工業實驗設計與品質改善分析

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September 8, 2016

日本工業界流行之名言錄

- 1960 年代

  不懂「統計」的工程師

  是二流工程師

- 1970 年代

  不懂「實驗設計」的工程師

  只是半個工程師

前言 (Introduction)

- 現況: 二次大戰後，日本展開復建工作時，它面臨著原料、生產設備、生產技術等嚴重短缺的困境。
- 結果: 日本竟然能在短短三、四十年內重新站起來，甚至把美國產業 (如：汽車業、電子業、照相業等) 逐一打敗，而成為工業強國，其成功的秘訣為何？

- 1980, May, NBC 製作一電視專輯探討此主題，其題目是：*If Japan can, why we can't?*

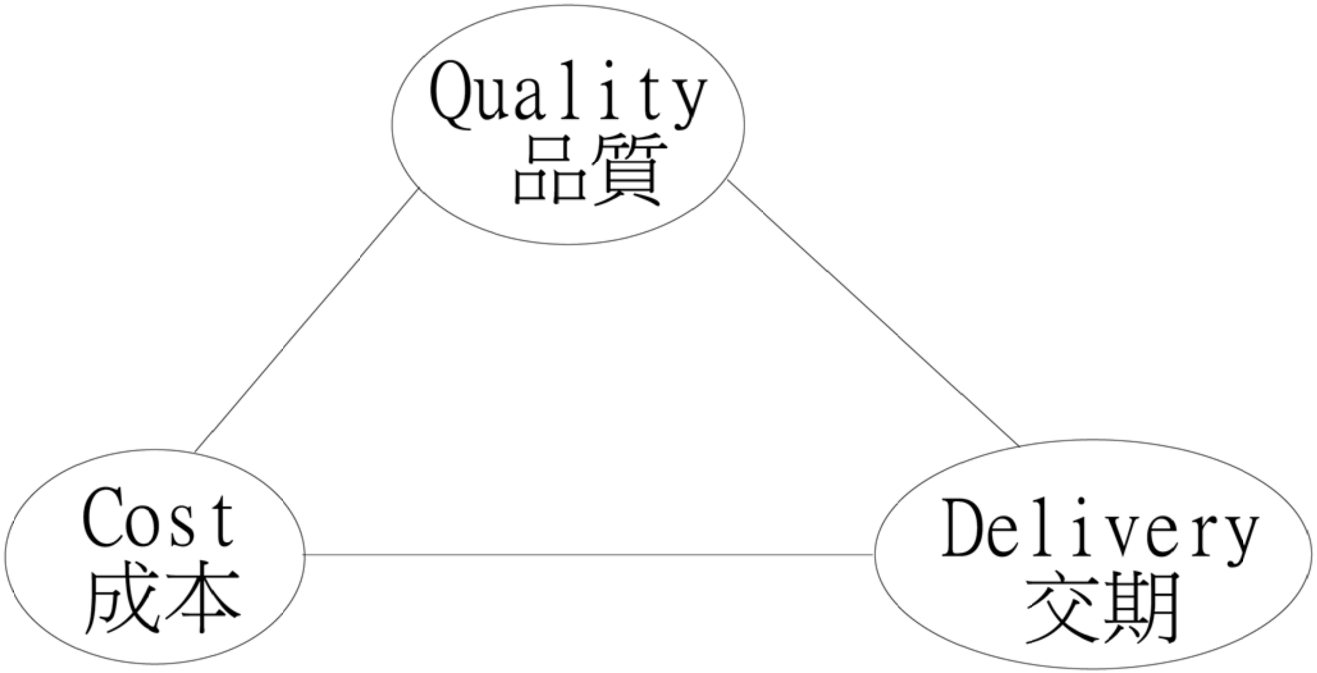
QCD 準則

- 一個企業賴以生存的三大要件是：

  (a) 品質（Quality）
  (b) 成本（Cost）
  (c) 交期（Delivery）

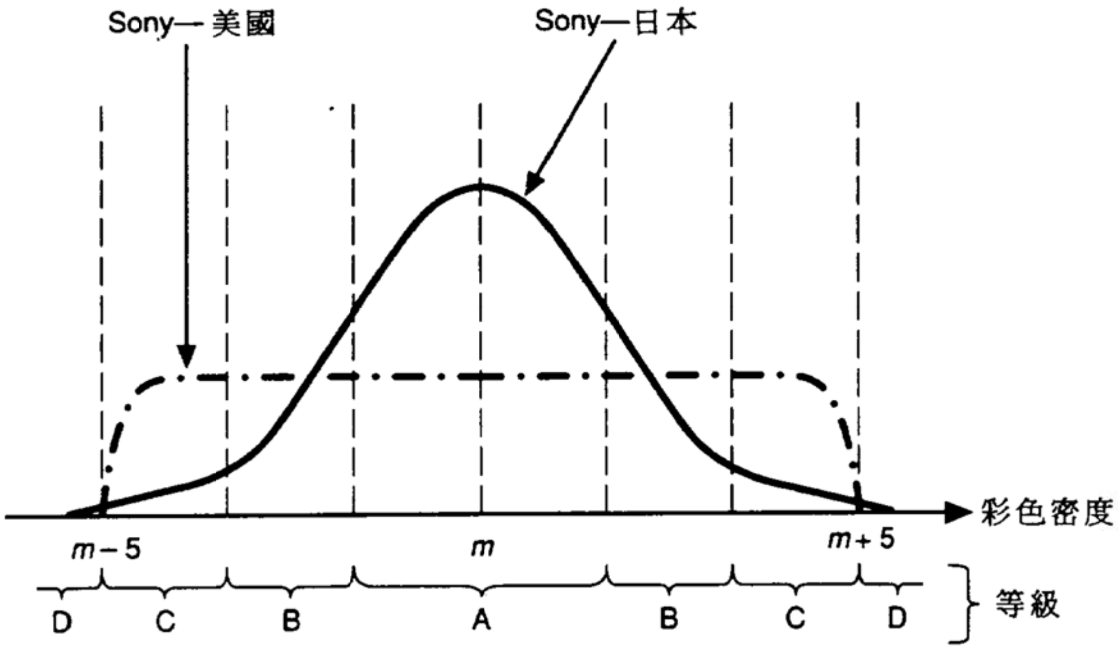
- 換句話說，如果一個企業能夠以最快的速度生產出讓消費者滿意的高品質（high quality）及低成本（low cost）之產品，則此企業將能永續經營下去。

企業競爭力之三大要素



實例一：電視機的色彩品質

- SONY-日本廠的產品品質及SONY-美國廠的產品品質分佈圖如下：



電視機彩色密度之分配圖 (The Asahi, April 17, 1979)

從消費者的角度來看此問題：

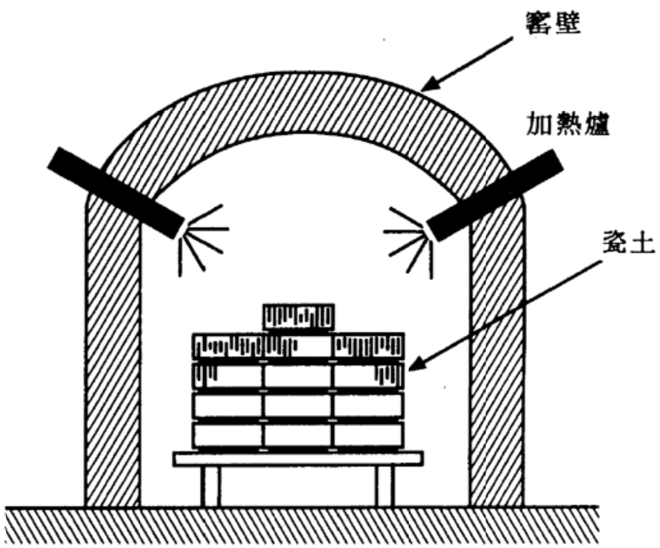
SONY-日本	SONY-美國
A（68%）	A（33%）
B（27%）	B（33%）
C（5%）	C（33%）
D（0.3%）	D（0%）

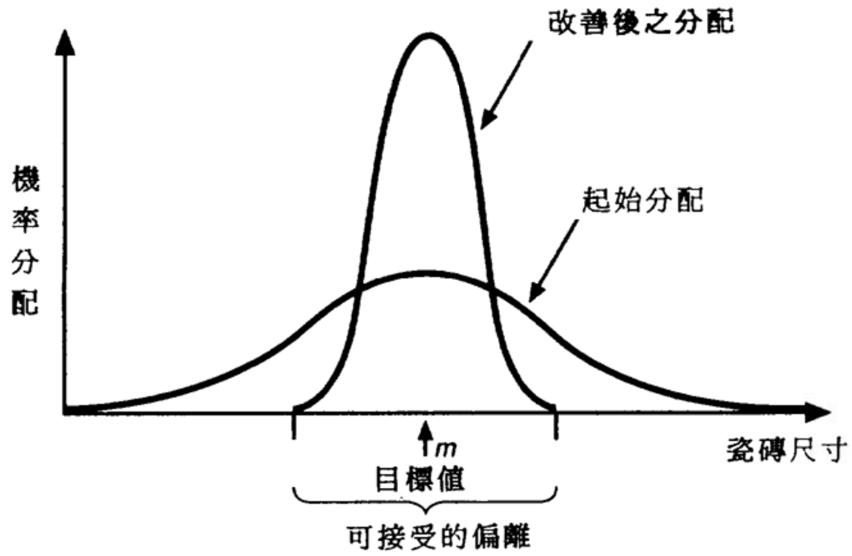
- A 級品：SONY-日本廠比SONY-美國廠顯著高
- C 級品：SONY-日本廠比SONY-美國廠顯著小

從生產者的角度來看：

- 從圖形來看，若為產品規格，則
（1）SONY-美國：沒有不良品。
（2）SONY-日本：有部分（比例很低）之不良品。
- 從合乎產品規格的角度來看SONY-美國的產品品質應該比SONY-日本的產品品質來得好，為何顧客反而對此持相反的看法?
- 生產者與消費者的想法有顯著差異存在。

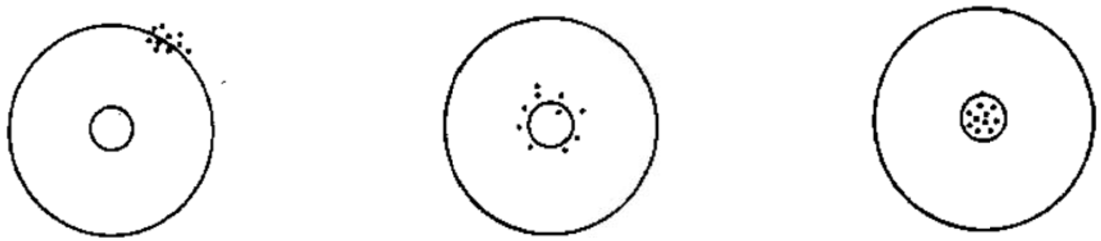
實例二：瓷磚製造例子





何謂「既精且準」產品?

- 既「精」且「準」之產品，它可用下圖說明之。

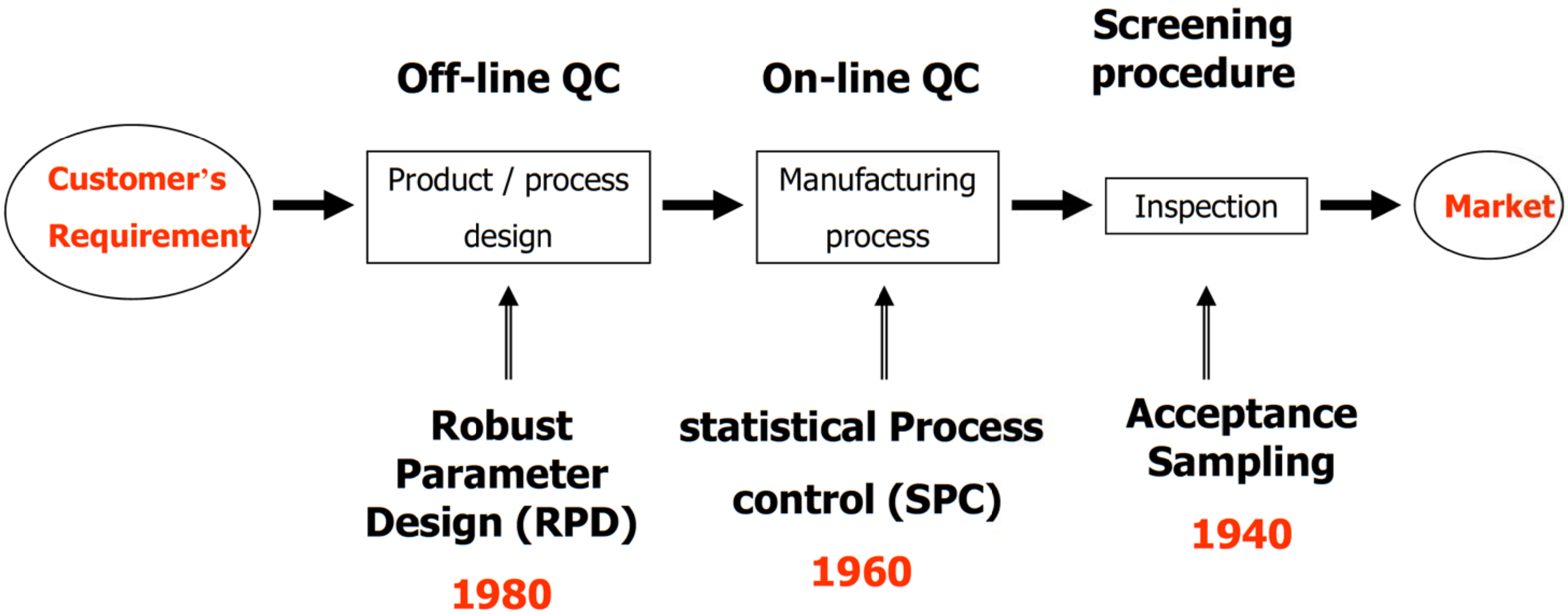


精確性 (Precision)　　準確性 (Accuracy)　　既精且準 (Accuracy & Precision)

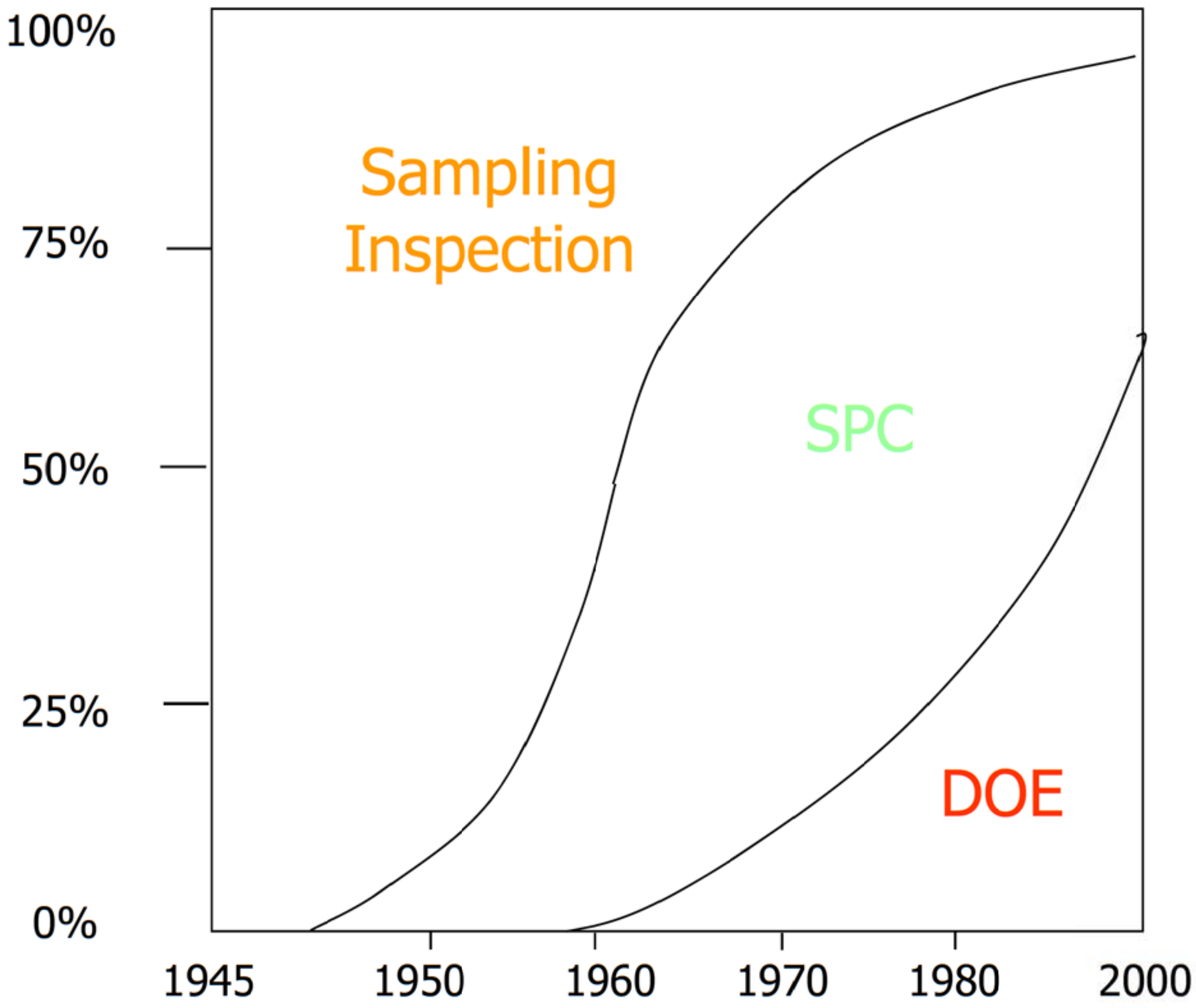
Some Key Issues

- 為何 SONY-日本廠可以生產出既「精」且「準」之產品?
- 為何日本瓷磚廠可以生產出 meet target且製程之變異達極少化之產品?

