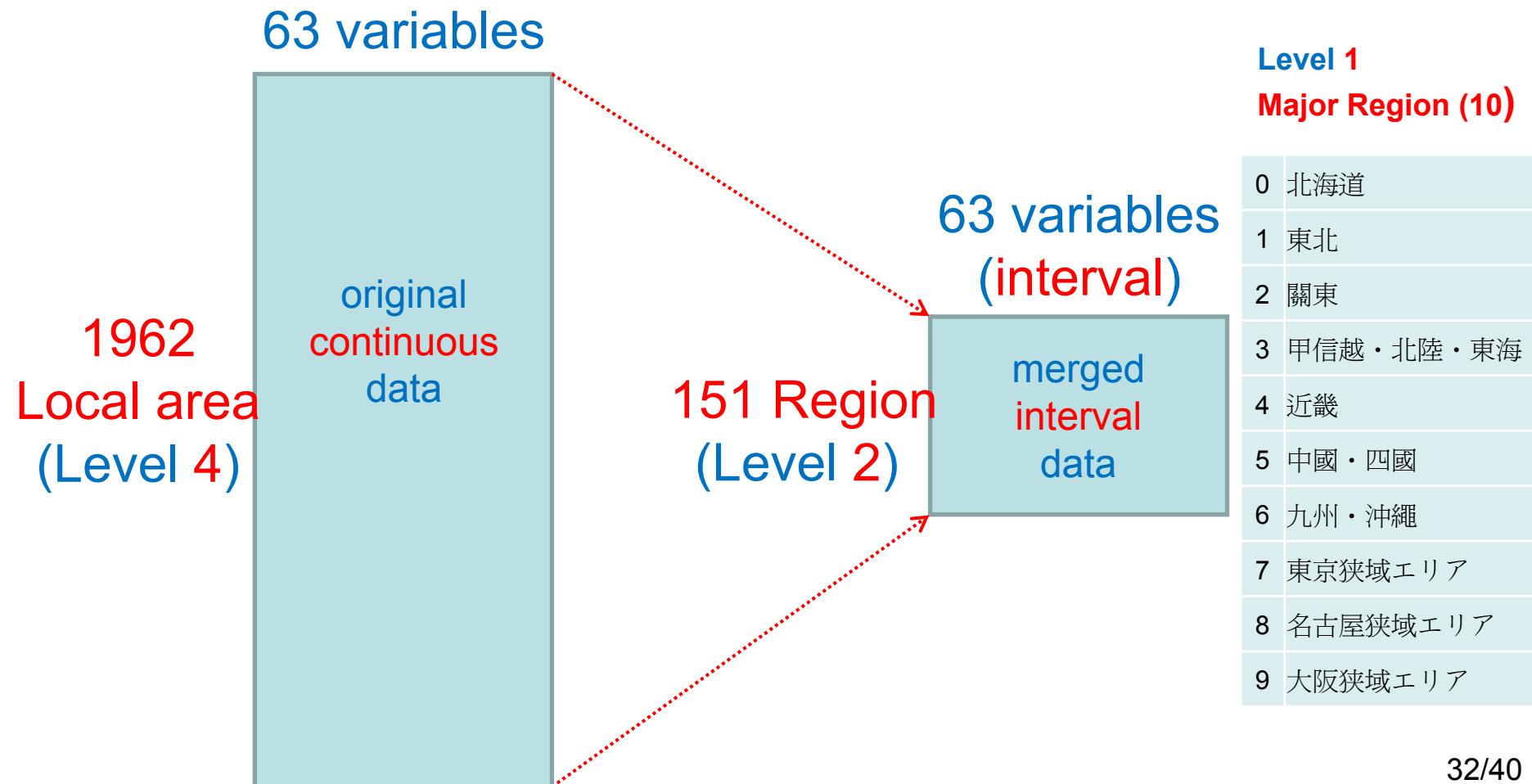
# Hierarchy for Regions of 民力 Data

Level 1	Level 2	Level 3	Level 4	Level 0
Major Region (10)	Region (151)	Major Area (821)	Area (1962)	都道府縣 (45)
東京狭域エリア	立川地域	あきる野都市圏	あきる野市	東京都
東京狭域エリア	立川地域	あきる野都市圏	日の出町	東京都
東京狭域エリア	立川地域	あきる野都市圏	檜原村	東京都
東京狭域エリア	立川地域	立川都市圏	立川市	東京都
東京狭域エリア	立川地域	羽村都市圏	羽村市	東京都
東京狭域エリア	立川地域	羽村都市圏	瑞穂町	東京都
東京狭域エリア	立川地域	国分寺都市圏	国分寺市	東京都
東京狭域エリア	立川地域	国立都市圏	国立市	東京都
東京狭域エリア	立川地域	東大和都市圏	東大和市	東京都
東京狭域エリア	立川地域	武藏村山都市圏	武藏村山市	東京都
東京狭域エリア	立川地域	青梅都市圏	青梅市	東京都
東京狭域エリア	立川地域	青梅都市圏	奥多摩町	東京都
東京狭域エリア	立川地域	青梅都市圏	小菅村	山梨縣
東京狭域エリア	立川地域	青梅都市圏	丹波山村	山梨縣
東京狭域エリア	立川地域	昭島都市圏	昭島市	東京都
東京狭域エリア	立川地域	福生都市圏	福生市	東京都
東京狭域エリア	都心拠点地域	千代田都市圏	千代田區	東京都
東京狭域エリア	都心拠点地域	中央都市圏	中央區	東京都
東京狭域エリア	都心拠点地域	中央都市圏	中央市	山梨縣
東京狭域エリア	都心拠点地域	中央都市圏	市川三郷町	山梨縣
東京狭域エリア	都心拠点地域	中央都市圏	身延町	山梨縣
東京狭域エリア	都心拠点地域	文京都市圏	文京區	東京都
東京狭域エリア	都心拠点地域	台東都市圏	台東區	東京都
東京狭域エリア	都心拠点地域	渋谷都市圏	渋谷區	東京都
東京狭域エリア	都心拠点地域	港都市圏	港區	東京都
東京狭域エリア	都心拠点地域	新宿都市圏	新宿區	東京都
東京狭域エリア	都心拠点地域	豊島都市圏	豊島區	東京都
近畿	熊野灘エリア	尾鷲都市圏	紀北町	三重縣
近畿	熊野灘エリア	尾鷲都市圏	尾鷲市	三重縣
近畿	熊野灘エリア	熊野都市圏	熊野市	三重縣
近畿	熊野灘エリア	熊野都市圏	北山村	和歌山縣
近畿	熊野灘エリア	熊野都市圏	御浜町	三重縣
近畿	熊野灘エリア	新宮都市圏	紀宝町	三重縣
近畿	熊野灘エリア	新宮都市圏	新宮市	和歌山縣
近畿	熊野灘エリア	新宮都市圏	那智勝浦町	和歌山縣
近畿	熊野灘エリア	新宮都市圏	太地町	和歌山縣
近畿	熊野灘エリア	新宮都市圏	古座川町	和歌山縣
近畿	熊野灘エリア	新宮都市圏	串本町	和歌山縣

Population_Basic_Resident_Registers	Number of private establishments Services
Household Basic Resident Registers	Total Number of Retail Stores
Population moved in	Annual Sales Stores
Population moved out	Annual Sales Retail Stores
Population by year 0-4	Number of restaurants
Population by year 5-9	Annual Shipments of industrial products
Population by year 10-14	Agricultural output value
Population by year 15-19	Taxable income
