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LOCALLY *D*-OPTIMAL DESIGNS FOR HIERARCHICAL RESPONSE EXPERIMENTS

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Abstract: Categorical responses with a hierarchical structure are common in social sciences, public health, and marketing. The continuation ratio model is one of the most common models used to characterize such hierarchical data. Despite the wealth of research on this model, few studies have considered its design in the data collection step. Here, we study locally *D*-optimal designs for models with general link functions under the partial proportional odds assumption. The necessary and sufficient conditions for the positive definiteness of the Fisher information matrix are derived, which show that a feasible design may contain fewer supports than the number of parameters in the model. Based on some deduced characteristics of the *D*-optimal criterion, an efficient algorithm is proposed to search for optimal designs that can deal with both discrete and continuous design fields. Lastly, numerical examples illustrate the advantages of the proposed designs over some existing designs.

Key words and phrases: Approximate design, continuation ratio model, general link functions, multinomial response.

1. Introduction

Categorical responses are common in scientific experiments. As such, design of experiments with categorical responses is becoming increasingly popular in many scientific disciplines. Some progress has been achieved in terms of both theory and algorithms; see, for example, Sitter and Wu (1993), Atkinson, Donev and Tobias (2007), Yang, Zhang and Huang (2011), Yang, Biedermann and Tang (2013), Yang, Tong and Mandal (2017), and Lukemire, Mandal and Wong (2019). Among the various types of categorical responses, ordinal responses with a hierarchical structure are often used. Examples include ratings of preferences in consumer choice experiments, tumor grades in drug testing, and animal fitness in ecology; see Agresti (2007) for details.

The continuation ratio (CR) model (O'Connell (2006)) is one of the most commonly applied models, and focuses on estimating the probabilities of successive stages when the lower stages are reached first (Fullerton (2009)). Mathemat-

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