Quality Control (QC) tools



Evolution of Quality Control Tools



Design of Experiments (DOE)

- Experiment: A Test or a series of tests.
- Goal of experiment:

  To understand and/or improve a system (or process)
- DOE:

  The process of planning and conducting the experiments and about analyzing the resulting data so that valid and objective conclusions are obtained.

DOE 常用方法之分類

Block variables / # of factors	No block variable	One block variables	More than one block variables
One factor (with multi-level) K-sample problem	Pooled-sample design (two-sample) Completely Randomized Design **(CRD)**	Paired-sample design (two-sample) Randomized Block Design **(RBD)**	Latin Square Design Greco-Latin Square Design
k (k>1*)* factors (with 2 or 3 levels) (small *k*)	**Full factorial design** **2^k factorial design** **3^k factorial design**	2^k design with one block variable 3^k design with one block variable	2^k design with two block variables 3^k design with two block variables
k (*k*>1) factors (with 2 & 3 levels) (large *k*)	**Fractional factorial design** **Orthogonal array (OA)**	(例 1) $L_{18}(2^1 3^7)$ 2 levels: 1 factors 3 levels: 7 factors	2^{k-p} in 2^q blocks

Four Categories of DOE

- Treatment Comparisons
  - (ANOVA Model)
- Variable Screening
  - (2-level and 3-level factorial design)
- System Robustness
  - (Robust parameter design)
- Response Surface Methodology
  - (Process optimization)

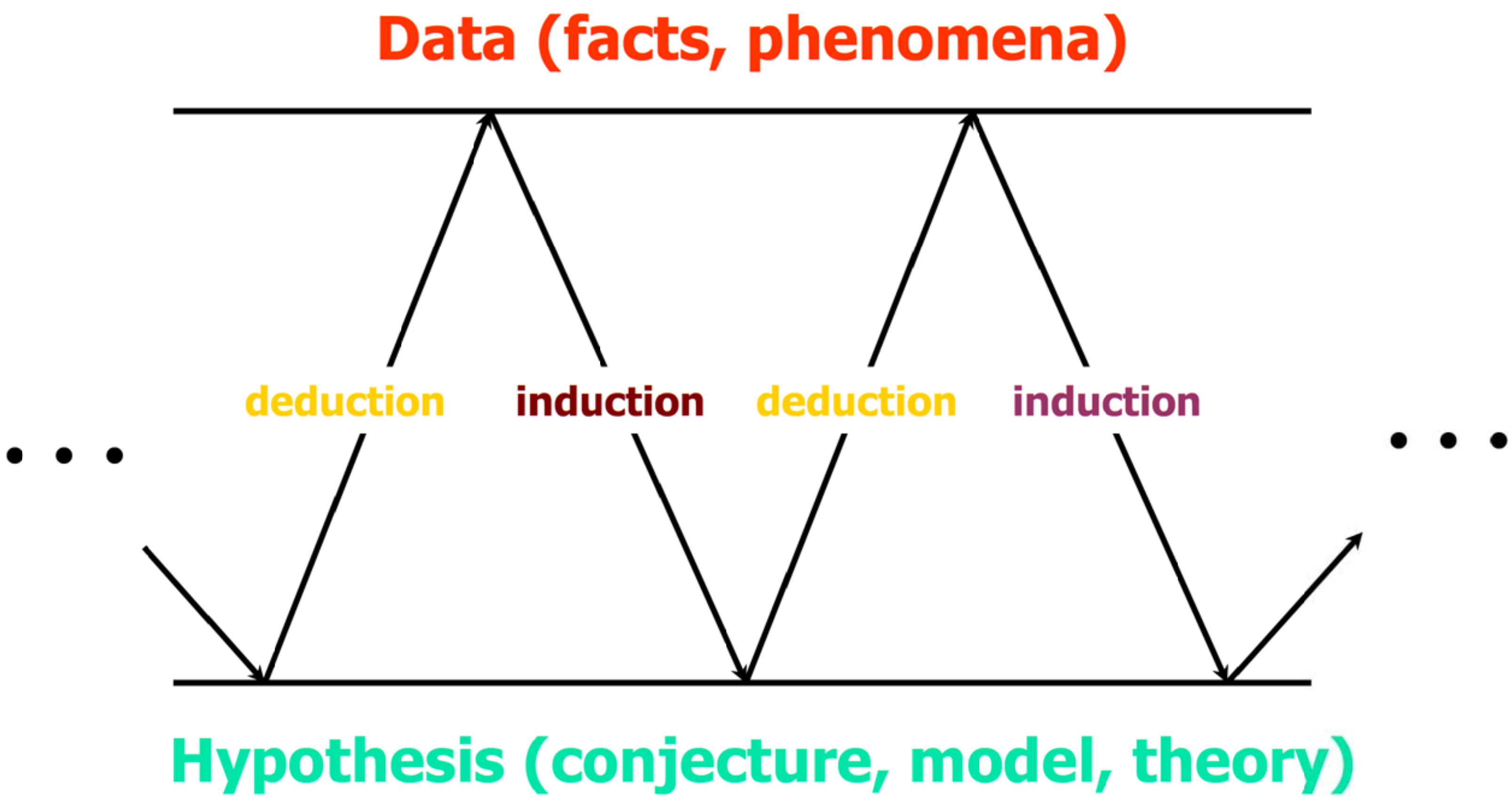
Science and Statistics

- Scientific Research– An iterative learning process

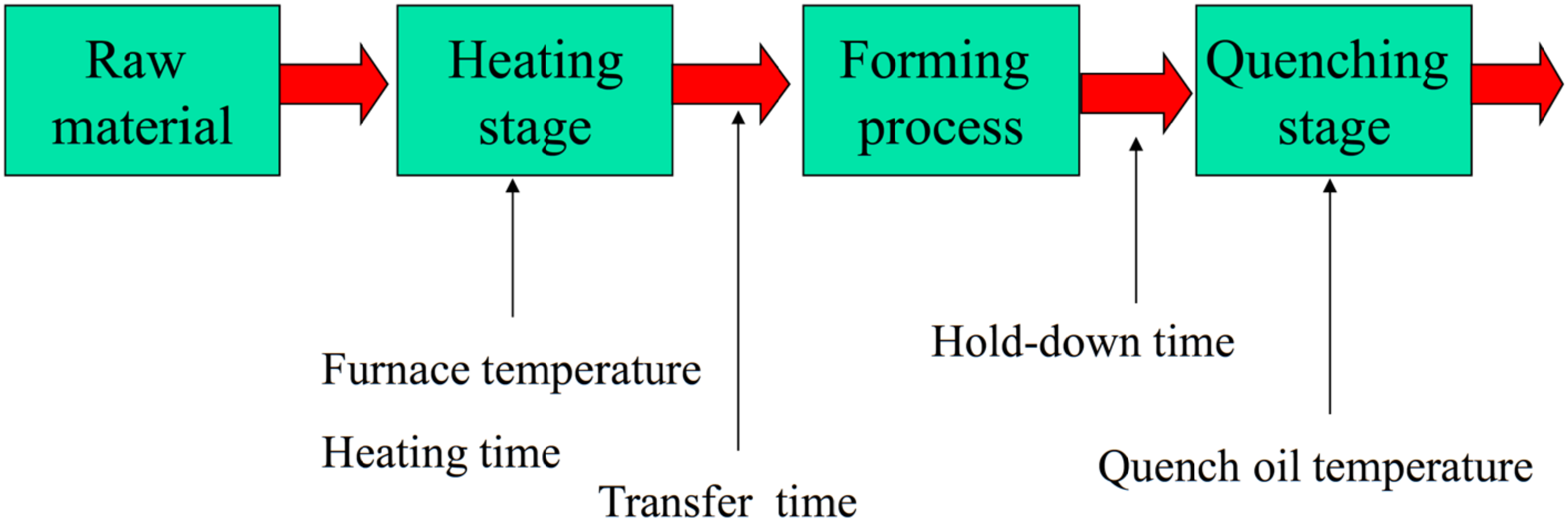
- The object of statistical method is to make that learning process as efficient as possible

Hypothesis 1 → Data 1 → hypothesis 2 → data 2 → …

The iterative learning process



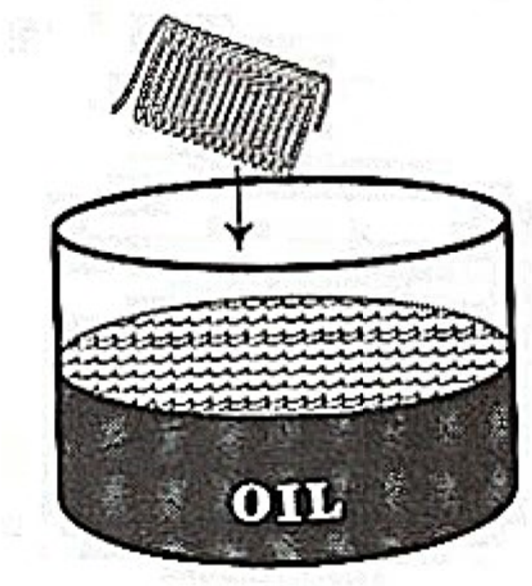
A Leaf Spring Experiment



彈簧製造實例說明

問題描述:

How to improve the design of springs so as to eliminate cracks, which occur during the quenching process?



典型的決策問題

- What is the best temperature (T) of the steel when it is immersed in the quenching oil?
- What is the best carbon content (C) of the steel?
- What is the best temperature of the quenching oil (O)?

現有參數設定 (Settings)

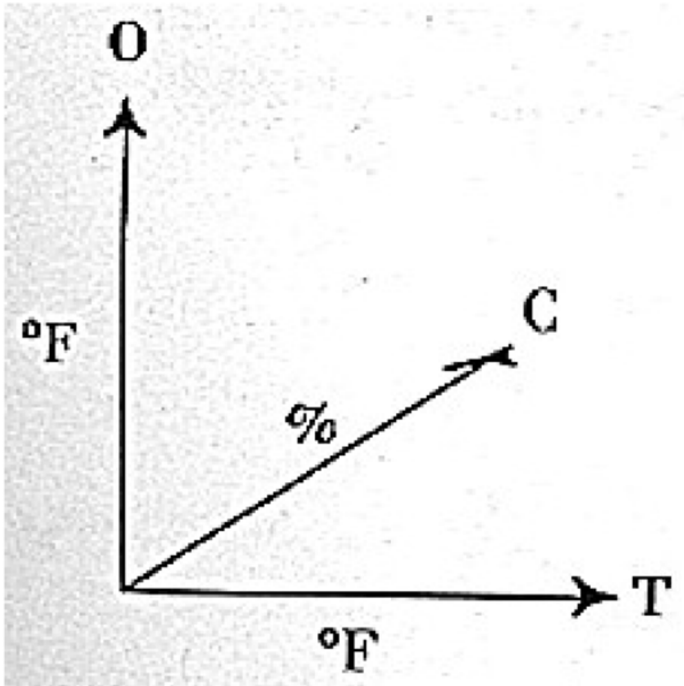
- Engineering handbooks provide ball park figures:

- **T=1450 °F**
  C=0.5%
  O=70 °F

T: Steel temperature

C: Carbon content

O: Oil temperature



「一次只變動一個因子」準則

- **Old Dogma:**

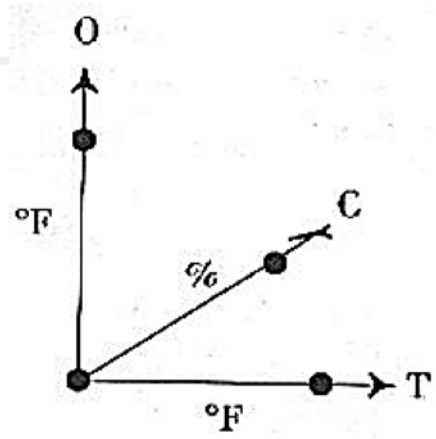
“Hold everything constant, and only vary one factor at a time.”

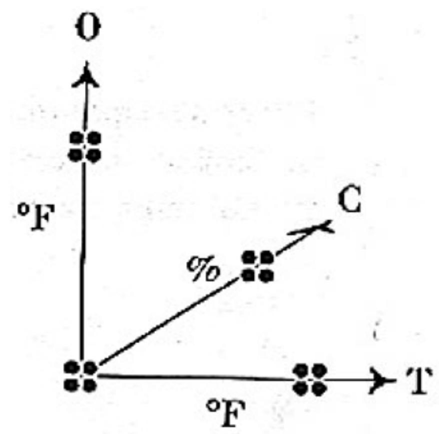
One-Factor-At-A-Time (O-F-A-A-T)

Replications in O-F-A-A-T

- If we were to repeat each run four times this format (one-factor-at-a-time) would require 4 x 4=16 trials.
- All we would know at the end is:

  the effect of each variable at one particular combination of settings of the other two.
- We would not know anything about interaction effects.





全因子設計(Factorial Design)

- Just eight runs to test all three factors and we get even more information than with one-factor-at-a-time experiments

- **Interaction effects**
- **Hidden replication**
- **Wider inductive basis**

T *steel temp.*	C *carbon content*	O *oil temp.*
low	*low*	*low*
high	*low*	*low*
low	high	*low*
high	high	*low*
low	*low*	high
high	*low*	high
low	high	high
high	high	high

Example 1:Manufacture of Springs

MINITAB - Untitled - [EXAMPLE1.MTW ***]

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↓	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	T	C	O	Data
1	1	1	1	1	1450	0.5	70	67
2	2	2	1	1	1600	0.5	70	79
3	3	3	1	1	1450	0.7	70	61
4	4	4	1	1	1600	0.7	70	75
5	5	5	1	1	1450	0.5	120	59
6	6	6	1	1	1600	0.5	120	90
7	7	7	1	1	1450	0.7	120	52
8	8	8	1	1	1600	0.7	120	87
9								

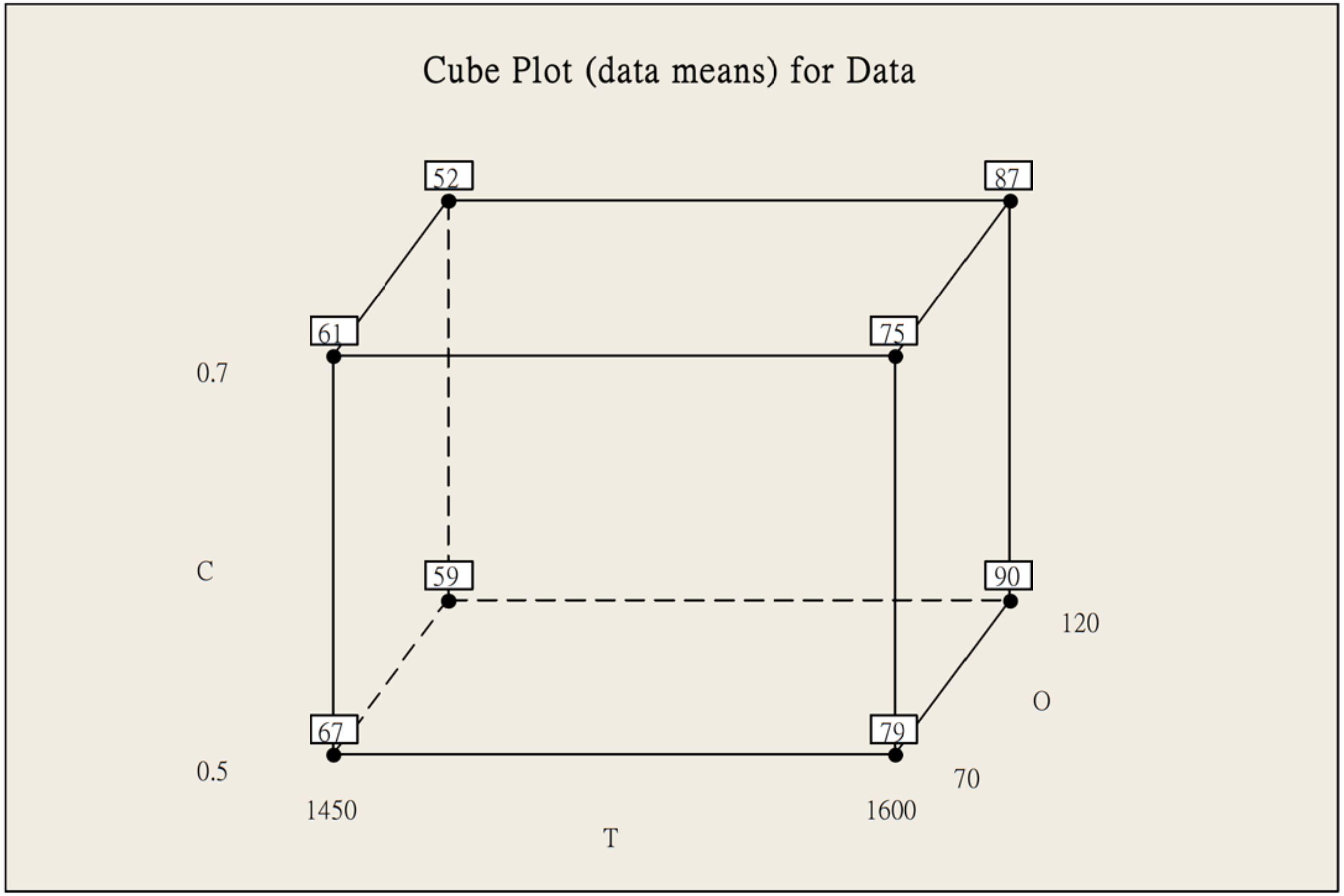
Example 1 (cont.)

