STABLE COMBINATION TESTS

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Abstract: This short note proposes a stable combination test as a natural extension of the Cauchy combination test. Similarly to the latter, the proposed test is simple to compute, enjoys good size, and has asymptotically optimal power even when the individual tests are not independent. Our simulations demonstrate that the proposed stable combination test may improve the Cauchy combination test if its parameters are chosen carefully.

Key words and phrases: Additive combination test, multiple hypothesis testing, stable distribution.

Liu and Xie (2020) recently proposed the Cauchy combination test (CCT), which originated from the observation that the standard Cauchy distribution is closed under convex combinations (Pillai and Meng (2016)). One of the most fascinating features of the CCT is its robustness to dependency among individual tests, despite its simple form. The standard Cauchy distribution is a member of the stable distribution family, which is also asymptotically closed under convex combinations. In this brief note, we investigate whether the favorable properties of the CCT can be extended to a stable combination test (SCT).

Let the \( p \)-values \( p_1, \ldots, p_n \) be uniform under the global null hypothesis \( H_0 = \bigcap_{i=1}^n H_i \). The SCT statistic is defined as

\[
T_{n;\alpha,\beta}(p) = a_{n;\alpha} \sum_{i=1}^n w_i W_{i;\alpha,\beta},
\]

where \( w_i \) are positive constants such that \( \sum_{i=1}^n w_i = 1 \), \( a_{n;\alpha} = \left( \sum_{j=1}^n w_j^{\alpha} \right)^{-1/\alpha} \) is the normalizing factor, and \( W_{i;\alpha,\beta} = F^{-1}(1 - p_i|\alpha,\beta) \). Here, \( F(\cdot|\alpha,\beta) \) is the distribution function of \( S(\alpha,\beta) \), which is the stable distribution with stability, skewness, scale, and location parameters \( \alpha, \beta, 1, \text{ and } 0 \), respectively, following Nolan’s (2020) S1 parametrization. We consider \( 0 < \alpha < 2 \), and \( -1 < \beta \leq 1 \) for

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