## Sliced Inverse Interval Estimation for Sufficient Dimension

Reduction: a Symbolic Data Analysis Approach

吳漢銘

國立政治大學統計學系

## Abstract

Sufficient Dimension Reduction (SDR) has become a central topic in statistics and machine learning, providing powerful tools for reducing the dimensionality of covariates in a nonparametric manner. This paper proposes a novel SDR method, Sliced Inverse Interval Estimation (SIIE), which draws inspiration from Symbolic Data Analysis (SDA) to improve the estimation of Effective Dimension Reduction (EDR) directions. Existing SDR approaches based on inverse conditional moments typically rely on single summary statistics, such as the mean, median, \$k\$th moment, variance, mean difference, or quantiles, thus neglecting much of the structural information contained within slices of the data. To address this limitation, SIIE introduces a new kernel matrix for central subspace estimation that incorporates interval information, yielding a richer representation of slice structures. We establish two theorems: under mild conditions, the directions estimated from inverse intervals span the EDR subspace, and the central subspace estimator is root-\$n\$ consistent. The effectiveness of SIIE is demonstrated through simulation studies and real data analyses, where it is compared with existing inverse conditional moments—based SDR methods. The results show that SIIE, through its novel use of inverse interval estimation, more effectively captures the conditional distributional features of the response given highdimensional covariates, thereby advancing both the theory and application of SDR. •

Keyword: Central subspace; Dimension reduction; Interval-valued data; Sliced inverse regression; Symbolic data analysis