Population by year 20-24	Local tax revenues
Population by year 25-29	Total expenditures
Population by year 30-34	Number of ownership car
Population by year 35-39	Number of educational facilities
Population by year 40-44	Number of Establishments Books Stationery Retail
Population by year 45-49	Number of libraries
Population by year 50-54	Number of nursery
Population by year 55-59	Number of community centers
Population by year 60-64	Number of hospitals
Population by year > 65	National manpower Composite Index
Municipal workers	Population index
New construction housing stock	Per capita level of national manpower
Total number of establishments	Population ratio populated area (Area)
Total number of employees, business	Daytime and nighttime population ratio
Number of private establishments	Basic Resident Registers population growth
The number of employees at private businesses	Population density (area)
Number of establishments Services	Census households (compared to single-person hous
Number of private establishments Services	Percentage of population < 15
Total Number of Retail Stores	Percentage of population 15-29
Annual Sales Stores	Percentage of population 30-44
Annual Sales Retail Stores	Percentage of population 45-64
Number of restaurants	Percentage of population > 65
Annual Shipments of industrial products	3 Industrial sector work force composition_1
Agricultural output value	3 Industrial sector work force composition_2
Taxable income	3 Industrial sector work force composition_3
Local tax revenues	Annual sales growth rate of retail shops
Total expenditures	Income gap
	Increase local tax revenues

