Mathematics, Tamkang University

Recent developments and applications of inference and design in degradation tests

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Outline

- Background of this study
- Optimal burn-in policy
- Optimal sample size allocation for ADT
- Optimal SSADT test plan
- Optimal design of accelerated-stress acceptance test
- Optimal design of accelerated destructive degradation test
- References

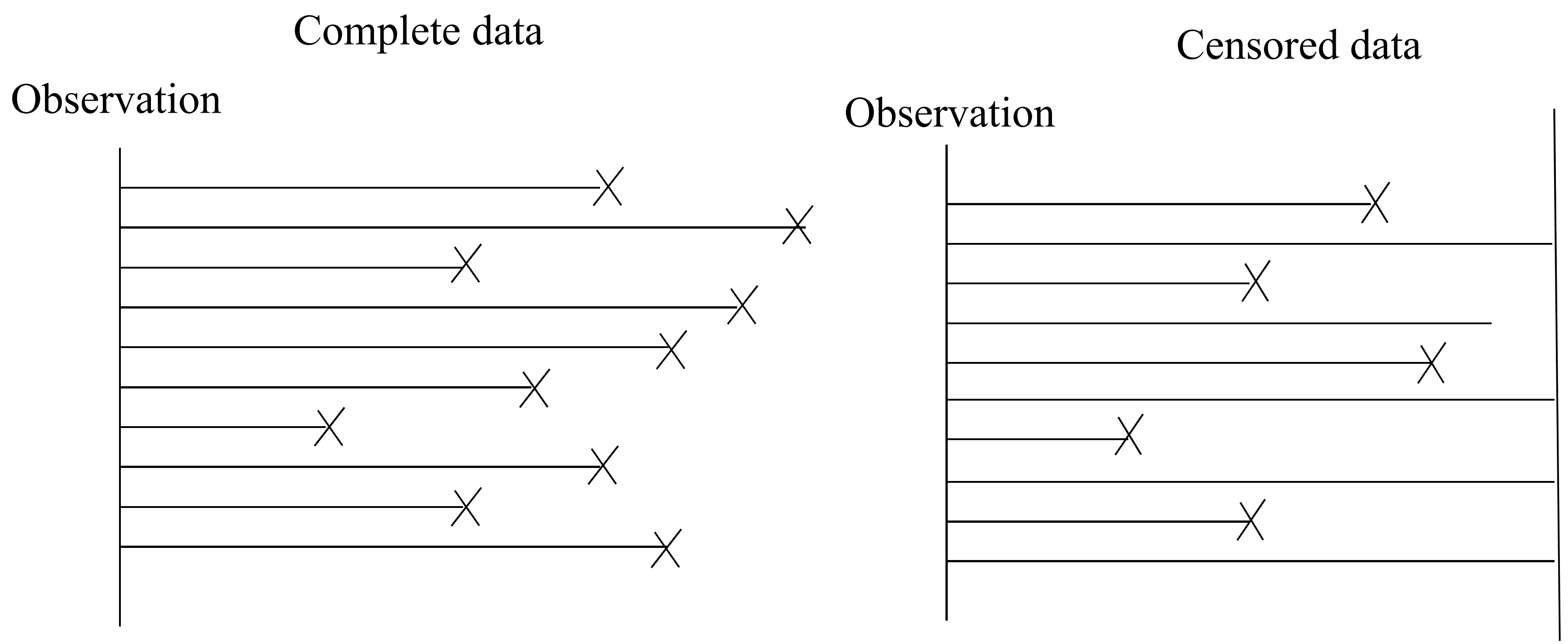
Background of this study

- **Quality**— "Fitness for use"
  Reliability: "Long-term quality"

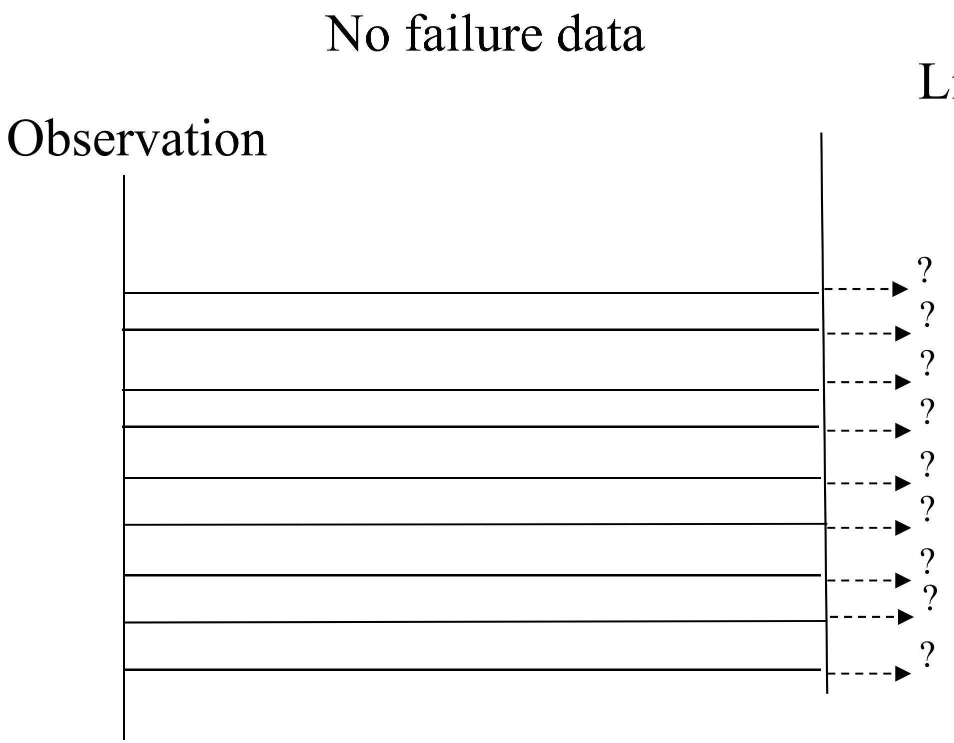
- **Why is reliability so important?**
  - Reputation
  - Warranty costs
  - Competitive advantage
  - Customer satisfaction
  - Repeat business