Factorial Fit: Data versus T, C, O

Estimated Effects and Coefficients for Data (coded units)

Term	Effect	Coef
Constant		71.250
T	23.000	11.500
C	-5.000	-2.500
O	1.500	0.750
T*C	1.500	0.750
T*O	10.000	5.000
C*O	-0.000	-0.000
T*C*O	0.500	0.250

Example 1 (cont.)



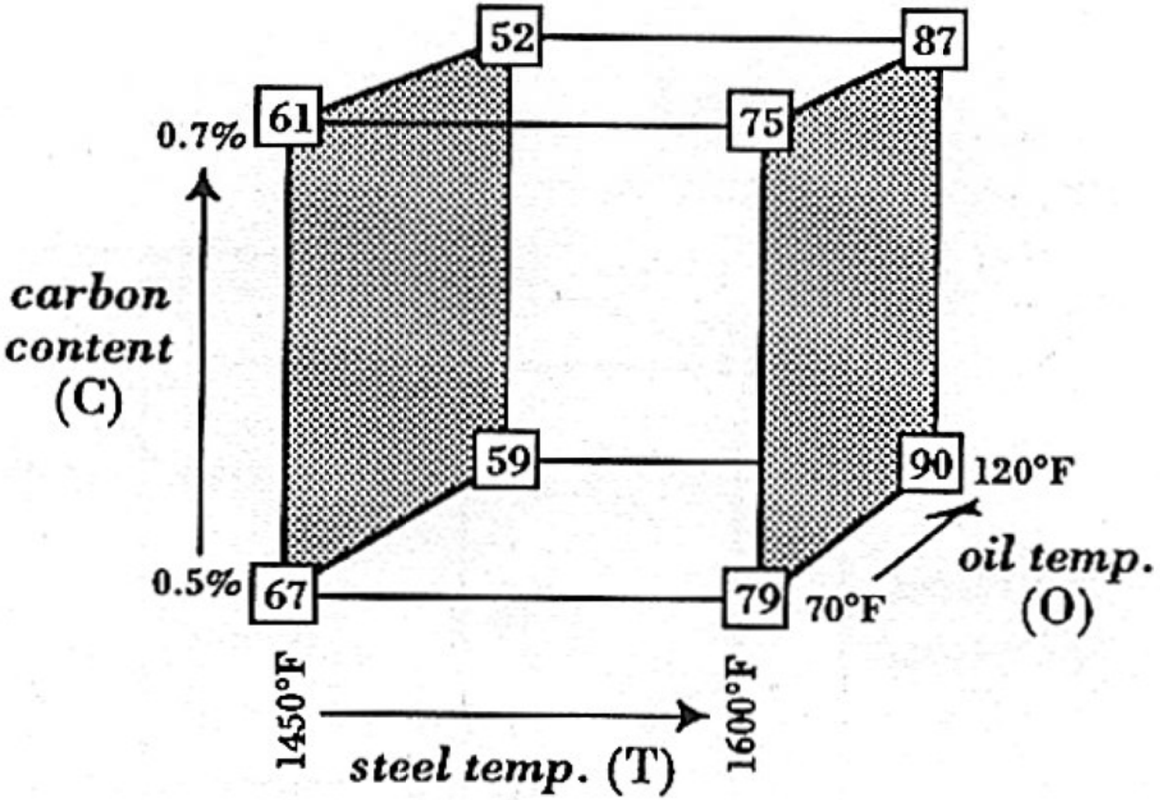
主效應 (Main Effects)

- Main effect:

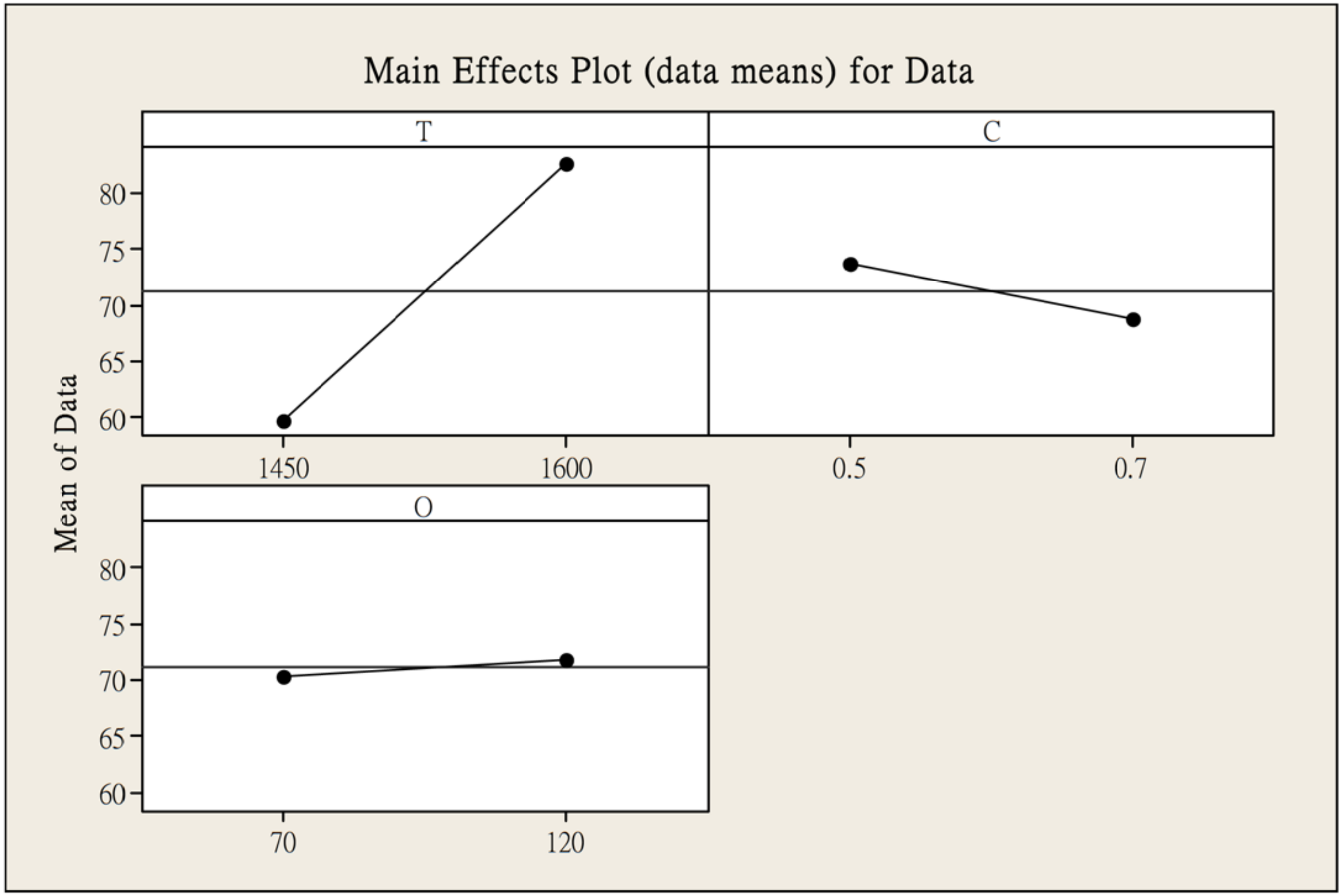
$$ME(A) = z(A^{+}) - z(A^{-})$$

$$= \frac{\sum_{j,k} z(A^{+}, j, k) - z(A^{-}, j, k)}{\#(j,k)}$$

Steel Temperature Effect



Example 1 (cont.)



交互效應 (Interaction Effects)

- Conditional main effect:

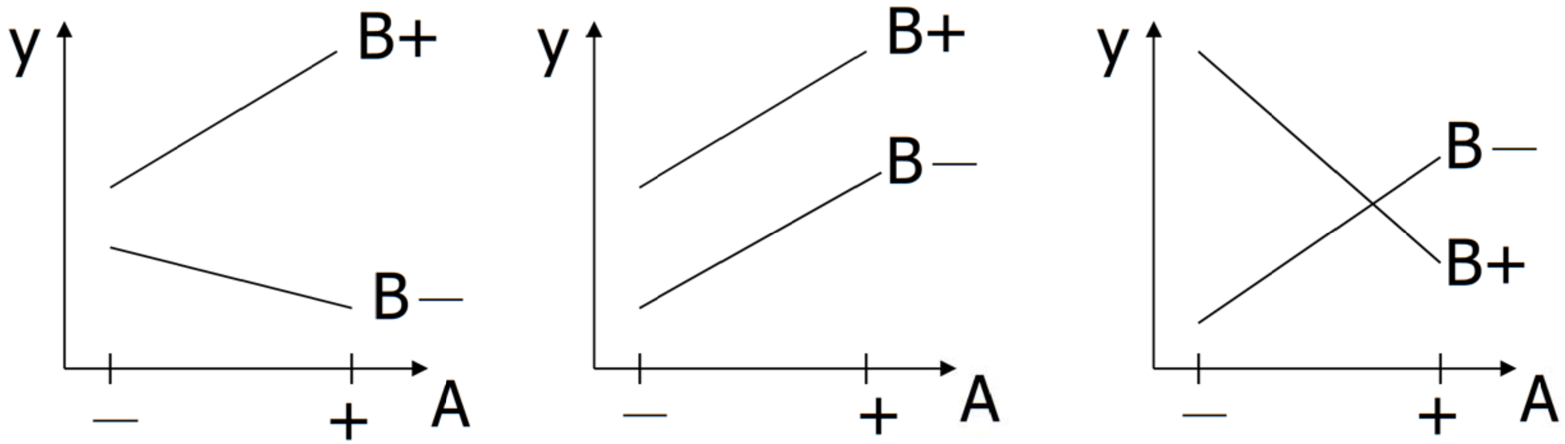
$$ME(A \mid B^{+}) = z(A^{+} \mid B^{+}) - z(A^{-} \mid B^{+})$$

- Interaction effect:

$$Int(A, B) = \frac{1}{2}\{ME(A \mid B^{+}) - ME(A \mid B^{-})\}$$

$$= \frac{1}{2}\{z(A^{+} \mid B^{+}) + z(A^{-} \mid B^{-}) - z(A^{+} \mid B^{-}) - z(A^{-} \mid B^{+})\}$$

Two-factor Interaction



	B—	B+
A—	10	15
A+	5	20

$Int(A,B) > 0$

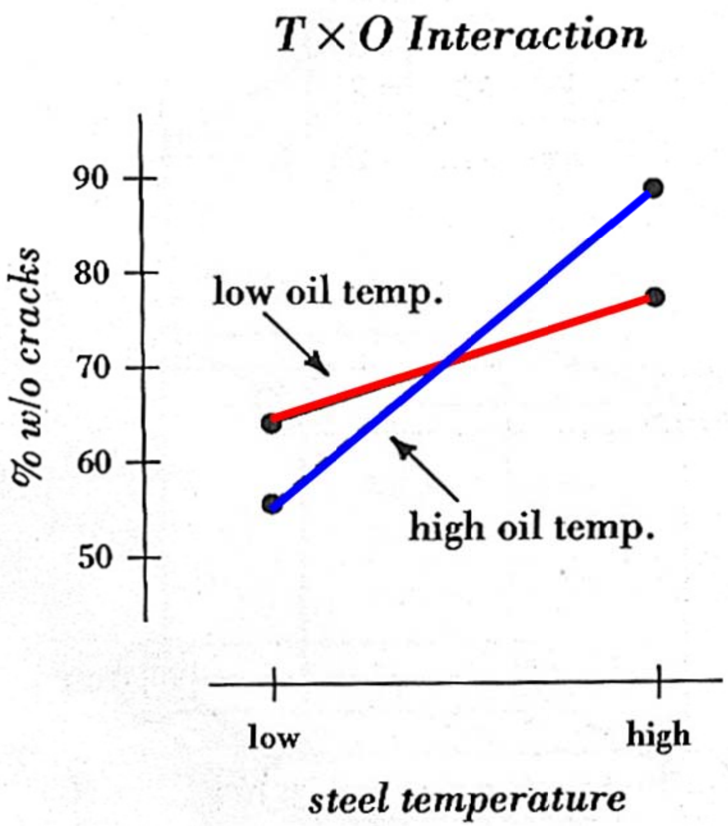
	B—	B+
A—	10	15
A+	15	20

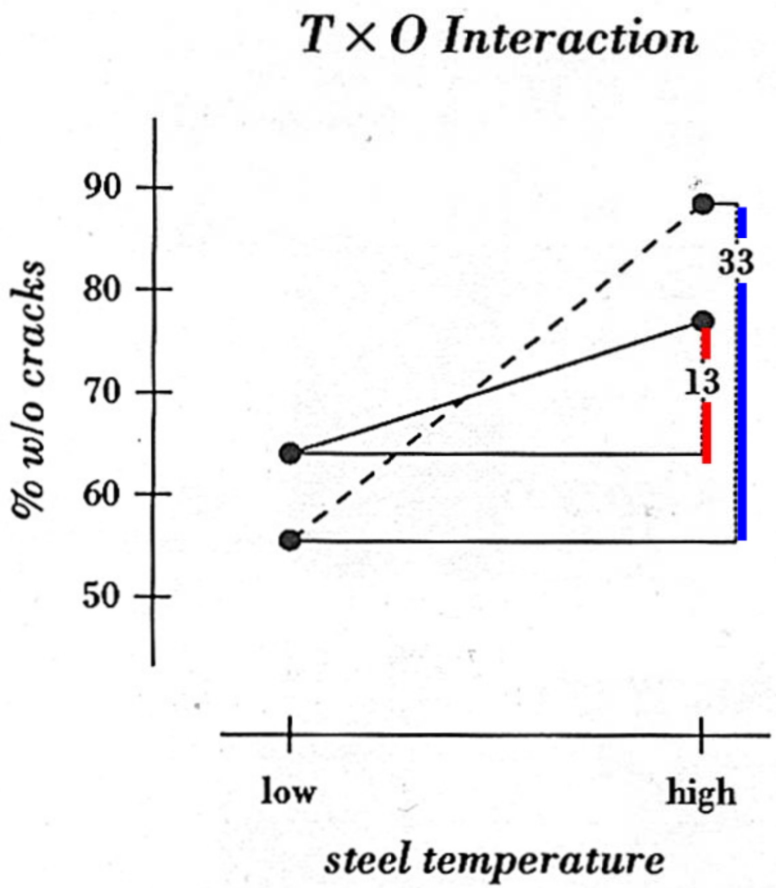
$Int(A,B) = 0$

	B—	B+
A—	5	20
A+	15	10

$Int(A,B) < 0$

Steel temperature effect depends on oil temperature.