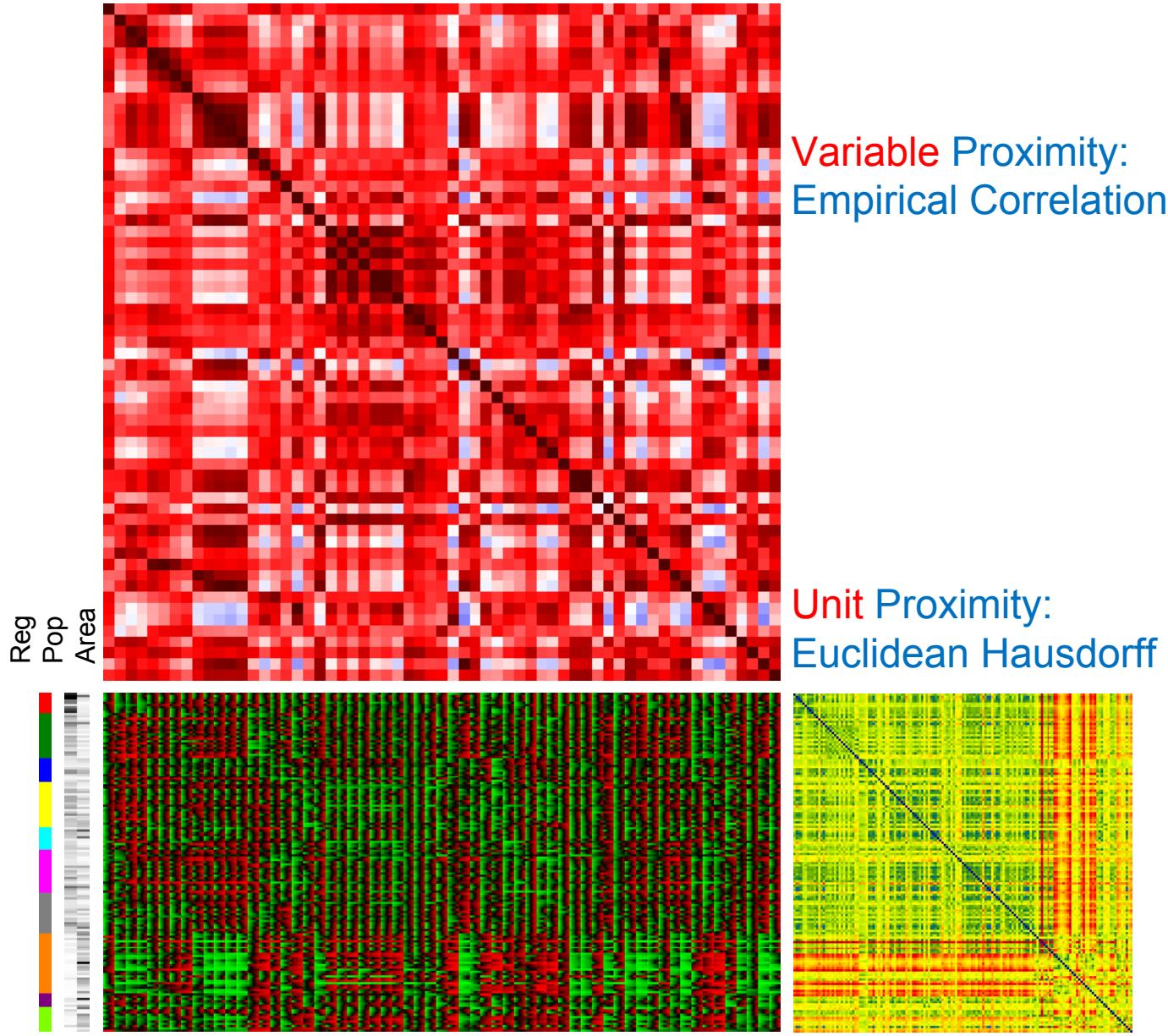
Data :

63 (continuous) variables for 1962 Local area (Level 4)  
were merged into  
63 (interval) variables for 151 Region (Level 2)