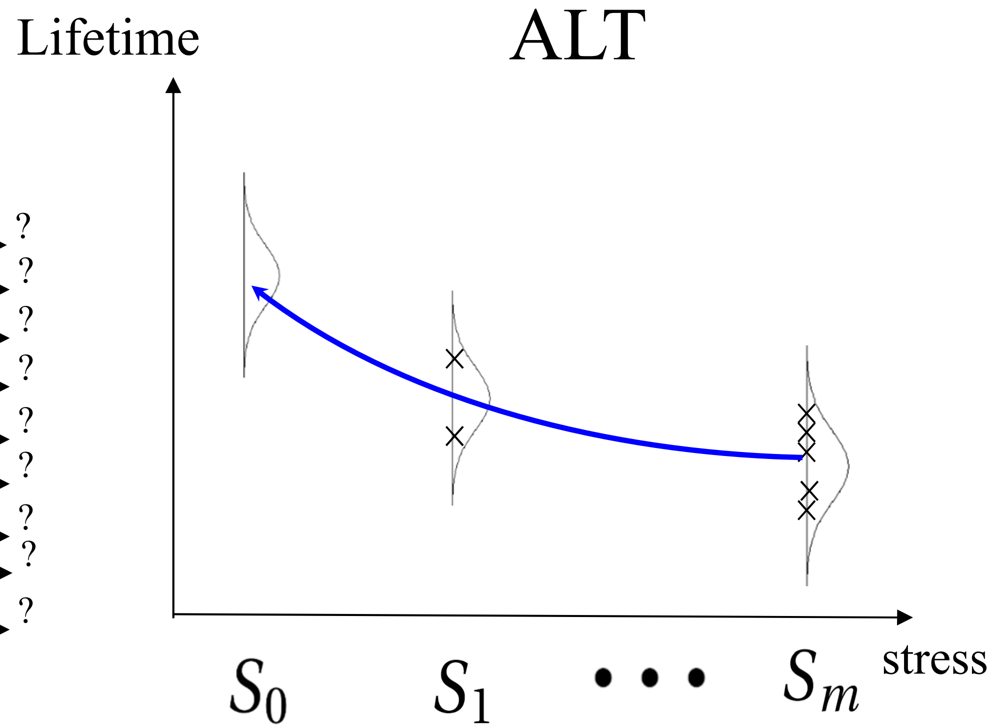
Lifetime model analysis

- This decision problem strongly depends on the pattern of reliability data

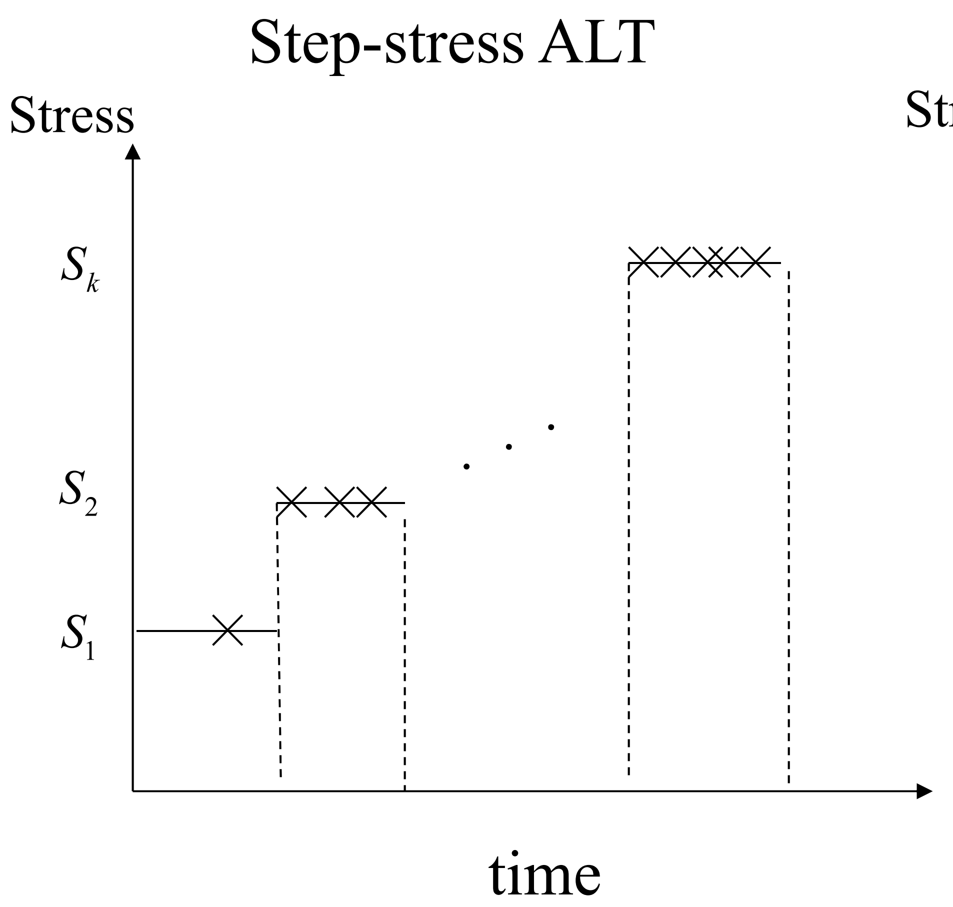


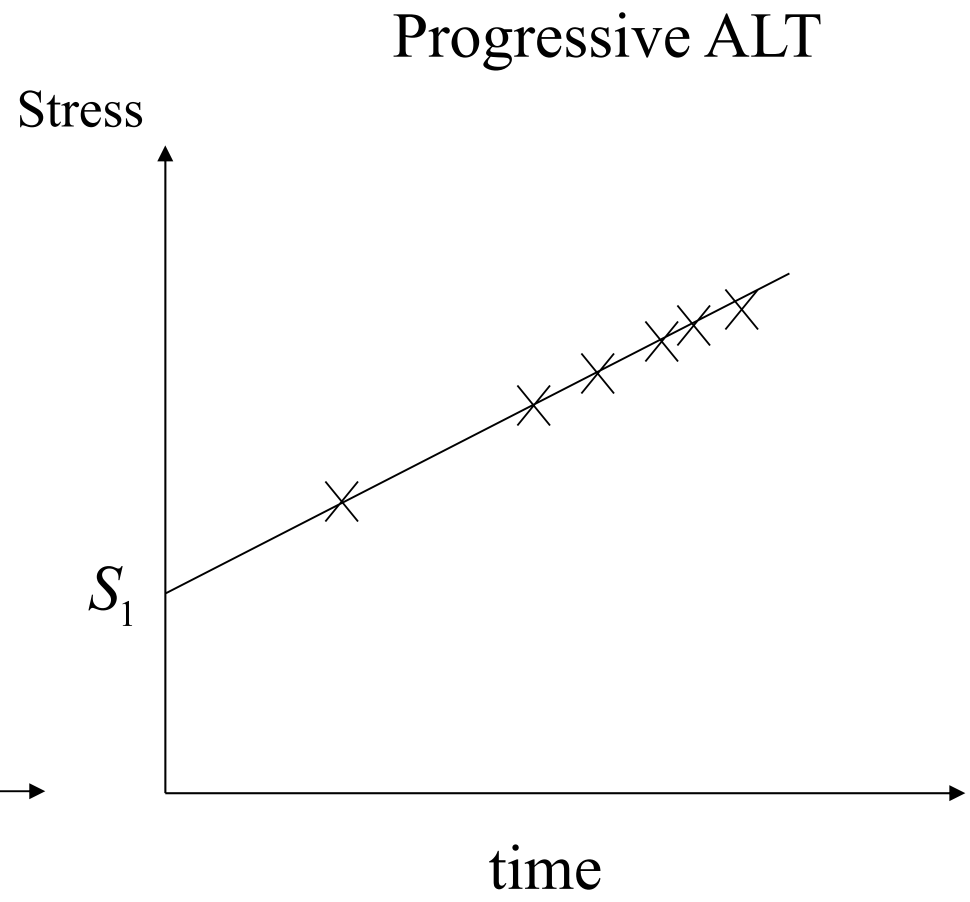
Accelerated Life Tests (ALT)





