

# **Adaptive Forward-Backward Splitting for Multi-Layer Convolutional Dictionary Learning**

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## **Abstract**

Multi-layer convolutional sparse coding (ML-CSC) is a powerful technique for obtaining parsimonious representations of signals in machine learning applications. However, achieving maximum parsimony without sacrificing data fidelity requires learning a dictionary from data, resulting in the non-convex non-smooth optimization problem of multi-layer convolutional dictionary learning (ML-CDL). Existing ML-CDL solvers derived from traditional dictionary learning employ the convex  $\ell_1$  function as the sparsity-inducing function to promote convergence; nevertheless, analyzing the convergence of candidate alternate optimization schemes is still challenging. In this paper, we reformulate the ML-CDL problem by replacing the convolution with the Hadamard product to reduce computational complexity and, to achieve this without destroying the consistency between the variables in the time and frequency domains, additional constraints are introduced to the reformulated problem. We also modify a forward-backward splitting algorithm to solve the reformulated ML-CDL problem using simultaneous coefficients and dictionary elements updates across all layers. In contrast, these variables are updated alternately in the previous approaches. The proposed algorithm includes a parameter adaptation scheme to improve optimization performance and ensure convergence. In numerical simulations, the proposed method outperforms existing methods by achieving convergence in fewer iterations and generating a smaller final functional value. The high quality of recovered images demonstrates the applicability of multi-layer sparse representation to image reconstruction.

Keyword: multi-layer convolutional sparse representation, dictionary learning, forward-backward splitting