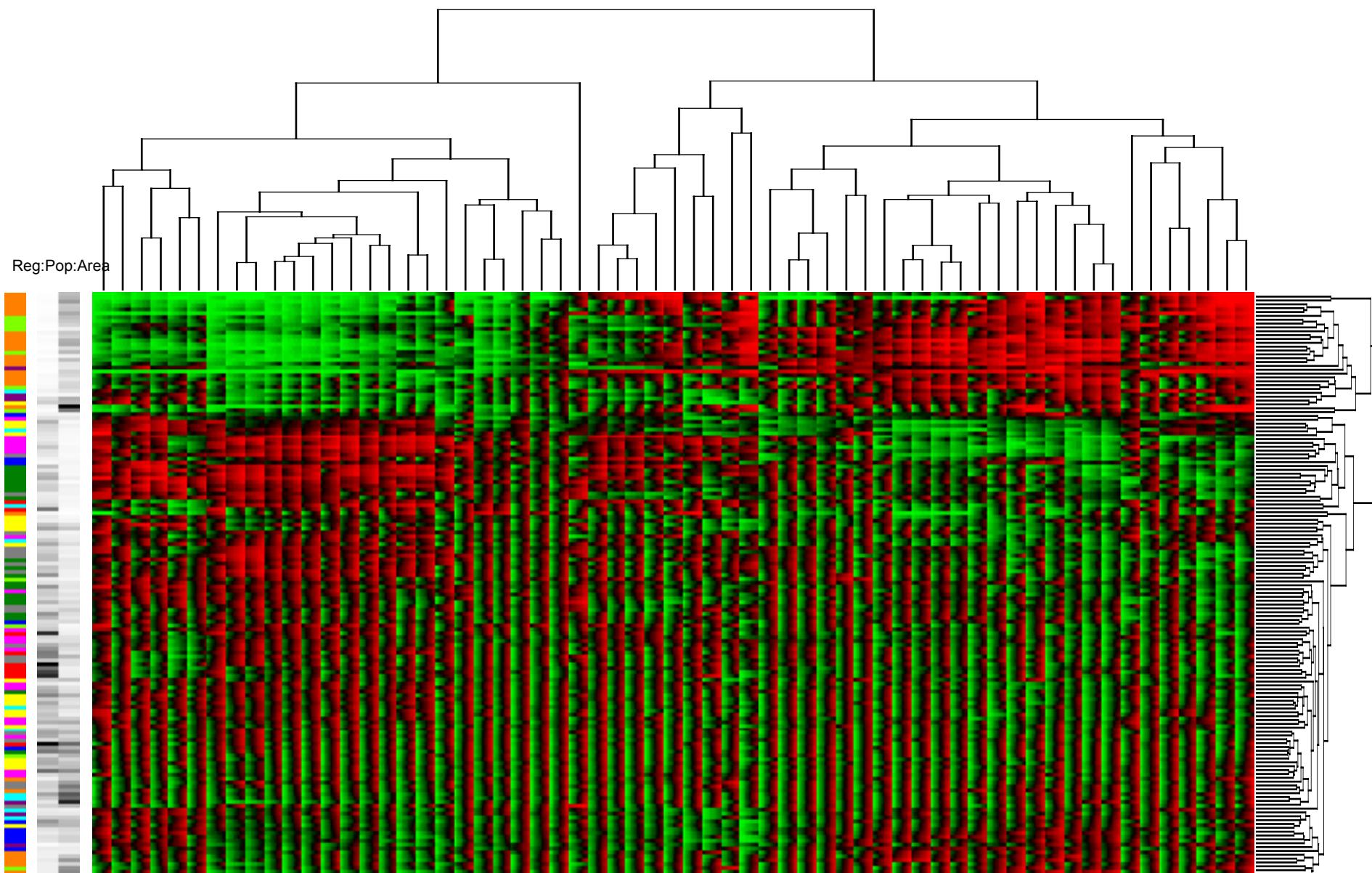


# Symbolic GAP for 民力2010 Data (Original Orders)

area_id	area_name
0	北海道
1	東北
2	關東
3	甲信越・北陸・東海
4	近畿
5	中國・四國
6	九州・沖繩
7	東京狭域エリア
8	名古屋狭域エリア
9	大阪狭域エリア