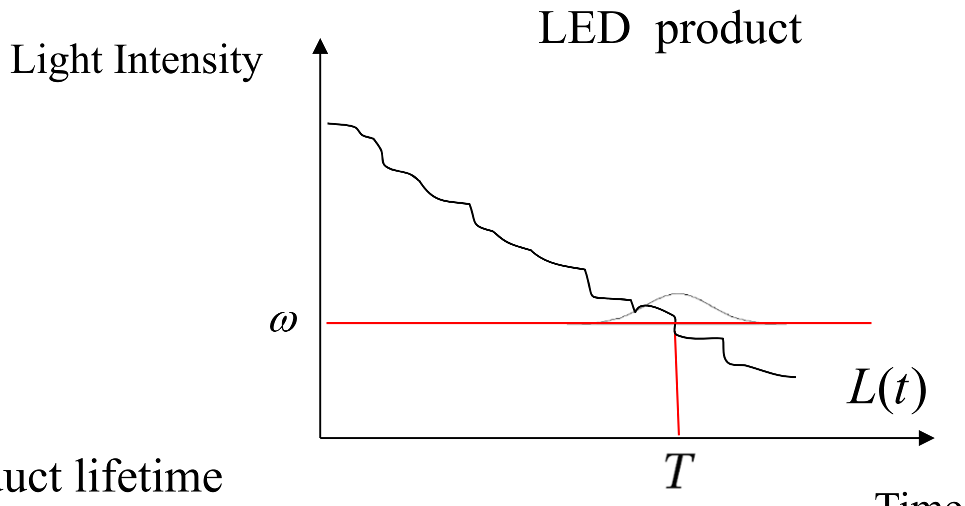
Step-Stress ALT and Progressive ALT



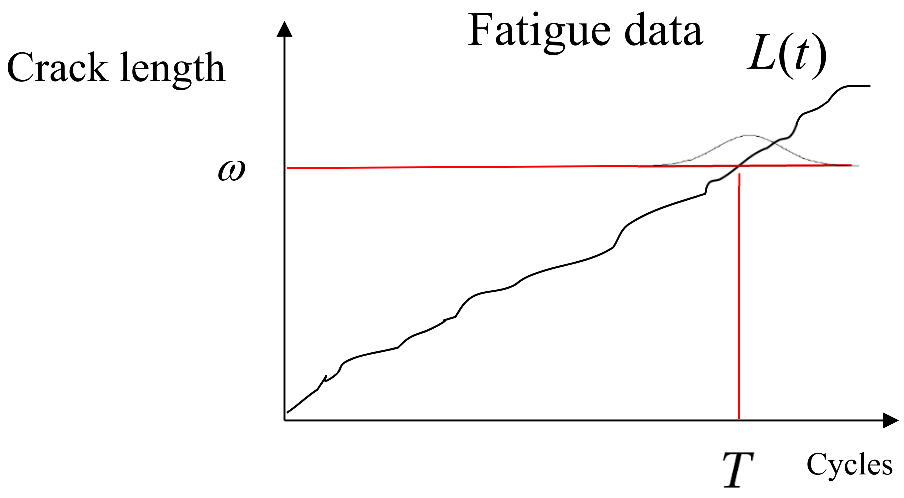


Degradation model analysis

- Quality characteristic (QC)
    - Related to reliability
    - Degradation over time
    - Failure level
- Collecting degradation data to assess product lifetime
    - 100p-th percentile
    - Mean-time-to failure (MTTF)
    - Mean residual life (MRL)
- Typical examples
    - LED (Light intensity)
    - Alloy fatigue (Crack size)



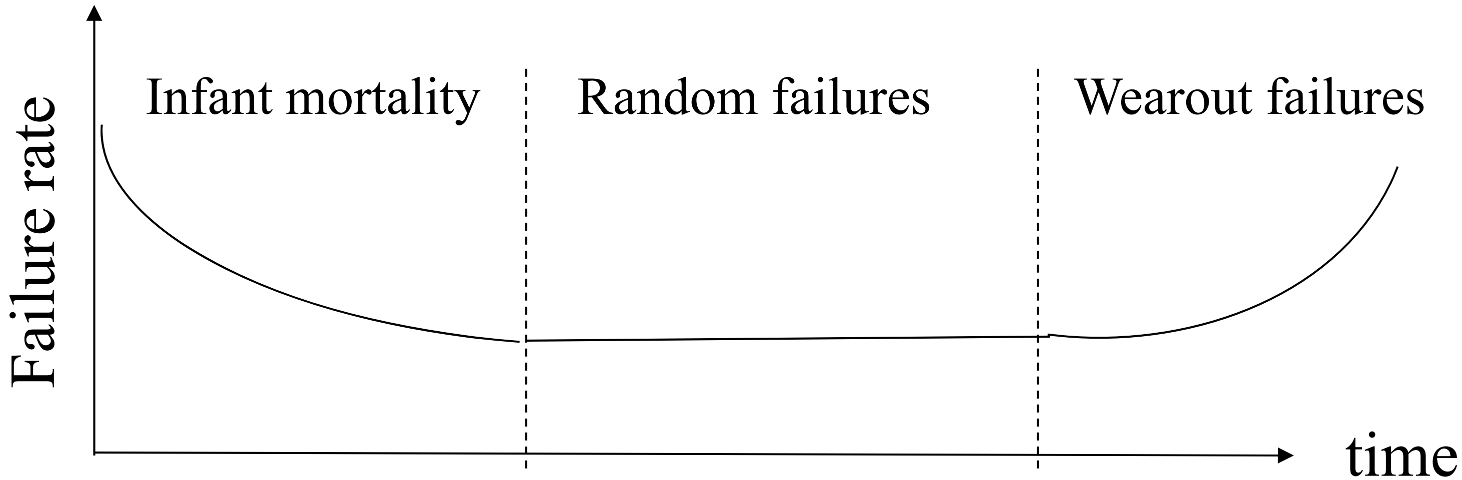
$$T = \inf\{t \mid L(t) \le \omega\}$$



$$T = \inf\{t \mid L(t) \ge \omega\}$$

Optimal burn-in policy

- **Bathtub curve**



- **Burn-in test**

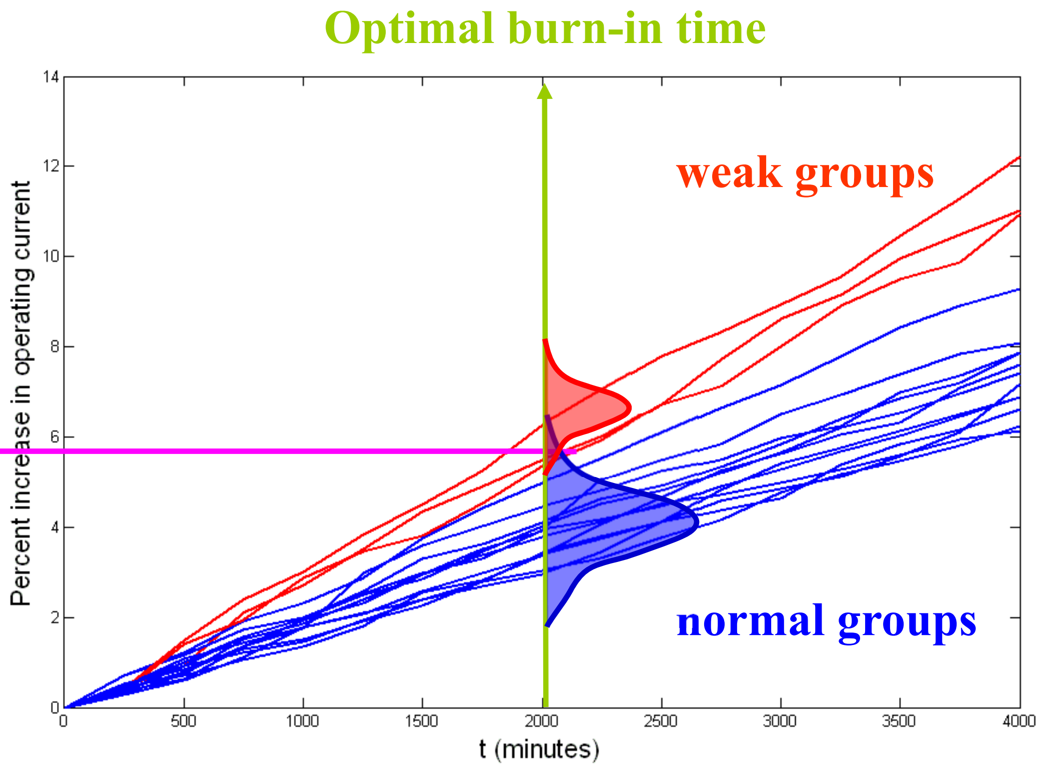
  The traditional burn-in test over a short time to collect time-to-failure is rather inefficient. How can we construct an optimal burn-in policy for highly reliable products in such a way that we can classify a product as weak or normal, and subsequently determine an optimal burn-in time?