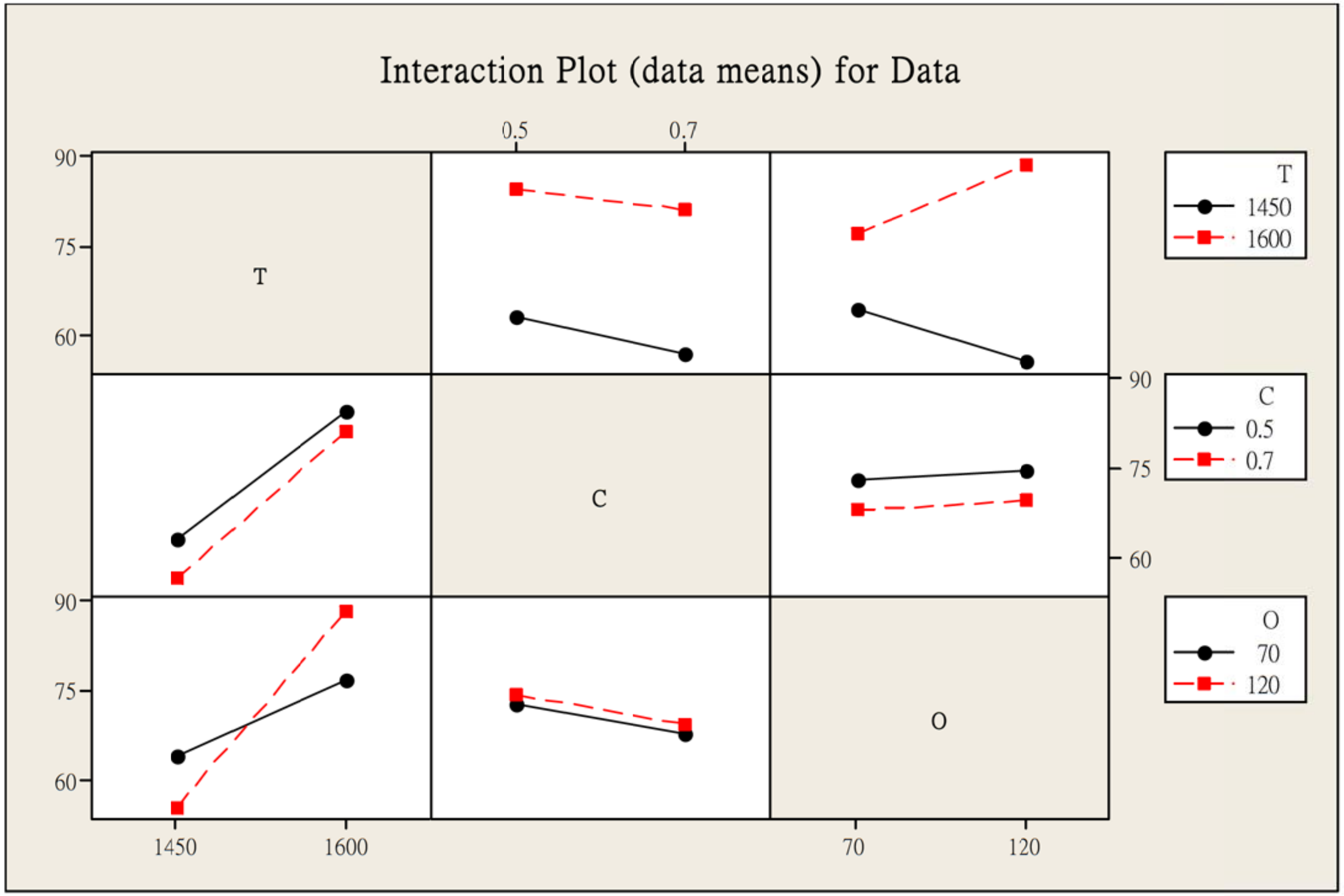




Interaction of factors T and O

Oil / Temp.	Low	High
Low	67% 61% 64%	59% 52% 55.5%
High	79% 75% 77%	90% 87% 88.5%

Example 1 (cont.)



Three-factor Interaction

- Int (A, B, C)={Int (A,B|C+)- Int (A,B|C-)}/2

- X=Z[(A,B,C)=(+,+,+);(+,-,-);(-,+,-);(-,-,+)]
- Y=Z[(A,B,C)=(-,-,-);(+,+,-);(+,-,+);(-,+,+)]

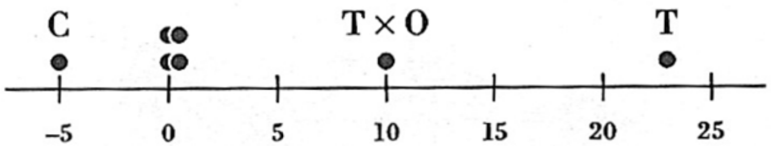
- Int(A,B,C)=(X-Y)/4

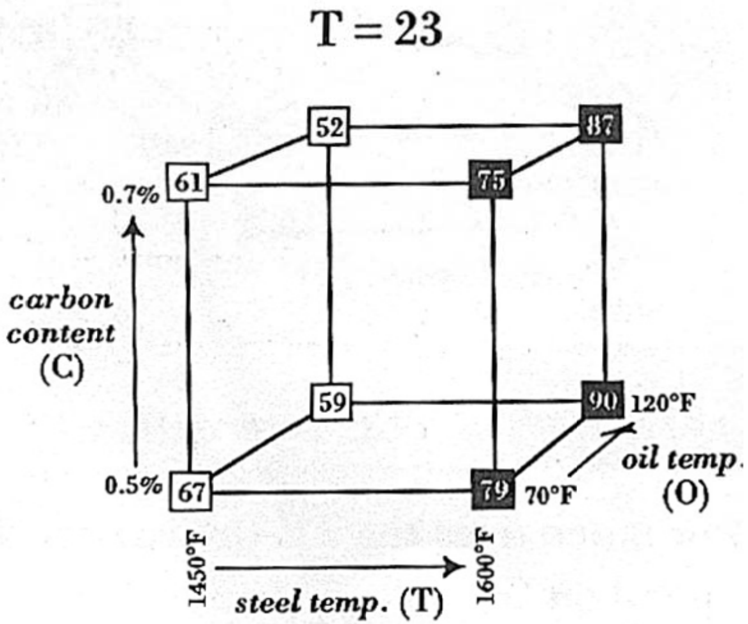
Summary

Run	Temp	Carbon	Oil	TC	TO	CO	TCO	Data
(1)	-	-	-	+	+	+	-	67%
a	+	-	-	-	-	+	+	79%
b	-	+	-	-	+	-	+	61%
ab	+	+	-	+	-	-	-	75%
c	-	-	+	+	-	-	+	59%
ac	+	-	+	-	+	-	-	90%
bc	-	+	+	-	-	+	-	52%
abc	+	+	+	+	+	+	+	87%
effect	23%	-5%	1.50%	1.50%	10%	0%	0.50%	

What did we learn about the hardening process from this experiment?

Effects	
T	23%
C	–5%
O	1.5%
T × C	1.5%
T × O	10%
C × O	0%
T × C × O	0.5%





Advantages of Factorial Designs

- Fewer runs than with one-factor-at-a-time experiments.
- Increased precision.
- Possibility of estimating interaction effects.
- Wider inductive basis.

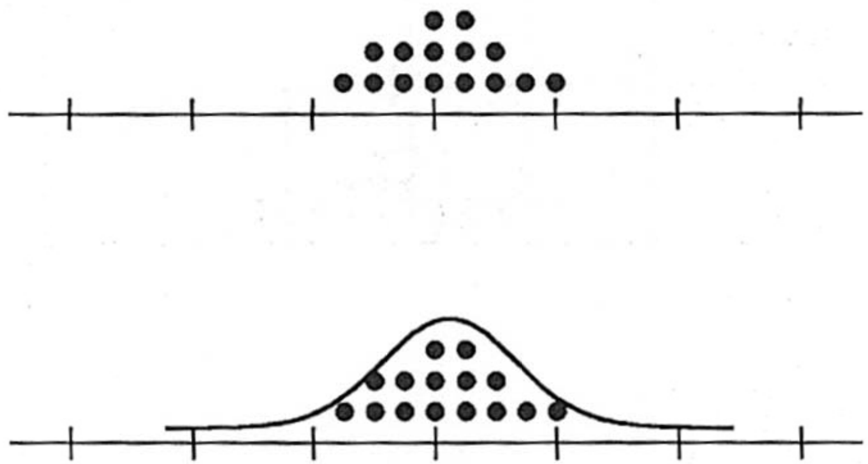
Summary of Full 2^k Design

- Two-level factorial experiments provide an important tool for engineers for product and process improvement.
- Factorial experiments are economical as compared with one-factor-at-a-time experiments since they reduce the experimental effort.
- Two-level factorials provide estimates of interaction effects.
- Experimental results using factorials have a wider inductive basis than one-factor-at-a-time experiments.

常態繪圖 (Normal Plots)

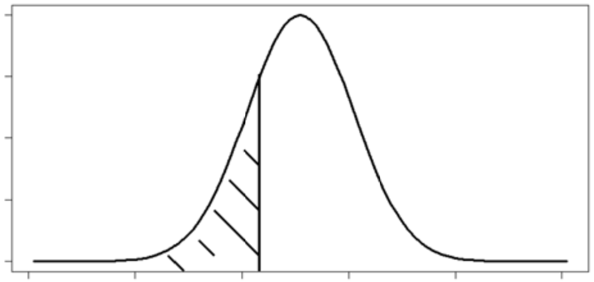
- Normal plots are simple to make and easy to interpret.

Fifteen normally distributed observations.

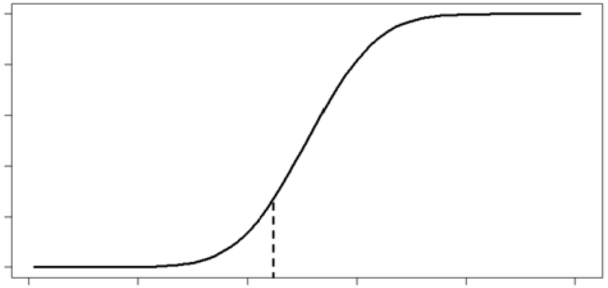


Concept of normal plots

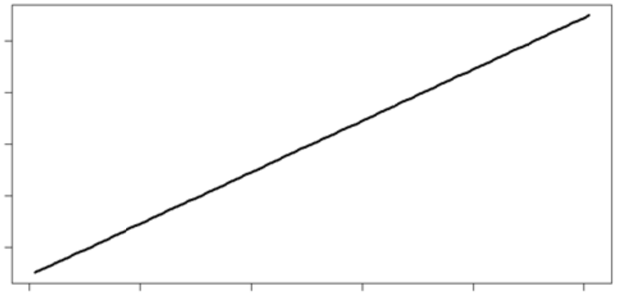
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



$$F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$



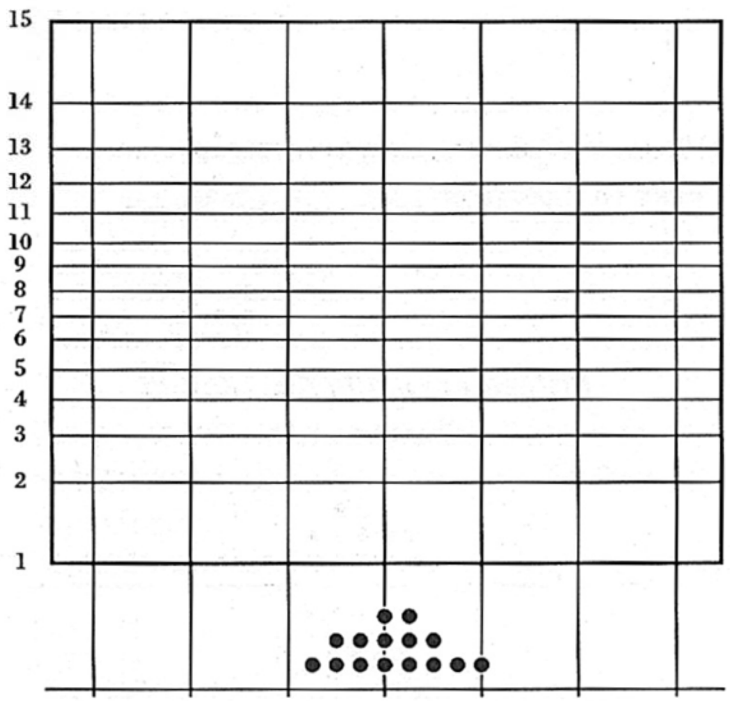
$$\Phi^{-1}\left(F(x)\right) = \frac{x-\mu}{\sigma}$$

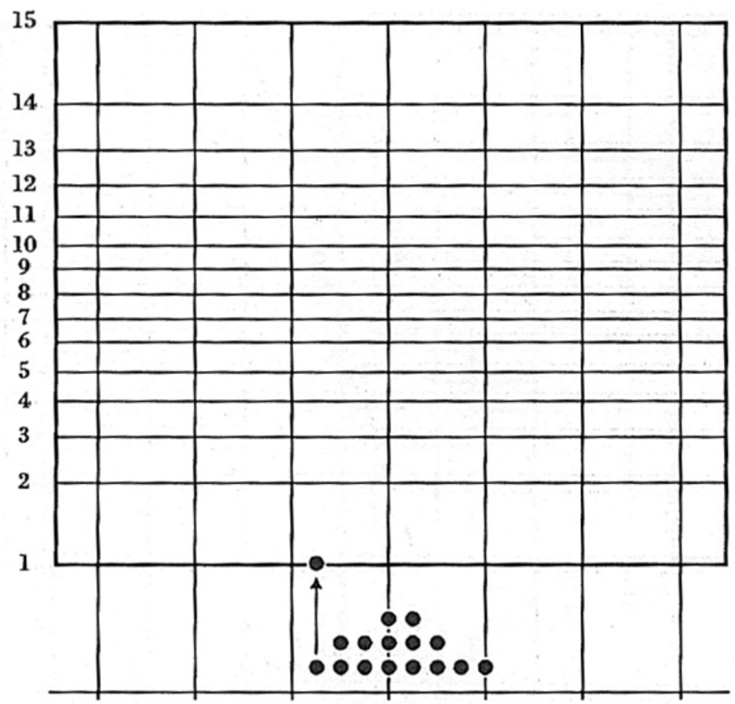


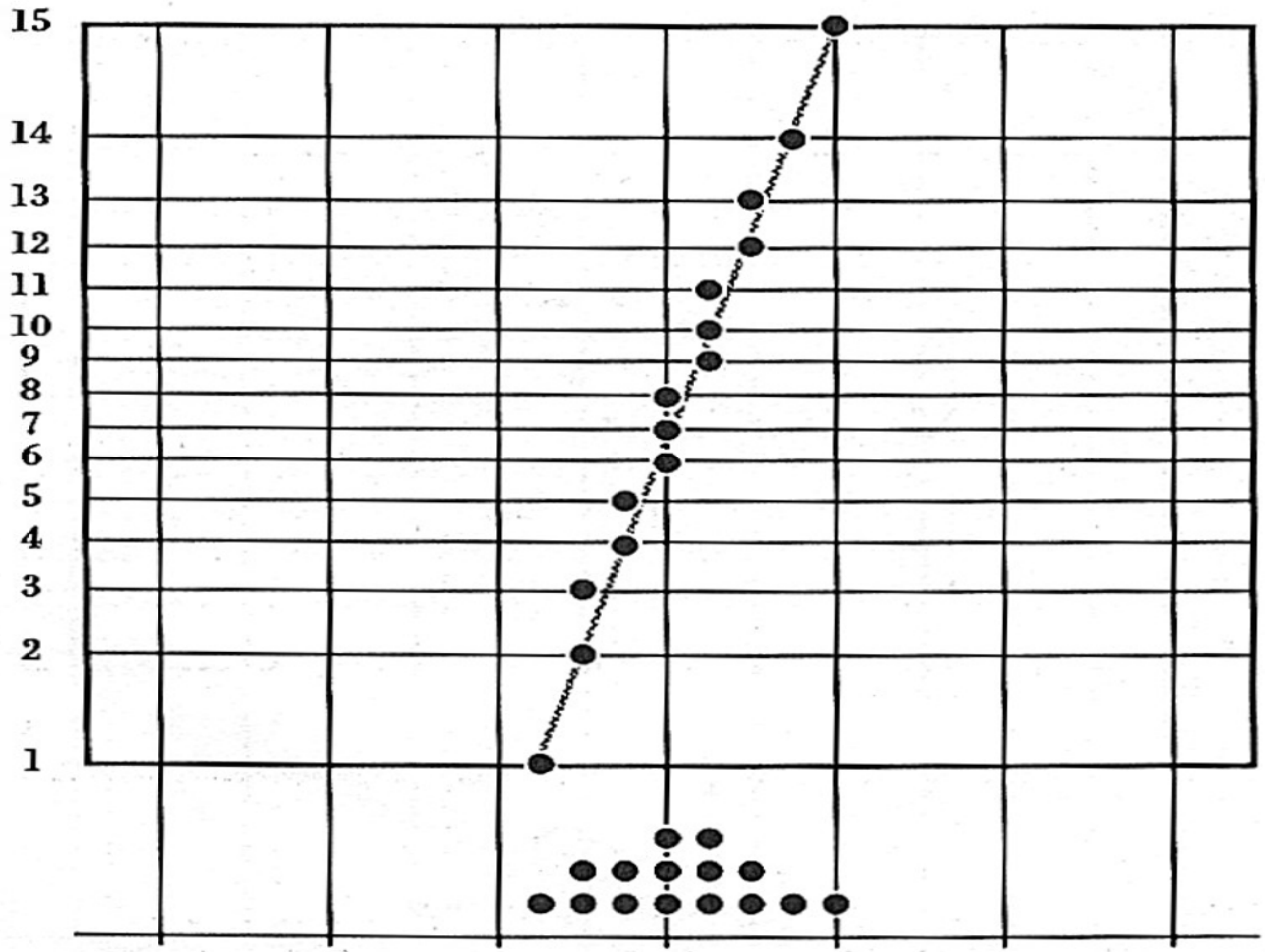
$$\hat{F}(x_{(j)}) = \frac{j}{n}, \forall j = 1, 2, \ldots, n,$$