# Symbolic GAP for 民力2010 Data (HCT\_R2E Orders)



# Cross Cultural Comparison of Rural Education Practice in China, Taiwan, and the US

1. MATHEMATICAL KNOWLEDGE TEST
  - Part A: Computation
  - Part B: Number Concepts and Equations
  - Part C: Geometry
  - Part D: Word Problems
2. AFTER SCHOOL LEARNING ACTIVITIES
3. ATTITUDES TOWARD LEARNING
4. SCHOOL CLASSMATES AND FRIENDS
5. HOME ENVIRONMENT
6. CREATIVITY—How creative are students
7. On the back side of this page,  
please use your pen or pencil to draw a picture of  
what an ALIEN--Creature from outer space--  
looks like

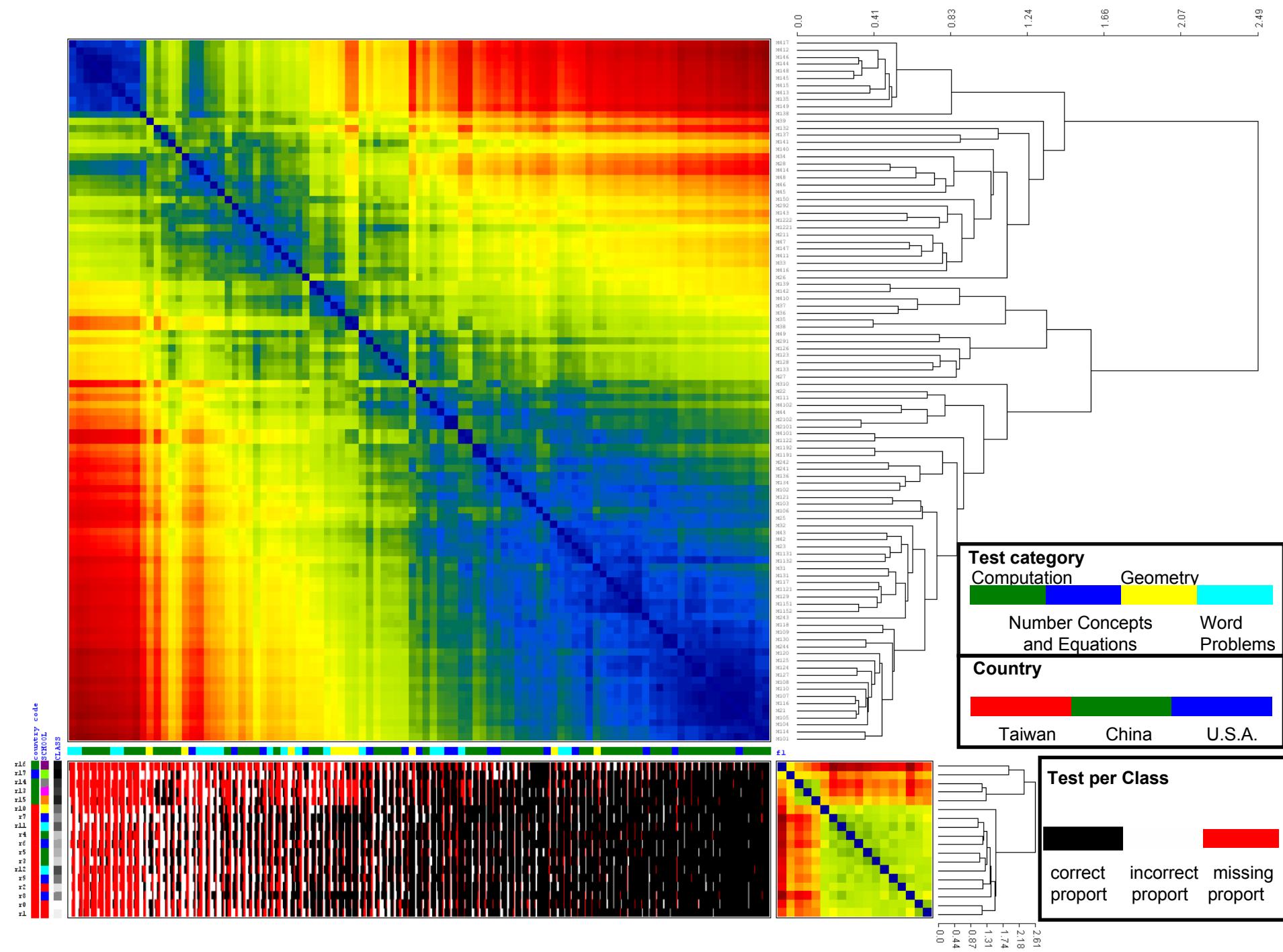
Benjamin, J. (2006) "Cross Cultural Comparison of Rural Education Practice in China, Taiwan, and the United States," American Educational Research Association Annual Conference.