Motivating example (Meeker & Escobar, 1998)

Decision rule
(Tseng & Tang, 2001)

R : An item is classified as
a normal item at time t
if and only if

Optimal cutoff point



- How to determine the optimal termination time of a burn-in test?
- How to derive the optimal cutoff point?

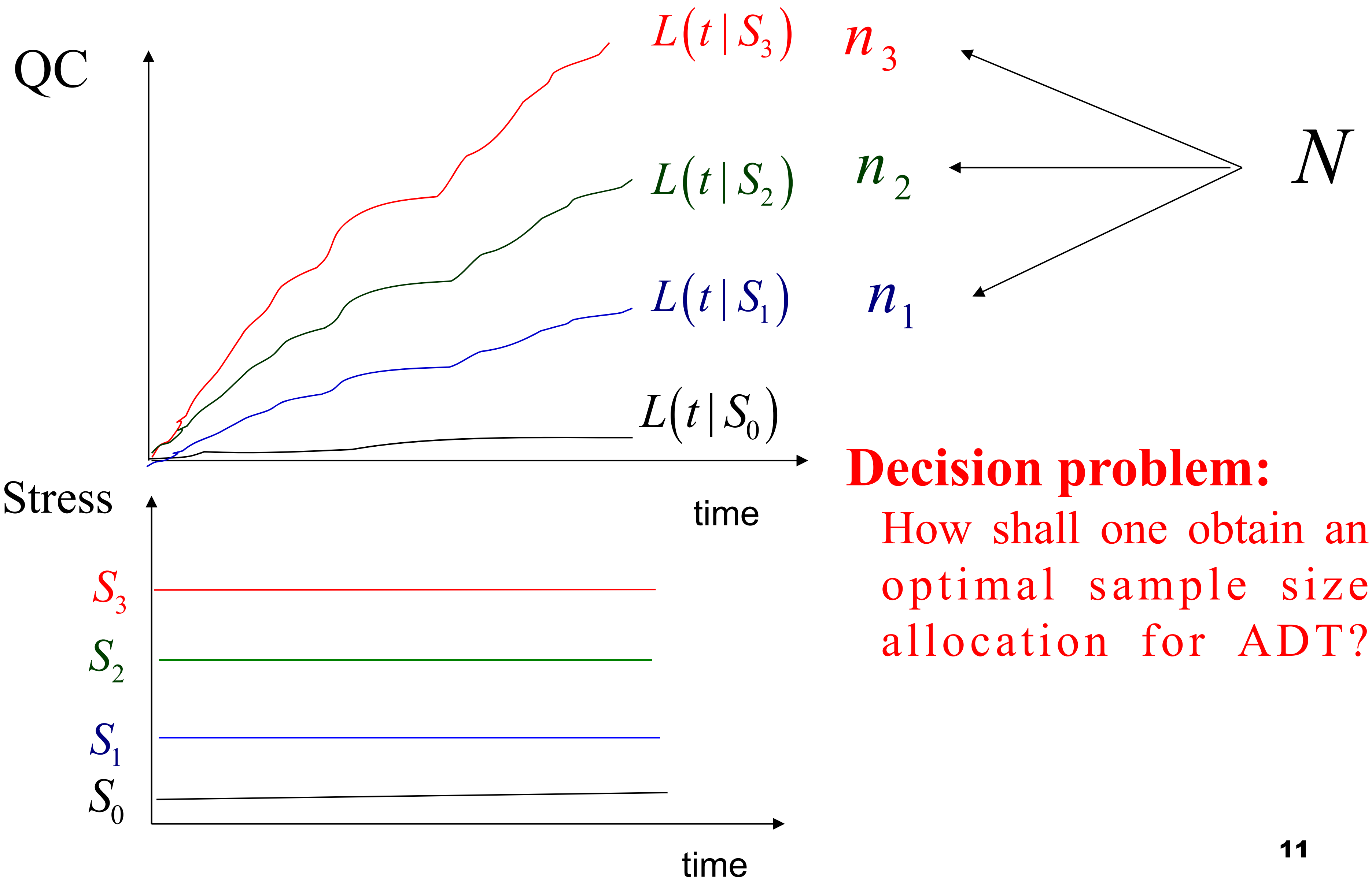
$$M_0 : \quad \Delta L(t) \sim \begin{cases} Ga(\Delta g_1(t), \nu_1) & \text{for weak group,} \quad (\text{ with } p) \\ Ga(\Delta g_2(t), \nu_2) & \text{for normal group,} \quad (\text{ with } 1-p) \end{cases}$$

where $\Delta g_1(t)\nu_1 > \Delta g_2(t)\nu_2 > 0$, $\Delta g_1(t) \neq \Delta g_2(t) > 0$, and $\nu_1 \neq \nu_2 > 0\ \forall t > 0$.

Remark

- Motivated by laser data, we first propose a mixed gamma process to describe the degradation path of the product. A decision rule for classifying a tested unit as typical or weak is presented, and the optimal cutoff point at each time has been derived analytically. Then, the optimal burn-in time is obtained by minimizing the total cost function.
- It will be of interest to find the optimal cutoff and optimal burn-in time simultaneously. The problem of classifying instances into multiclass can be considered. And, This topic on the decision rule for other degradation tests is of great interest.

Accelerated degradation tests (ADT)



Decision problem:

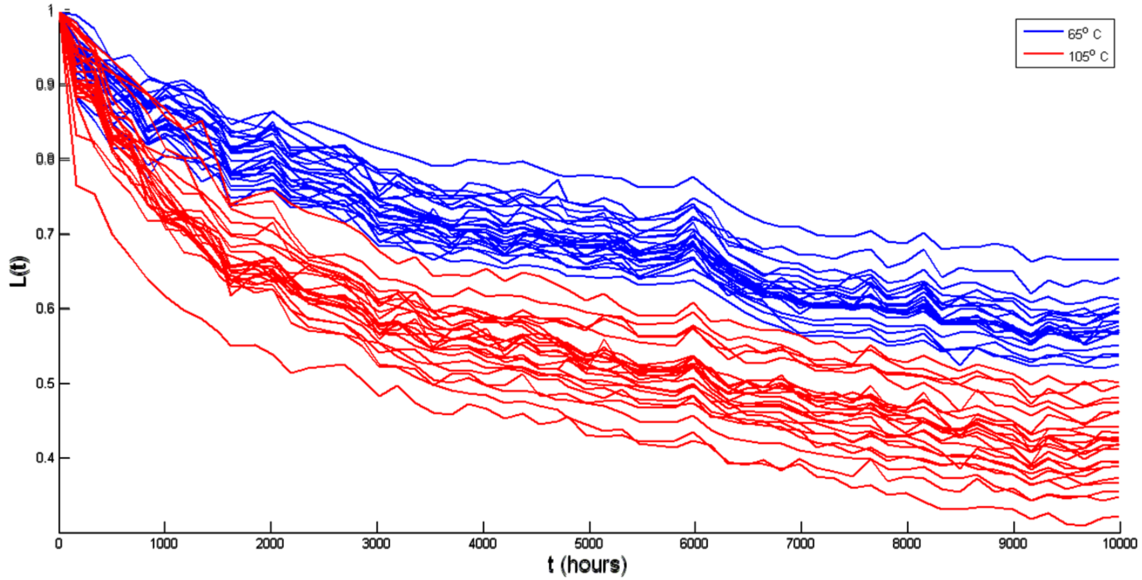
How shall one obtain an optimal sample size allocation for ADT?