where $x_{(1)} \le x_{(2)} \le \cdots \le x_{(n)}$

Normal Probability Paper



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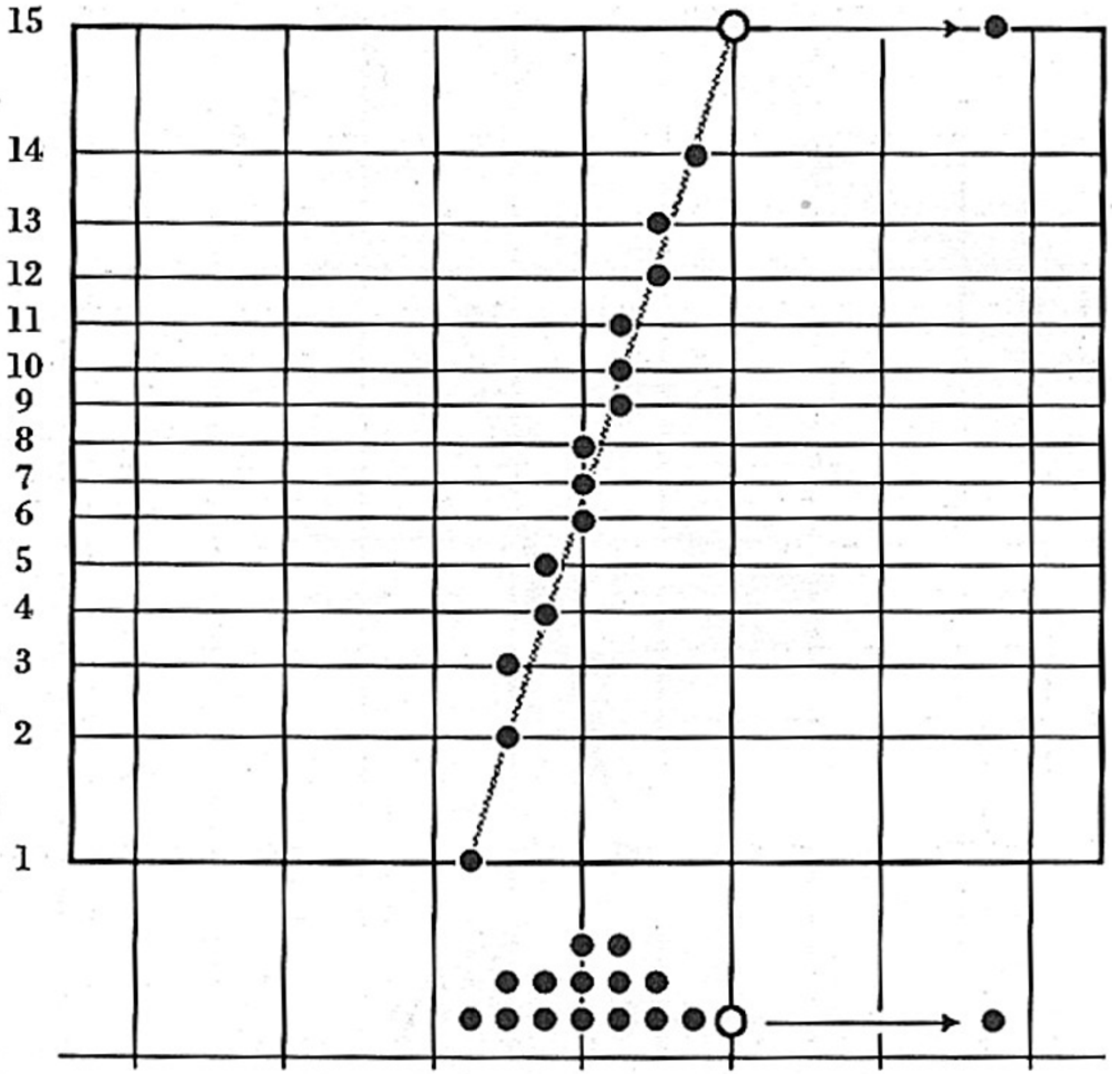
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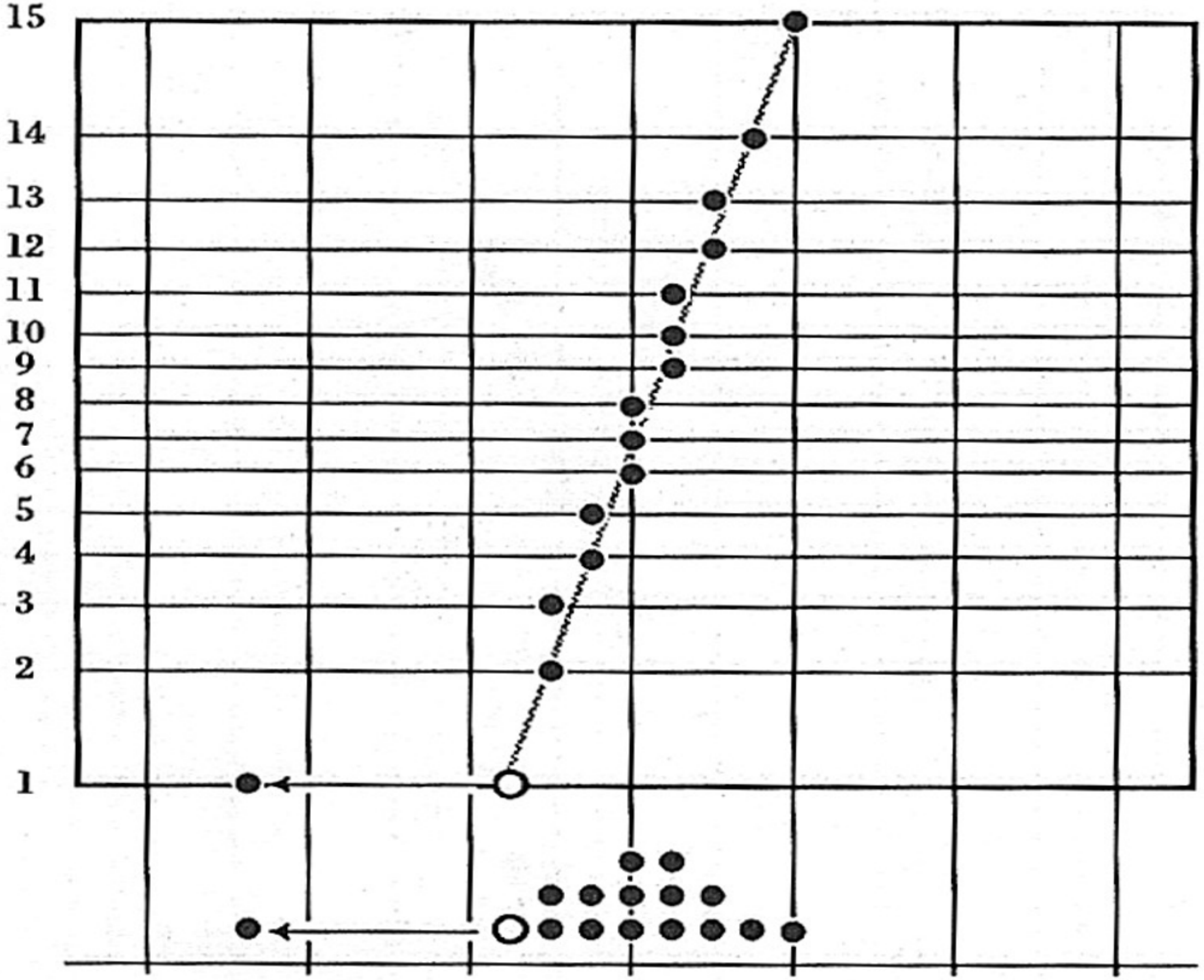
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實例二: Chemical Process

	factor	−	+
1	catalyst charge (lb)	10	15
2	temperature (°C)	220	240
3	pressure (psi)	50	80
4	concentration (%)	10	12

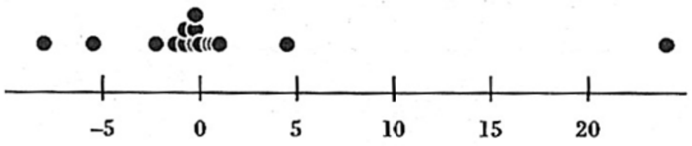
$$2^4 = 16 \quad runs$$

1	2	3	4	*conversion* (%)
−	−	−	−	71
+	−	−	−	61
−	+	−	−	90
+	+	−	−	82
−	−	+	−	68
+	−	+	−	61
−	+	+	−	87
+	+	+	−	80
−	−	−	+	61
+	−	−	+	50
−	+	−	+	89
+	+	−	+	83
−	−	+	+	59
+	−	+	+	51
−	+	+	+	85
+	+	+	+	78

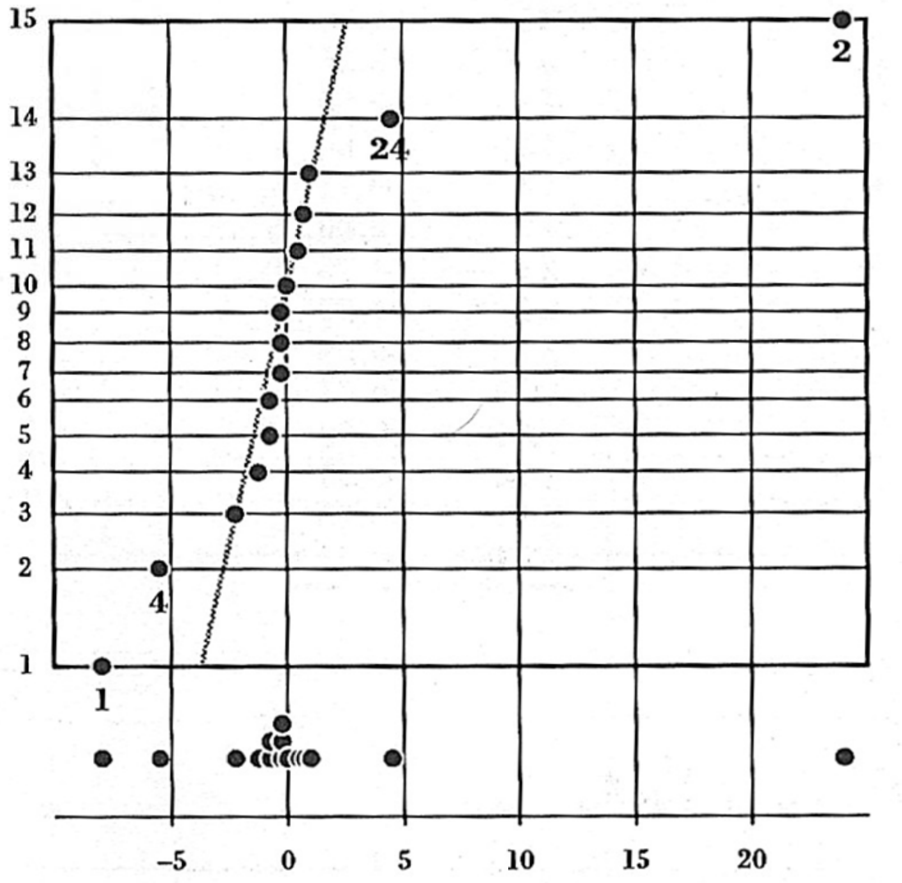
1	2	3	4	12	13	14	23	24	34	123	124	134	234	1234	% con-version
−	−	−	−	+	+	+	+	+	+	−	−	−	−	+	71
+	−	−	−	−	−	−	+	+	+	+	+	+	−	−	61
−	+	−	−	−	+	+	−	−	+	+	+	−	+	−	90
+	+	−	−	+	−	−	−	−	+	−	−	+	+	+	82
−	−	+	−	+	−	+	−	+	−	+	−	+	+	−	68
+	−	+	−	−	+	−	−	+	−	−	+	−	+	+	61
−	+	+	−	−	−	+	+	−	−	−	+	+	−	+	87
+	+	+	−	+	+	−	+	−	−	+	−	−	−	−	80
−	−	−	+	+	+	−	+	−	−	−	+	+	+	−	61
+	−	−	+	−	−	+	+	−	−	+	−	−	+	+	50
−	+	−	+	−	+	−	−	+	−	+	−	+	−	+	89
+	+	−	+	+	−	+	−	+	−	−	+	−	−	−	83
−	−	+	+	+	−	−	−	−	+	+	+	−	−	+	59
+	−	+	+	−	+	+	−	−	+	−	−	+	−	−	51
−	+	+	+	−	−	−	+	+	+	−	−	−	+	−	85
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	78
8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	

Estimated Effects

	Estimated Effects
1	–8.00
2	24.00
3	–2.25
4	–5.50
12	1.00
13	0.75
14	0.00
23	–1.25
24	4.50
34	–0.25
123	–0.75
124	0.50
134	–0.25
234	–0.75
1234	–0.25



Normal Plot of Estimated Effects



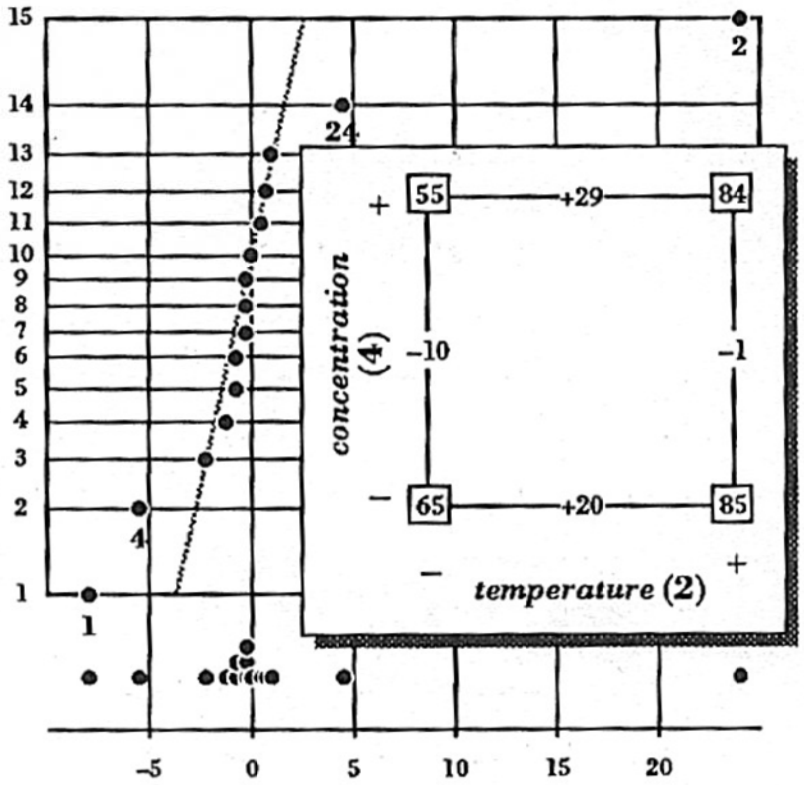
1 = catalyst charge

2 = temperature

3 = pressure

4 = concentration

Potential Factors

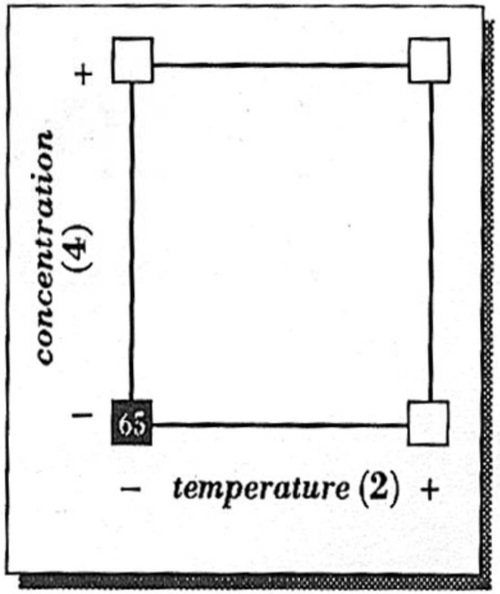


The numbers in the four corners of the square diagram are computed by projecting down the design into factors 2 and 4.

