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FUNCTIONAL HORSESHOE SMOOTHING FOR FUNCTIONAL TREND ESTIMATION

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Abstract: As a result of developments in instruments and computers, functional observations are becoming increasingly prevalent. However, few existing methodologies can flexibly estimate the underlying trends with valid uncertainty quantification for a sequence of functional data (e.g., functional time series). In this work, we develop a locally adaptive smoothing method, called functional horseshoe smoothing, by introducing a shrinkage prior to the general order of differences of functional variables. This allows us to capture abrupt changes by making the most of the shrinkage capability, and to assess uncertainty by using a Bayesian inference. The fully Bayesian framework allows us to select the number of basis functions using the posterior predictive loss. We provide theoretical properties of the model, which support the shrinkage ability. Furthermore, by taking advantage of the nature of functional data, the proposed method can handle heterogeneously observed data without data augmentation. Simulation studies and a real-data analysis demonstrate that the proposed method has desirable properties.

Key words and phrases: Functional time series, MCMC, shrinkage prior, tail robustness, trend filtering.

1. Introduction

The recent development of measuring instruments and computers has made it possible to obtain high-dimensional data in various fields. However, analyzing such data using a classical multivariate analysis requires a huge number of parameters making it difficult to extract valuable information from the data. A promising methodology to solve these problems is functional data analysis (FDA), which treats and analyzes high-dimensional data as a curve (function). Functional versions for various branches of statistics have been provided; see Ramsay and Silverman (2005), Kokoszka and Reimherr (2017), and Horváth and Kokoszka (2012).

The traditional FDA approach for independent functional data has recently been extended to time series. In fact, for functional time series data, the standard stationary model for multivariate data has been extended (e.g., Besse, Cardot and Stephenson (2000); Klepsch and Klüppelberg (2017); Klepsch, Klüppelberg and Wei (2017); Hörmann, Horváth and Reeder (2013); Gao, Shang and Yang (2019);

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