Motivating Example (LED - GaAlAs)

- QC: Standardized light intensity
- At use stress : $S_0 = 298.15\text{K}\ (25^{\circ}\text{C})$
- Failure level: 50%
- Two higher stress level :

  $S_1 = 338.15\text{K}\ (65^{\circ}\text{C})$

  $S_2 = 378.15\text{K}\ (105^{\circ}\text{C})$
- Sample size : $n_1 = 25$, $n_2 = 24$
- Frequency : 336 hr
- Inspection numbers : 29



Decision Problem:

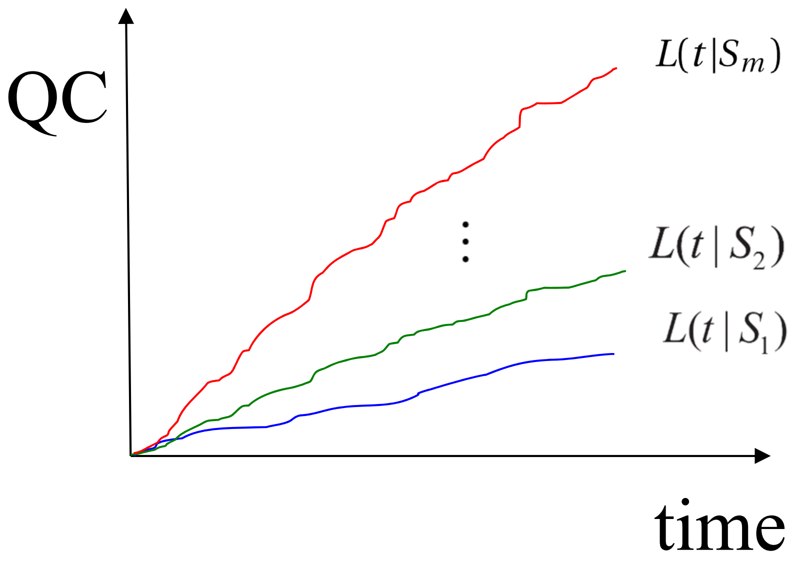
For planning such an accelerated degradation experiment, pre-fixed inspection time and termination time, we have the feasibility of choosing (n_1, n_2) under the constraint $\sum_{l=1}^{2} n_l = 49$. Hence, it arises an interesting decision problem: "How shall one obtain an optimal sample size allocation for the ADT ?"

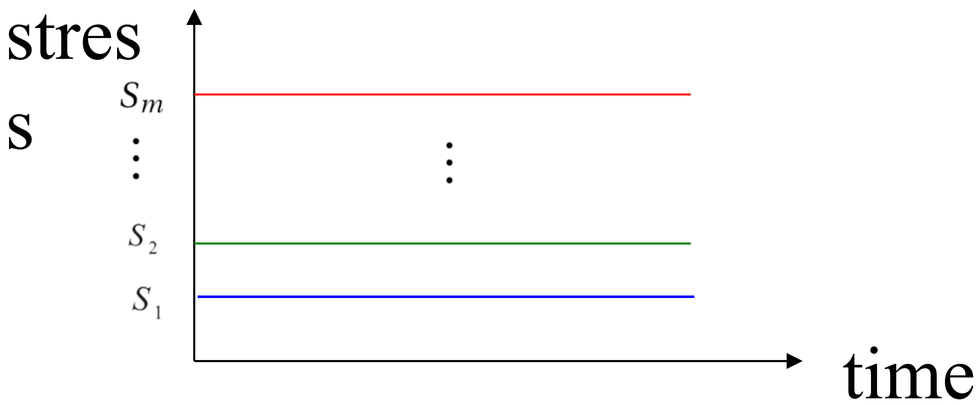
Remark

- By using three different optimality criteria, the optimal allocation procedures of tested units are derived. Furthermore, we also use relative efficiency to measure the performance of a specific sample size allocation.
- In practical application, we can consider the general degradation model to address the optimal allocation. Besides, the best compromise test plans can be derived between these optimality criteria.

Motivation of step-stress ADT

Accelerated degradation tests





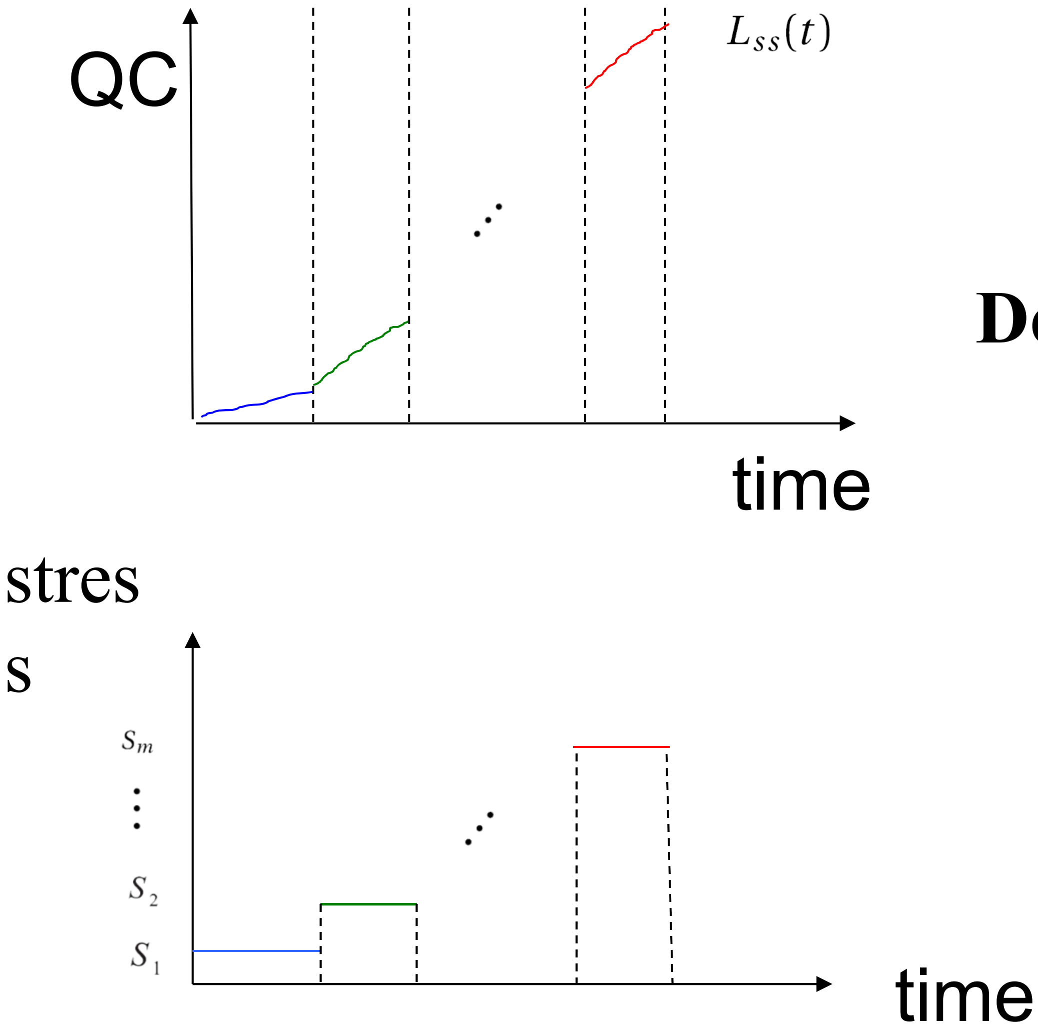
- **Advantage:**

  Degradation data can be collected under higher levels of stress within a reasonable period of time so as to allow extrapolation of the reliability information at the use condition.

- **Disadvantage**

  For a newly developed product or an expensive product, it may not be possible to have so many test units and ovens on hand.