(See Below)

Projection to 2-dimensions

1	2	3	4	conversion (%)
−	−	−	−	71
+	−	−	−	61
−	+	−	−	90
+	+	−	−	82
−	−	+	−	68
+	−	+	−	61
−	+	+	−	87
+	+	+	−	80
−	−	−	+	61
+	−	−	+	50
−	+	−	+	89
+	+	−	+	83
−	−	+	+	59
+	−	+	+	51
−	+	+	+	85
+	+	+	+	78



$$\frac{71 + 61 + 68 + 61}{4} = 65$$

實例說明（2^8 design）

- Main Effects: C(8,1)=8
- Two-factor interactions: C(8,2)=28
- Three-factor interactions: C(8,3)=56
- Four-factor interactions: C(8,4)=70
- Five-factor interactions: C(8,5)=56
- …

Can we conduct a subset of the design if we are only interesting in Main effects and two-factor interaction effects?

建構出 2^{k-p} 的因子設計

- Write down a full factorial design with (k-p) factors; and set p generators in terms of these (k-p) factors.
- Example 2^{8-4} with 4 generators:

  E=BCD, F=ACD, G=ABC, H=ABD

  I=BCDE=ACDF=ABCG=ABDH

  =ABEF=…

常用部份因子設計表

DESIGN DIRECTORY														
		Number of Factors k												
# Runs	Block	3	4	5	6	7	8	9	10	11	12	13	14	15
8	1	2^3	2^{4-1}_{IV}	2^{5-2}_{III}	2^{6-3}_{III}	2^{7-4}_{III}								
8	2	2^3_{IV}	2^{4-1}_{III}	2^{5-2}_{III}	2^{6-3}_{III}									
8	4	2^3_{III}	2^{4-1}_{III}											
16	1		2^4	2^{5-1}_{V}	2^{6-2}_{IV}	2^{7-3}_{IV}	2^{8-4}_{IV}	2^{9-5}_{IV}	2^{10-6}_{III}	2^{11-7}_{III}	2^{12-8}_{III}	2^{13-9}_{III}	2^{14-10}_{III}	2^{15-11}_{III}
16	2		2^4_{V}	2^{5-1}_{IV}	2^{6-2}_{IV}	2^{7-3}_{IV}	2^{8-4}_{III}	2^{9-5}_{III}	2^{10-6}_{III}	2^{11-7}_{III}	2^{12-8}_{III}	2^{13-9}_{III}	2^{14-10}_{III}	
16	4		2^4_{III}	2^{5-1}_{III}	2^{6-2}_{III}	2^{7-3}_{III}	2^{8-4}_{III}	2^{9-5}_{III}	2^{10-6}_{III}	2^{11-7}_{III}	2^{12-8}_{III}			
16	8		2^4_{III}	2^{5-1}_{III}	2^{6-2}_{III}	2^{7-3}_{III}	2^{8-4}_{III}							

Four Categories of DOE

- Treatment Comparisons
  - (ANOVA Model)
- Variable Screening
  - (2-level and 3-level factorial design)
- System Robustness
  - (Robust parameter design)
- Response Surface Methodology
  - (Process optimization)

Constrained Optimization

- We want to find adjust factors so that the following equation can be held:

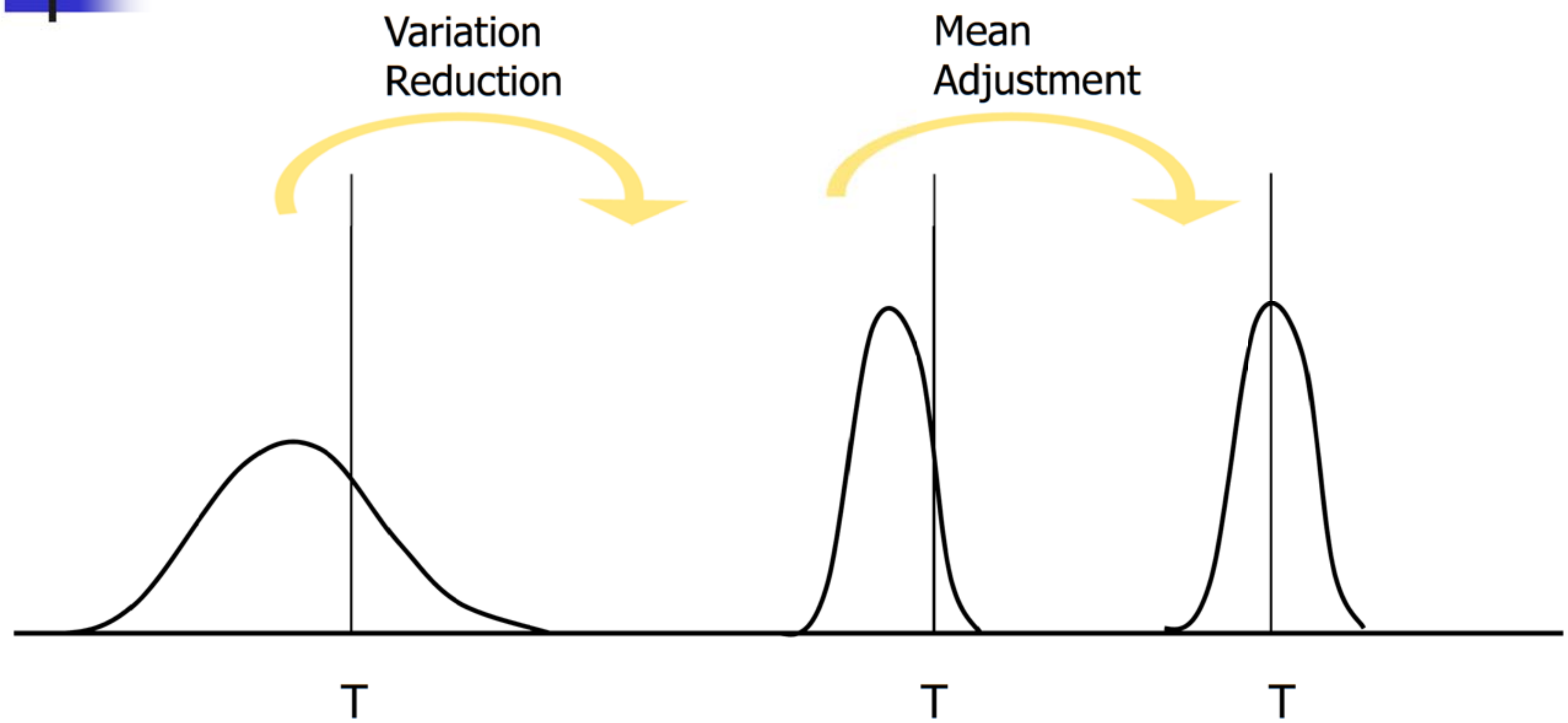
$$\begin{aligned} &\textit{Minimize } \sigma^2(Z) \\ &\textit{Subject to } \mu(\mathrm{Z}) = \tau \end{aligned}$$

where

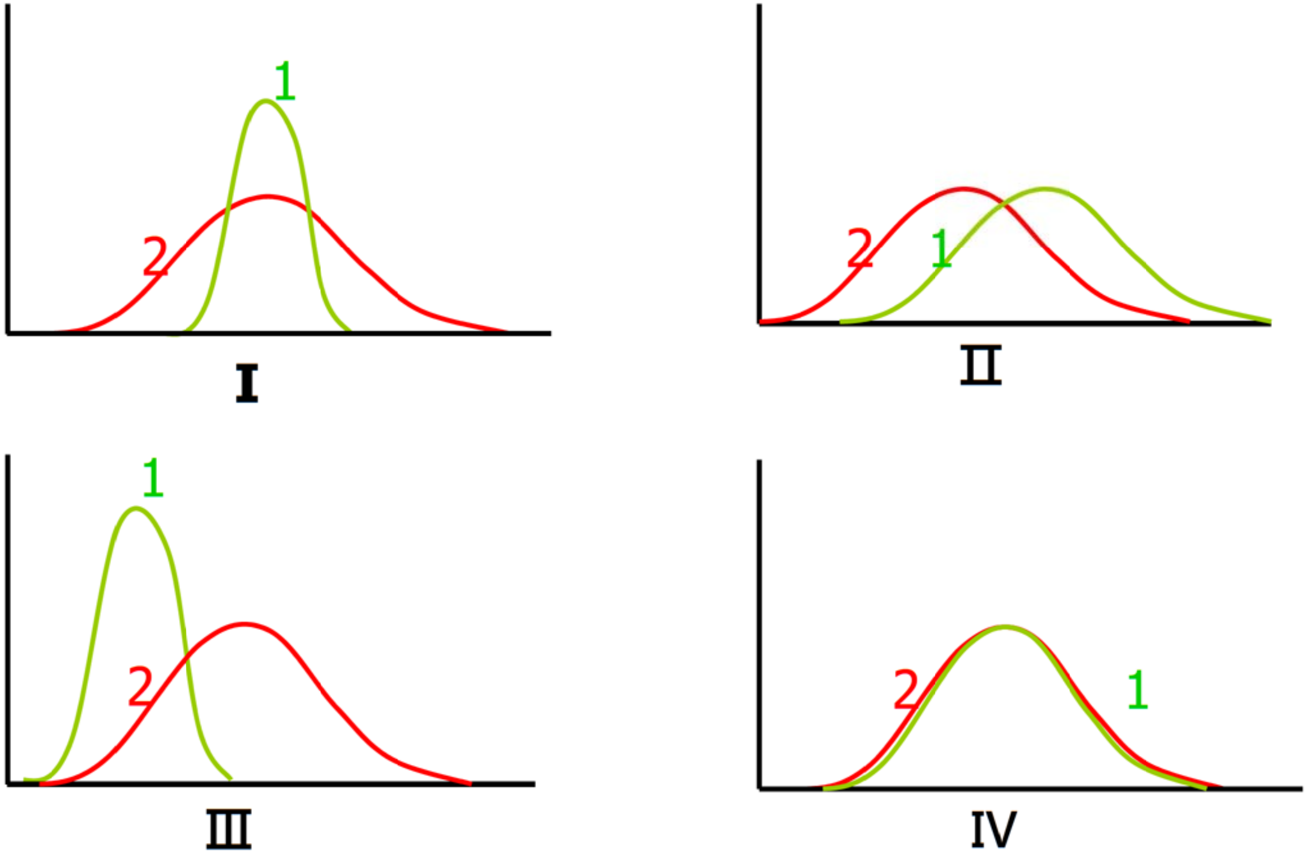
$$\mu(Z) = E\{Y(N;Z)\}$$

$$\sigma^2(Z) = Var\{Y(N;Z)\}$$

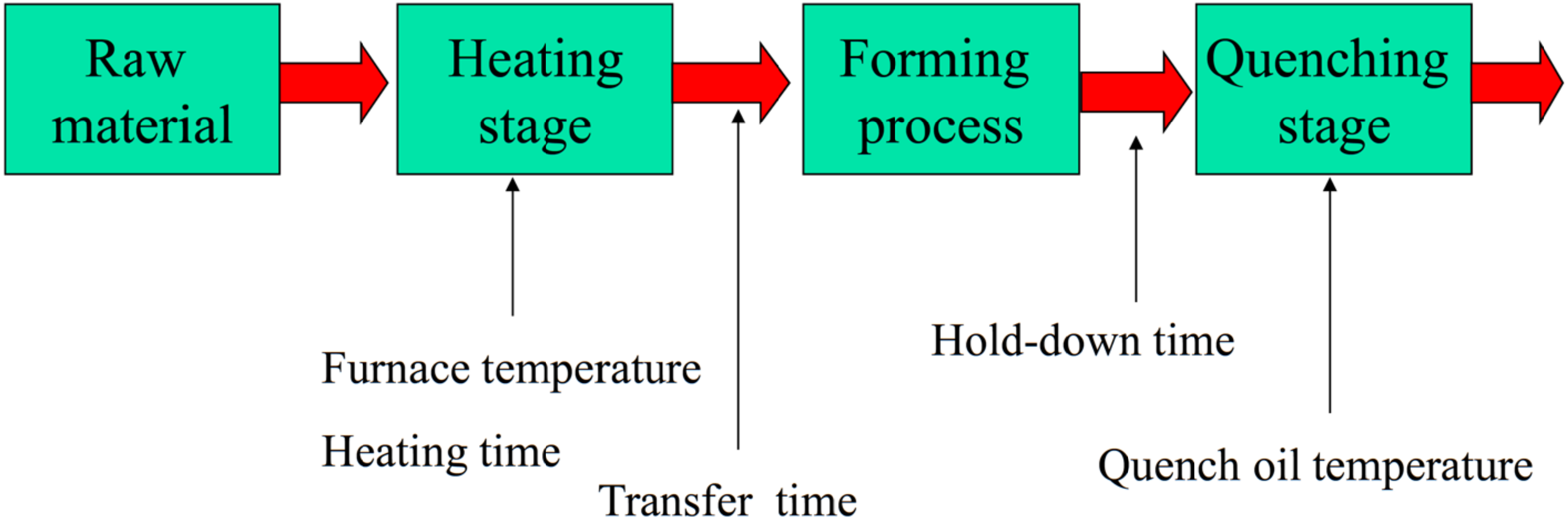
Robust parameter design



Four possible patterns



A Leaf Spring Experiment



Five key factors

- Quench oil temperature (Q)
- Furnace temperature (B)
- Heating time (C)
- Transfer time (D)
- Hold-down time (E)

Factor and level

	Level	
Factor	-	+
Q. Quench oil temperature (°F)	130-150	150-170
B. High heat temperature (°F)	1840	1880
C. Heating time (seconds)	23	25
D. Transfer time (seconds)	10	12
E. Hold down time (seconds)	2	3

Design matrix for FH Data

Q	B	C	D	E	Free Height			yi-bar	si-sq	ln si-sq
-1	-1	-1	-1	-1	7.56	7.62	7.44	7.5400	0.0084	-4.7795
-1	-1	-1	1	1	7.50	7.56	7.50	7.5200	0.0012	-6.7254
-1	-1	1	-1	1	7.94	8.00	7.88	7.9400	0.0036	-5.6268
-1	-1	1	1	-1	7.78	7.78	7.81	7.7900	0.0003	-8.1117
-1	1	-1	-1	1	7.56	7.81	7.69	7.6867	0.0156	-4.1583
-1	1	-1	1	-1	7.59	7.56	7.75	7.6333	0.0104	-4.5627
-1	1	1	-1	-1	7.69	8.09	8.06	7.9467	0.0496	-3.0031
-1	1	1	1	1	8.15	8.18	7.88	8.0700	0.0273	-3.6009
1	-1	-1	-1	-1	7.18	7.18	7.25	7.2033	0.0016	-6.4171
1	-1	-1	1	1	7.50	7.56	7.50	7.5200	0.0012	-6.7254
1	-1	1	-1	1	7.32	7.44	7.44	7.4000	0.0048	-5.3391
1	-1	1	1	-1	7.50	7.25	7.12	7.2900	0.0373	-3.2888
1	1	-1	-1	1	7.81	7.50	7.59	7.6333	0.0254	-3.6717
1	1	-1	1	-1	7.63	7.75	7.56	7.6467	0.0092	-4.6849
1	1	1	-1	-1	7.56	7.69	7.62	7.6233	0.0042	-5.4648
1	1	1	1	1	7.88	7.88	7.44	7.7333	0.0645	-2.7406

Find Optimal Settings

- The optimal settings can be solved by the following programming

$$\begin{aligned}&\text{minimize} \quad Var(Y)\\&\text{subject to} \quad E(Y) = target = 8.0\end{aligned}$$