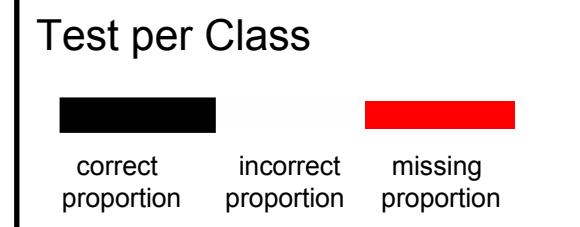
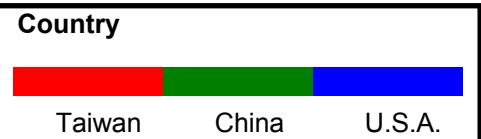
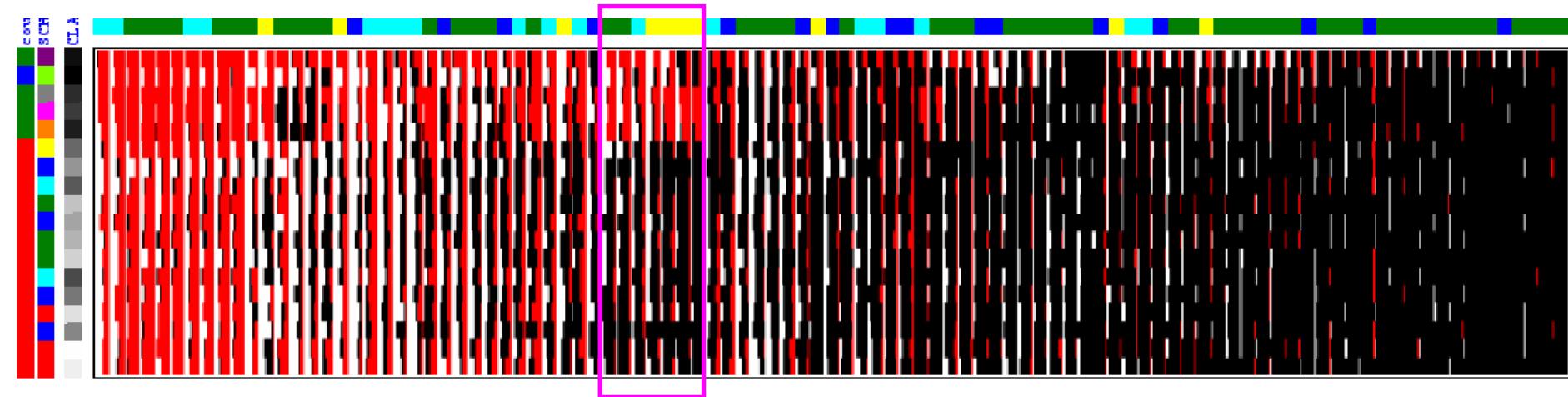
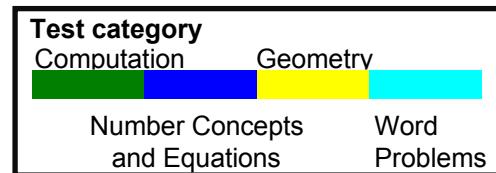
Madigan, T., Benjamin, J., and Shieh, S. H. (2006) "Rural Education Practices and Outcomes in China and the United States," 101 Annual Meeting of the American Sociological Association, Montreal, Quebec.

	<b>China</b>	<b>Taiwan</b>	<b>USA</b>
School	3	5	2
Class	3	13	2
students	272	361	95

<b>Mathematical Knowledge Test</b>	<b>category</b>	<b>number of testing</b>
	Computation	50
	Number Concepts and Equations	11
	Geometry	10
	Word Problems	17

	<b>number of testing</b>
<b>After School Learning Activities</b>	3
<b>Attitudes Toward Learning</b>	5
<b>School Classmates And Friends</b>	6
<b>Home Environment</b>	12
<b>Creativity</b>	8





# Summary

- We have extended our matrix visualization methods on symbolic data with **continuous** data types.
- We will also try to extend our matrix visualization methods to other symbolic data types (**categorical**).

*Thank you for your attention !*