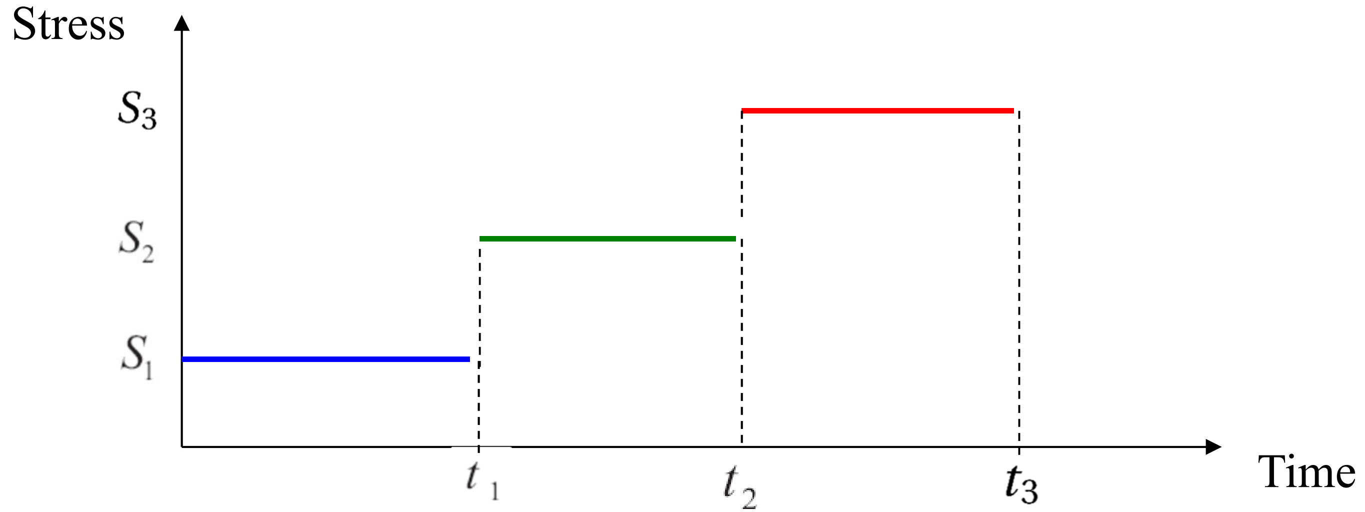
Step-stress ADT (Tseng and Wen, JQT, 2000)



Decision problem:

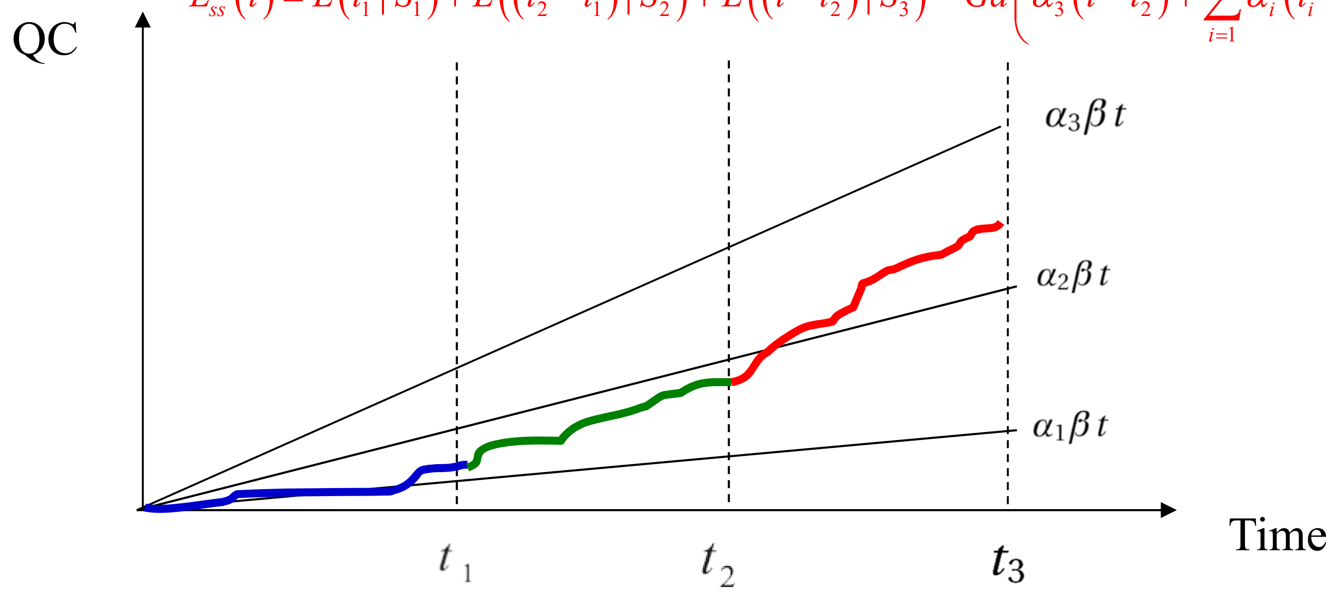
- Sample size
- Measurement frequency
- Number of measurements for each stress level

Step-stress accelerated degradation model



- Assume that $L(t \mid S_i) \sim Ga(\alpha_i t, \nu),\ i = 1,2,3$

$$L_{ss}(t) = L(t_1 \mid S_1) + L((t_2 - t_1) \mid S_2) + L((t - t_2) \mid S_3) \sim Ga\left(\alpha_3(t - t_2) + \sum_{i=1}^{2} \alpha_i (t_i - t_{i-1}), \nu\right)$$



Remark

- By minimizing the approximate variance of the estimated MTTF subject to the total experimental cost not exceeding a pre-specified budget, we found the optimal settings of the sample size, number of measurements for each stress level, and measurement frequency numerically.
- Instead of the enumeration search, the optimal settings of the sample size, number of measurements for each stress level, and measurement frequency may be of interest to derive theoretically. And, the SSADT degradation models based on other stochastic processes or methodology such as nonparametric or semi-parametric approach and the corresponding the optimal test plans would be worth comparing and discussing.