Defining relation

- 2_{IV}^{5-1} design with I=BCDE

$$I = BCDE$$

$$\begin{aligned} B &= CDE, & BC &= DE, \\ C &= BDE, & BD &= CE, \\ D &= BCE, & BE &= CD, \\ E &= BCD, & BQ &= CDEQ, \\ Q &= BCDEQ, & CQ &= BDEQ, \\ & & DQ &= BCEQ, \\ & & EQ &= BCDQ, \end{aligned}$$

$$BCQ = DEQ,\ BDQ = CEQ,\ BEQ = CDQ$$

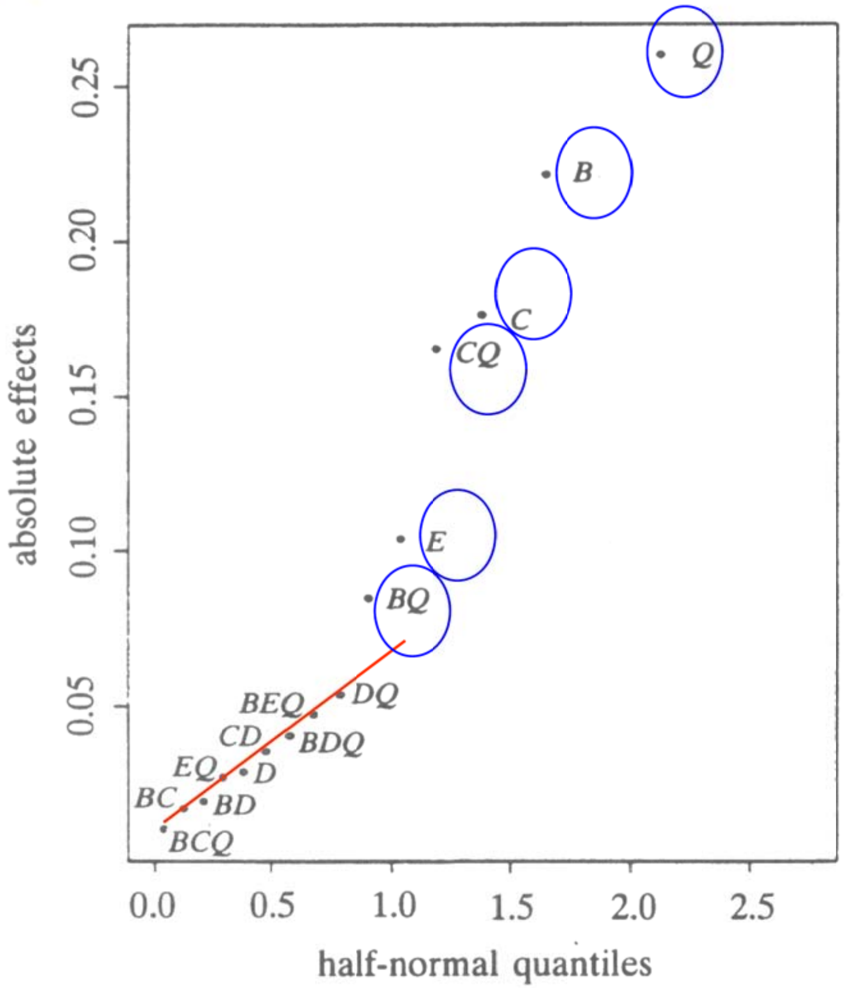
Factorial effects

Effect	y-bar	ln s-sq
B	0.221	1.891
C	0.176	0.569
D	0.029	-0.247
E	0.104	0.216
Q	-0.260	0.280
BQ	0.085	-0.589
CQ	-0.165	0.598
DQ	0.054	1.111
EQ	0.027	0.129
BC	0.017	-0.002
BD	0.020	0.425
CD	-0.035	0.670
BCQ	0.010	-1.089
BDQ	-0.040	-0.432
BEQ	-0.047	0.854

Analysis of location effects

Half-normal probability plot

$$\hat{y} = 7.636 + 0.1106x_B + 0.0519x_E$$
$$+ 0.0881x_C - 0.1298x_Q$$
$$+ 0.0423x_B x_Q - 0.0827x_C x_Q$$

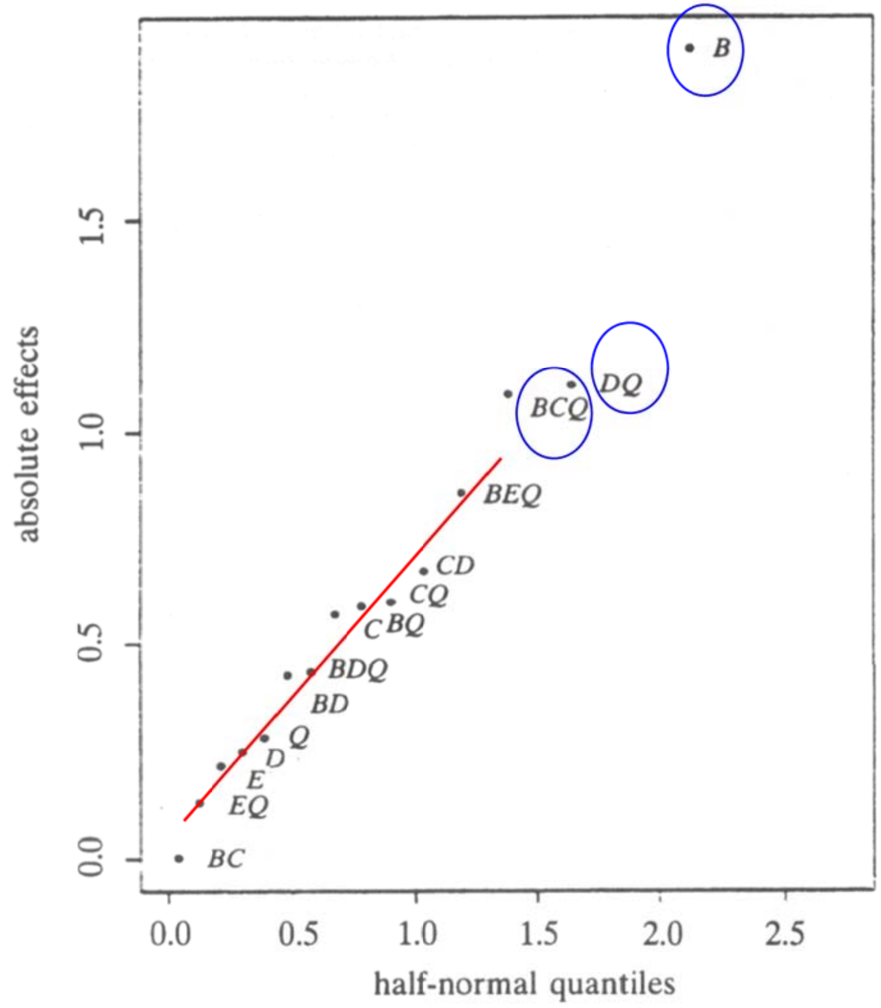


Analysis of dispersion effects

Half-normal probability plot

(α=0.25)

$$\hat{z} = \ln \hat{\sigma}^2$$
$$= -4.9313 + 0.9455 x_B$$
$$+ 0.5556 x_D x_Q - 0.5445 x_B x_C x_Q$$

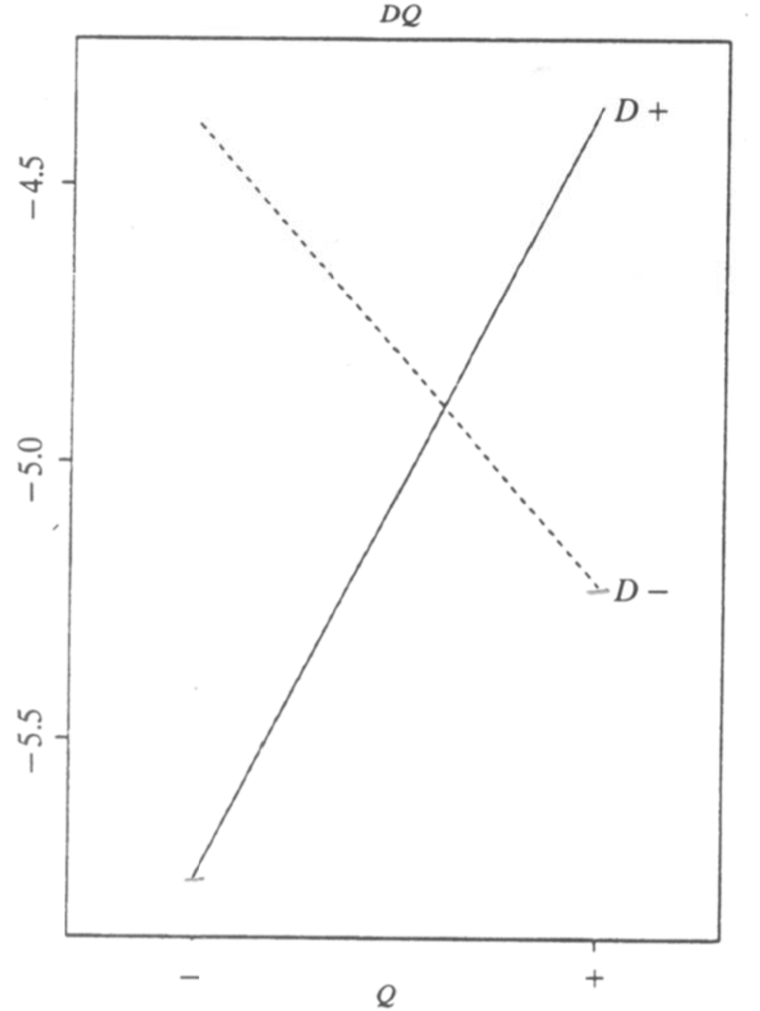


Step I: Variation reduction

$$\hat{z} = \ln \hat{\sigma}^2$$
$$= -4.9313 + 0.9455 x_B + 0.5556 x_D x_Q - 0.5445 x_B x_C x_Q$$

- Set $x_B = -1; x_D x_Q = -1; x_B x_C x_Q = 1$
- Then the process variation will be minimized.
- ($x_B = -1; x_C = 1; x_D = 1; x_Q = -1$) is the optimal settings for minimizing the variation

D*Q Interaction plots for dispersion



Step II: Adjusted the mean

- Under $BCDQ = (-,+,+,-)$, the equation in location model reduces to

$$\hat{y} = 7.6360 + 0.1106(-1) + 0.0519x_E + 0.0881(+1)$$
$$- 0.1298(-1) + 0.0423(-1)(-1)\text{-}0.0827(+1)(-1)$$
$$= 7.8683 + 0.0519x_E$$

Find Optimal setting (cont.)

- By solving $\hat{y} = 8.0$, x_E should be set as

$$\frac{8.0 - 7.8683}{0.0519} = 2.54$$

- The un-coded value for factor E is

$$2.5 + 2.54(0.5) = 3.77$$

Conclusion

- Optimal setting is

$$BCDQ = (-,+,+,-)$$

and E=2.54

- The un-coded value of these factors are:

  Furnace temperature = 1840 度F

  Heating time = 25 秒

  Transfer time = 12 秒

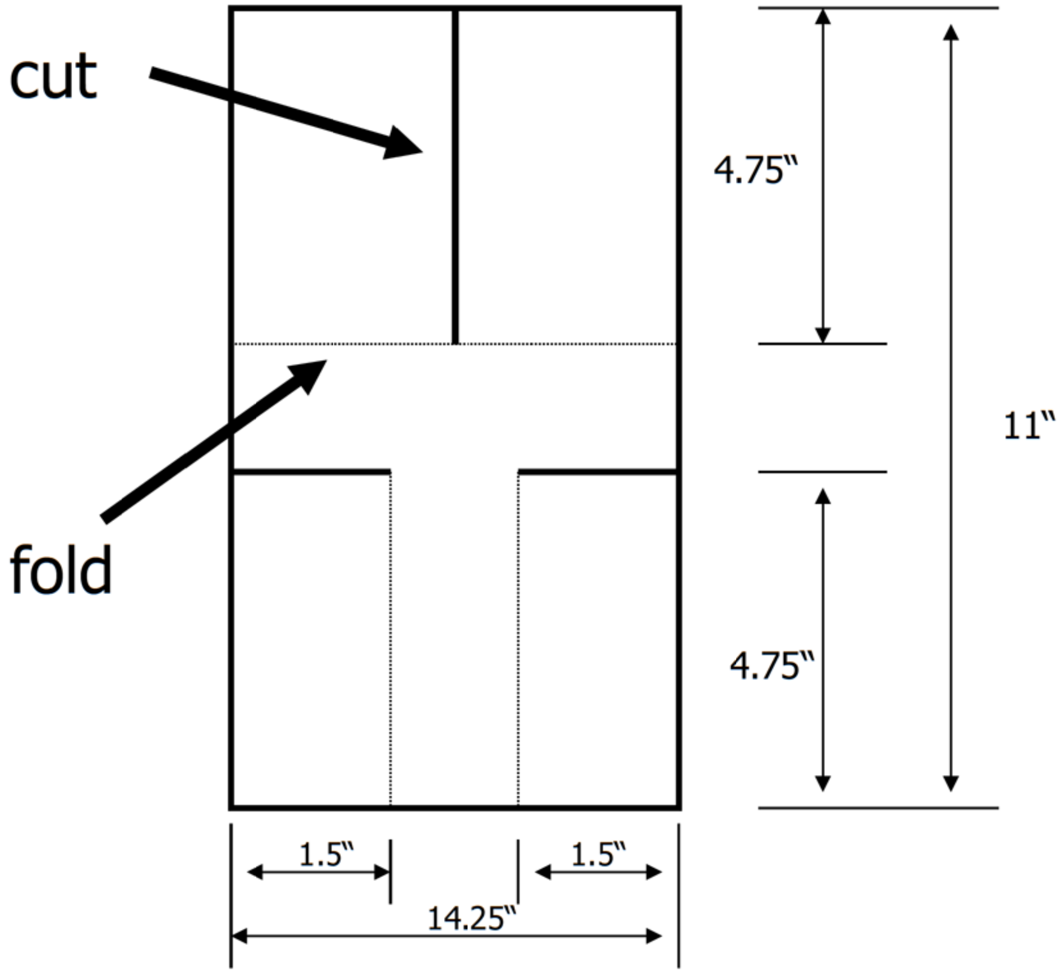
  Hold-down time = 3.77 秒

  Quench oil temperature = 130-150 度F

Appendix

Real Example of Fractional Factorial Design

A Helicopter Experiment



Helicopter 個案研究

- A long brainstorming session ensues, during which are proposed a great many helicopter design factors which might affect the flight time.
- The list is finally narrowed down to eight factors, which will be studied in a designed experiment.

Helicopter 實驗之重要因子

- The team's suggested factors

factor		-	+
Paper Type	(P)	Regular	Bond
Wing Length	(W)	3.00"	4.75"
Body Length	(L)	3.00"	4.75"
Body Width	(B)	2.00"	1.25"
Paper Clip	(C)	No	Yes
Fold	(F)	No	Yes
Taped Body	(T)	No	Yes
Taped Wing	(M)	No	Yes

Eight factors in sixteen runs: 2_{IV}^{8-4}

Random order	Standard order	P	W	L	B	C	F	G	H	response
7	1	-	-	-	-	-	-	-	-	2.5
13	2	+	-	-	-	+	-	+	+	2.9
4	3	-	+	-	-	+	+	-	+	3.5
9	4	+	+	-	-	-	+	+	-	2.7
1	5	-	-	+	-	+	+	+	-	2
12	6	+	-	+	-	-	+	-	+	2.3
15	7	-	+	+	-	-	-	+	+	2.9
3	8	+	+	+	-	+	-	-	-	3
6	9	-	-	-	+	-	+	+	+	2.4
16	10	+	-	-	+	+	+	-	-	2.6
14	11	-	+	-	+	+	-	+	-	3.2
5	12	+	+	-	+	-	-	-	+	3.7
11	13	-	-	+	+	+	-	-	+	1.9
10	14	+	-	+	+	-	-	+	-	2.2
2	15	-	+	+	+	-	+	-	-	3
8	16	+	+	+	+	+	+	+	+	3

Alais Structure of 2^{8-4} Design

```
Alias Information for Terms in the Model.
Totally confounded terms were removed from the analysis.

P + W*L*T + W*C*F + W*B*M + L*B*F + L*C*M + B*T*C + T*F*M
W + P*L*T + P*C*F + P*B*M + L*B*C + L*F*M + B*T*F + T*C*M
L + P*W*T + P*B*F + P*C*M + W*B*C + W*F*M + B*T*M + T*C*F
B + P*T*C + P*L*F + P*W*M + W*L*C + W*T*F + L*T*M + C*F*M
T + P*W*L + P*B*C + P*F*M + W*B*F + W*C*M + L*C*F + L*B*M
C + P*B*T + P*W*F + P*L*M + W*L*B + W*T*M + L*T*F + B*F*M
F + P*L*B + P*W*C + P*T*M + W*B*T + W*L*M + L*T*C + B*C*M
M + P*W*B + P*L*C + P*T*F + W*T*C + W*L*F + L*B*T + B*C*F
P*W + L*T + B*M + C*F
P*L + W*T + B*F + C*M
P*B + W*M + L*F + T*C
P*T + W*L + B*C + F*M
P*C + W*F + L*M + B*T
P*F + W*C + L*B + T*M
P*M + W*B + L*C + T*F
```

The calculated effects

Paper Type	(P)	0.125
Wing Length	(W)	0.775
Body Length	(L)	-0.4
Body Width	(B)	0.025
Paper Clip	(C)	0.05
Fold	(F)	-0.1
Taped Body	(T)	-0.15
Taped Wing	(M)	0.175

interacing factors	effect
PW	-0.175
PL	0.05
PB	0.125
PC	0.1
PF	-0.2
PT	-0.05
PM	0.175

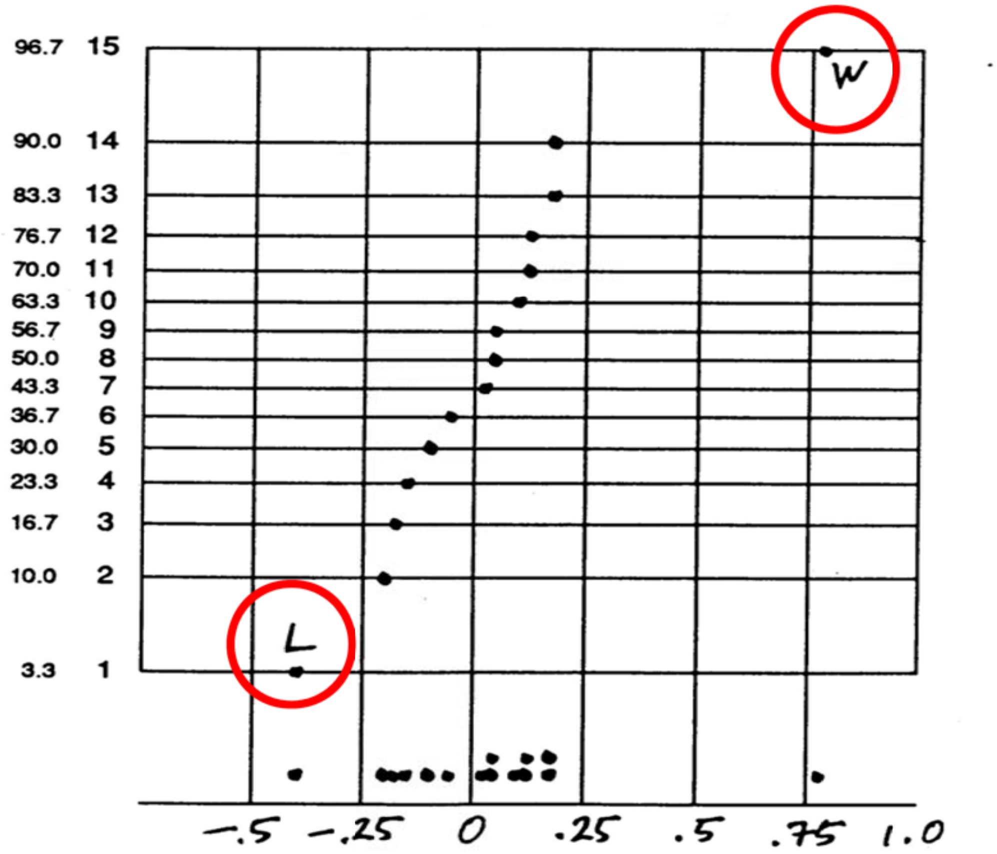
Example 3 (cont.)

Factorial Fit: response versus P, W, L, B, T, C, F, M

Estimated Effects and Coefficients for response (coded units)

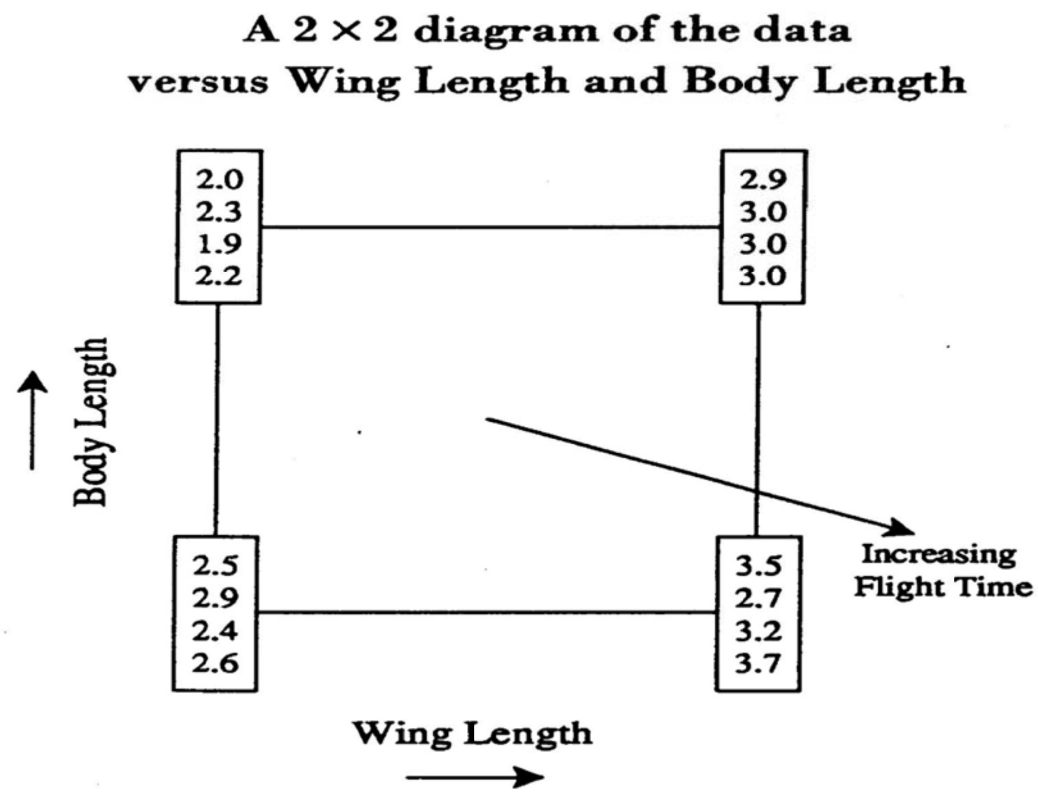
Term	Effect	Coef
Constant		2.7375
P	0.1250	0.0625
W	0.7750	0.3875
L	-0.4000	-0.2000
B	0.0250	0.0125
T	0.0500	0.0250
C	-0.1000	-0.0500
F	-0.1500	-0.0750
M	0.1750	0.0875
P*W	-0.1750	-0.0875
P*L	0.0500	0.0250
P*B	0.1250	0.0625
P*T	0.1000	0.0500
P*C	-0.2000	-0.1000
P*F	-0.0500	-0.0250
P*M	0.1750	0.0875

常態機率繪圖



96.7 15
90.0 14
83.3 13
76.7 12
70.0 11
63.3 10
56.7 9
50.0 8
43.3 7
36.7 6
30.0 5
23.3 4
16.7 3
10.0 2
3.3 1
W
L
-.5 -.25 0 .25 .5 .75 1.0

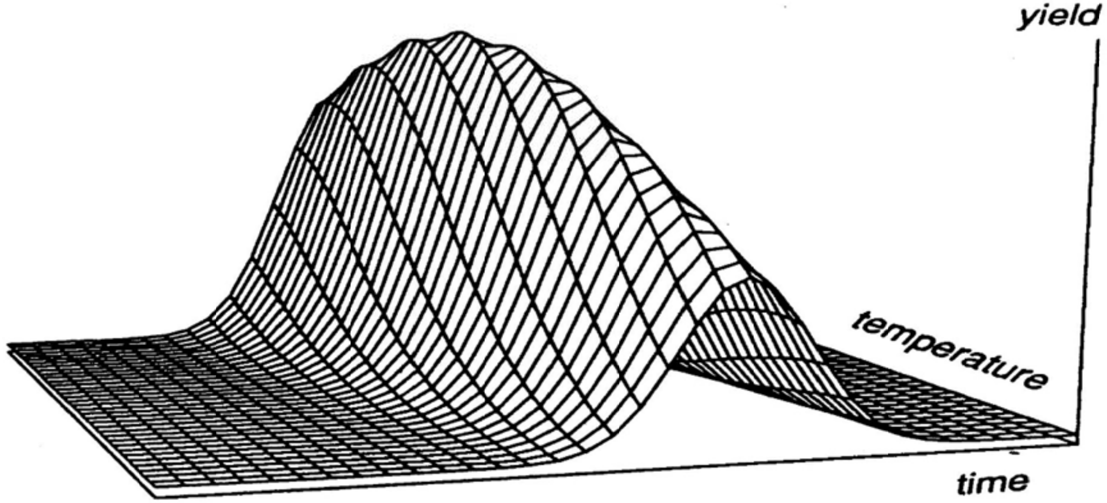
投影到二度空間之結果


A 2 × 2 diagram of the data
versus Wing Length and Body Length
2.0
2.3
1.9
2.2
2.9
3.0
3.0
3.0
Body Length
2.5
2.9
2.4
2.6
3.5
2.7
3.2
3.7
Increasing
Flight Time
Wing Length

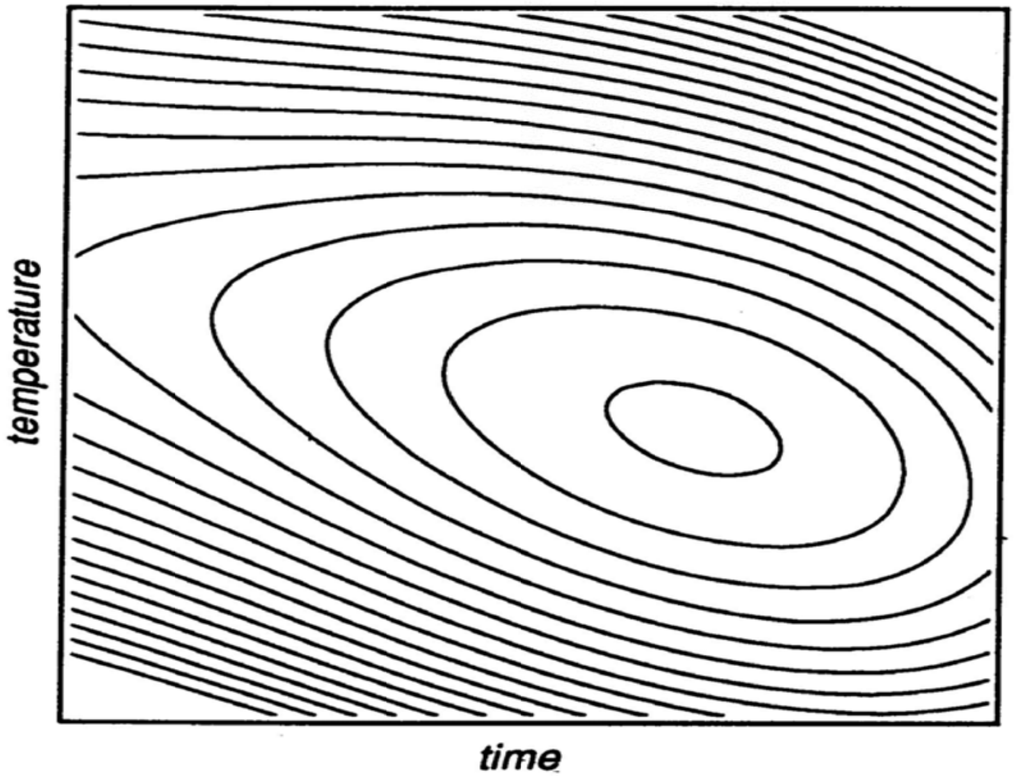
Process Optimization-RSM

- Screen out trivial factors
- Compute a path of steepest ascent
- Conduct experimental runs along the path, until a near optimal point is found
- Use a Second-order Model to obtain the optimal solution.

A plot of Y = F (temp, time)



Contour plots



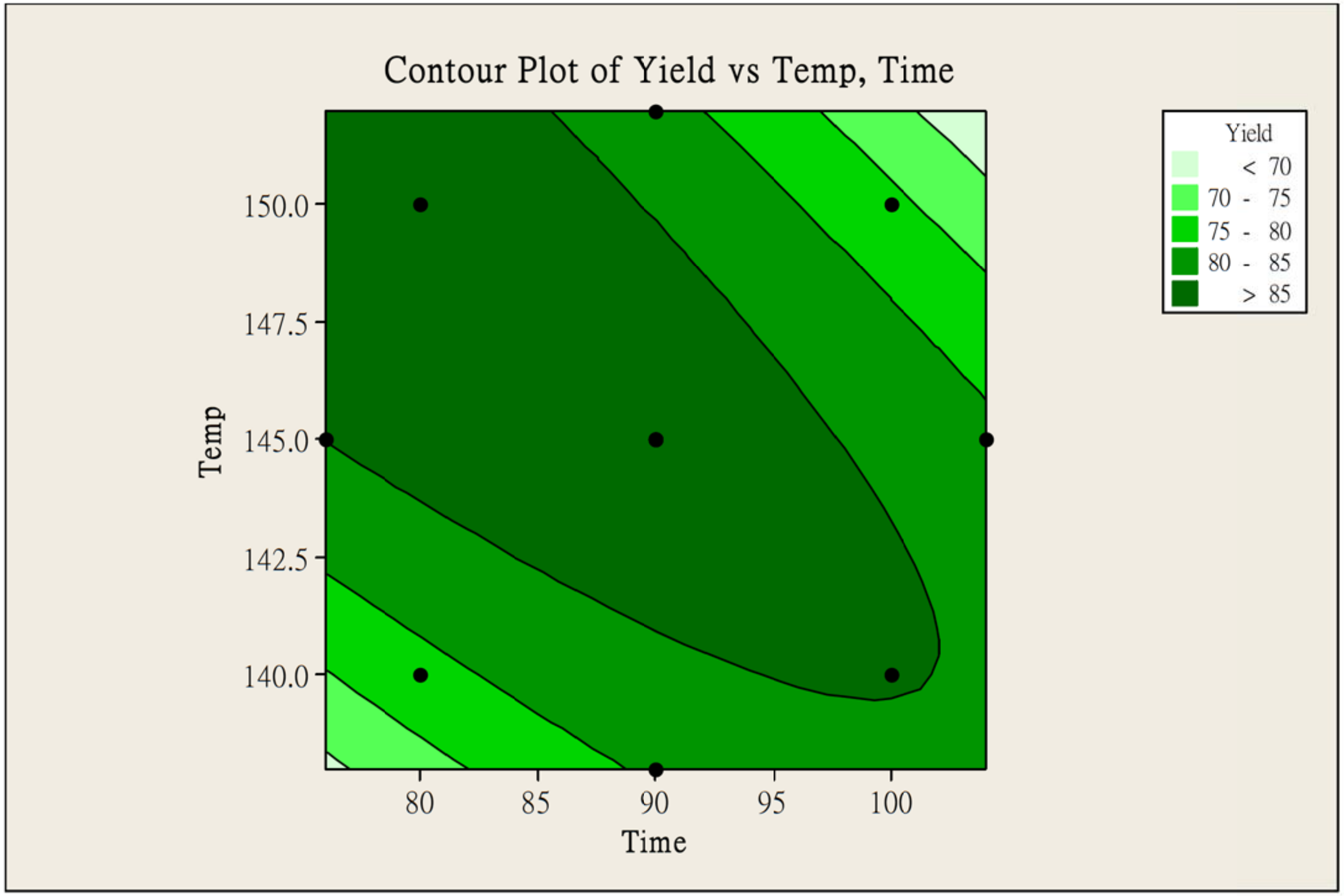
Regression model

- Second-order fitting

$$\hat{y} = 87.4 - 1.394x_1 + 0.369x_2 - 2.147x_1^2 - 4.875x_1x_2 - 3.116x_2^2$$

$$= 87.4 + (-1.394, 0.369)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} x_1 & x_2 \end{pmatrix}\begin{bmatrix} -2.147 & -2.4375 \\ -2.4375 & -3.116 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Example



Contour Plot of Yield vs Temp, Time
Temp
150.0
147.5
145.0
142.5
140.0
80
85
90
95
100
Time
Yield
< 70
70 - 75
75 - 80
80 - 85
> 85