Optimal design of accelerated-stress acceptance test

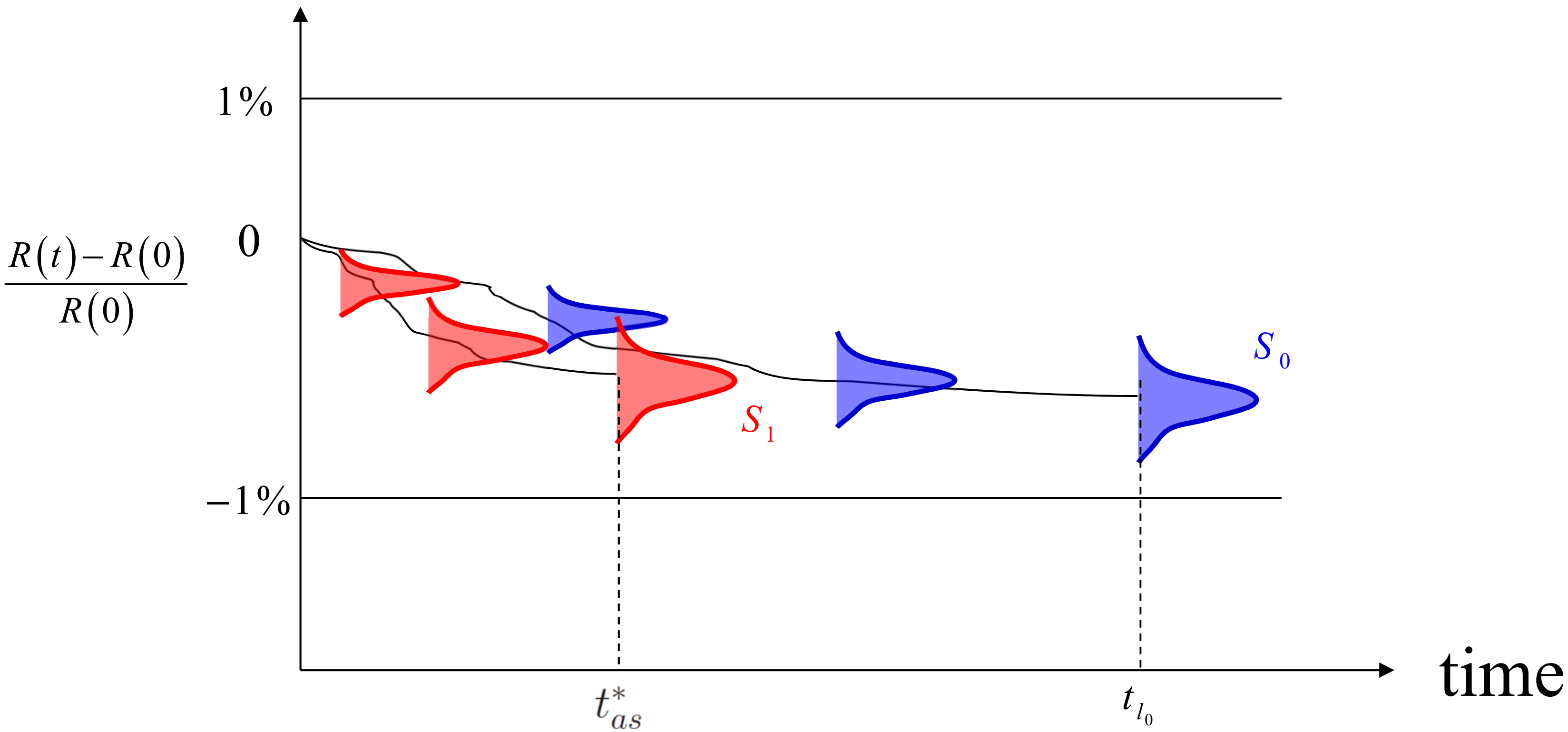
- **Acceptance test**

  is an important **inspection procedure** in reliability engineering and manufacturing. Traditionally, this type of testing involve chemical tests, physical tests, or performance tests.

- **Why is acceptance test so important**?
  - Determining whether some pertinent characteristic of the unit **meets the specification requirements** of customers.
  - Gives the customer **confidence** that the purchased unit/equipment possesses the specified features and requirements.
  - The **satisfaction and repeat business** of customers is increased, since they will be more confident that the requirements are met.

Motivation of the Study

- To **shorten** the acceptance testing time, one may consider conducting an acceptance experiment at a **higher temperature** (say, $S_1 = 125^oC$) to **accelerate** the chemical degradation process. How can one determine the **optimal accelerated-stress acceptance testing time** (denoted by t_{as}^*) in this accelerated test such that the performance of the relative changes in resistance under the new accelerated stress level S_1 will be **close to** the one under original stress S_0 at time t_{l_0} ?



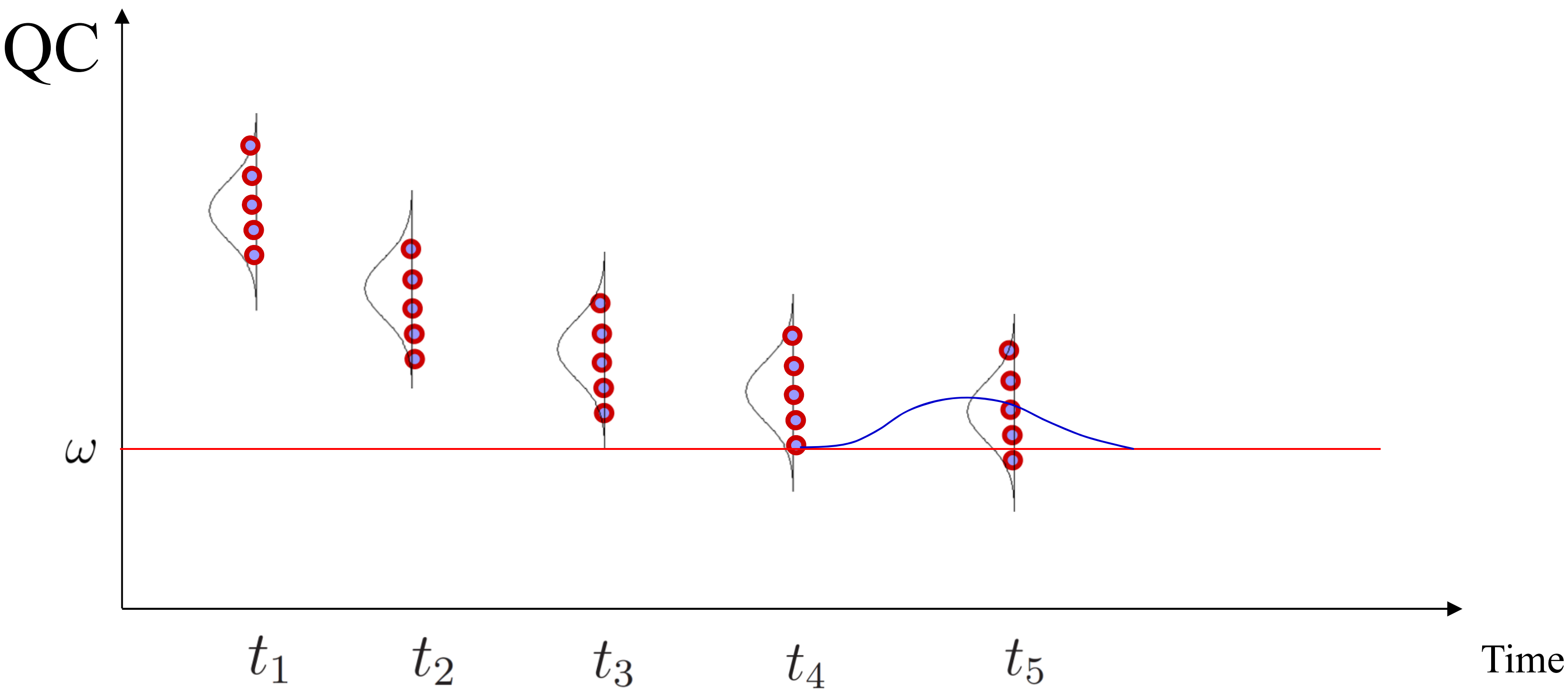
Remark

- Motivated by resistor data, an accelerated-stress acceptance test based on a Wiener degradation model has been proposed here to shorten the acceptance testing time. As illustrated in Section V, the acceptance testing time under the accelerated test is considerably better than the traditional acceptance testing time. We have determined the optimal test plan, including the total sample size, measurement frequency, and the number of measurements, for an accelerated-stress acceptance test.
- For some poor-quality resistors in the batch, their relative changes in resistance would be decayed or diffused much faster than would the good-quality ones. For such poor-quality batches, the probability of acceptance is the consumer's risk, and the probability of rejection of a good-quality batch is the producer's risk. It is important to balance these two risks by incorporating their respective costs in the proposed decision model.

Optimal design of Accelerated Destructive Degradation Test

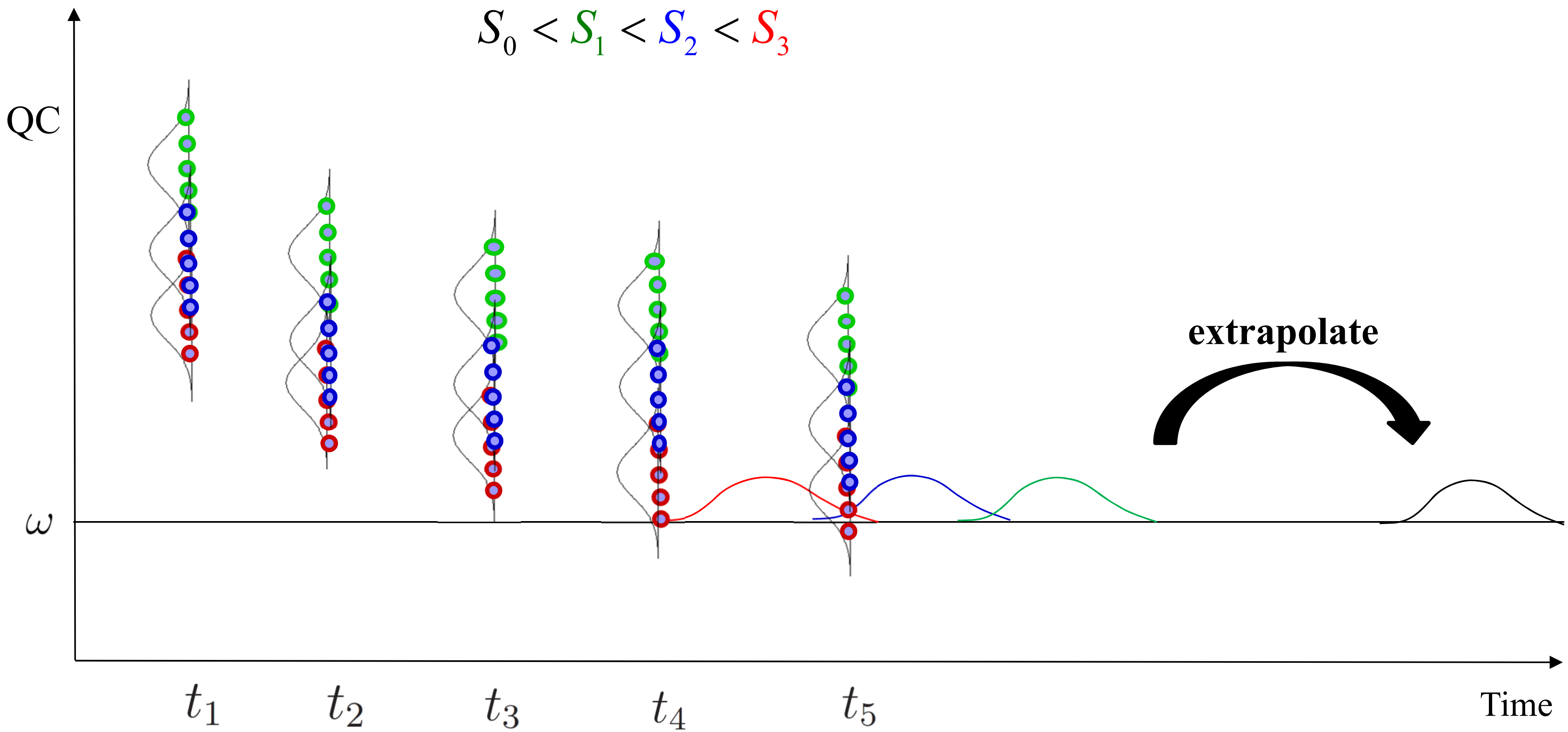
- But, the **degradation** measurement process in some specific applications will end up **destroying** the physical/ mechanical characteristic of tested units, and consequently **only one** measurement can be made on the QC of each unit.
- Ex. dielectric breakdown strength, adhesive bond, polymer material, etc.

  - **Total sample size:** 25 tested units
  - **Destructive measurements time :** t_1, t_2, t_3, t_4, t_5



Accelerated Destructive Degradation Tests (ADDT)

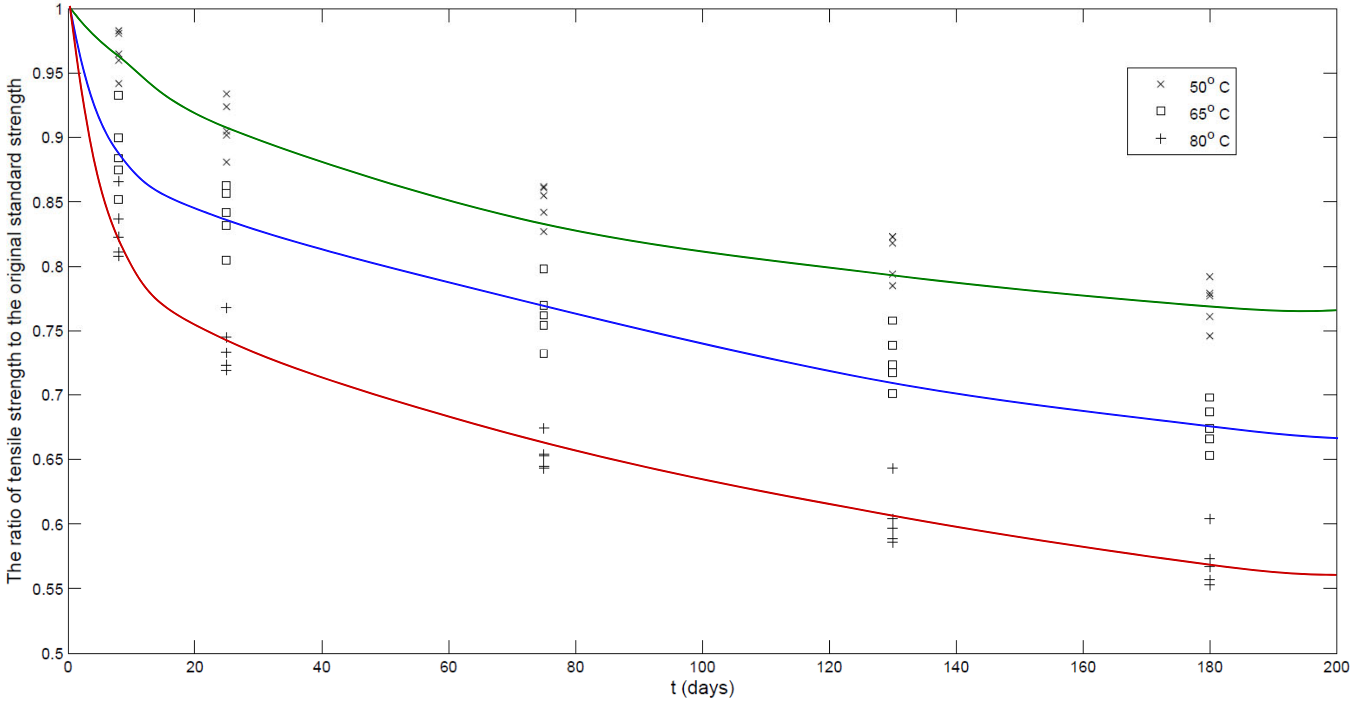
- However, for some highly reliable products, degradation process may be **too slow** to obtain a good estimate **within a reasonable period of time**.
- For this reason, degradation data are generally collected from tests under **higher levels** of stress so that **extrapolation** may be used to obtain the reliability information at the use condition.



A Motivating Example (Yang, 2007)

Polymer material

- **QC :** ratio of the tensile strength to the original standard strength
- **Failure level :** 60%
- **Use condition :** S_0=25^{0}C
- **Three higher accelerated condition:** S_1=50^{0}C, S_2=65^{0}C, S_3=80^{0}C
- **Each accelerated condition :** 25 coupons
- **Destructive measurement time :** 8, 25, 75, 130, 180 days
- **Each measurement time during each test:** 5 coupons



Nonlinear Degradation path

$$L_k(t_j|S_i) = e^{-\beta_i t_j^{\alpha_i}} + \varepsilon_{ijk}$$

Remark

- Motivated by the polymer data, we propose a nonlinear ADDT model wherein the measurement error follows a normal or skew-normal distribution. The expressions for a product's lifetime distribution and the 100pth percentile under this model are derived analytically. The optimal choices for total sample size, measurement frequency, and the number of measurements at each stress level can be obtained.

- The unit-to-unit variability may be taken into consideration in degradation modeling and analysis. And, a possible generalization is the skew-distribution under which skewness and excess kurtosis may take larger values. Like its symmetric counterpart, it has a degree of freedom parameter that makes it possible to model both higher moments and the tails of the distribution.

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