FUNCTIONAL RESPONSE
QUANTILE REGRESSION MODEL

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Abstract: We propose a new functional response quantile regression model, and
develop a data-driven estimation procedure to estimate the quantile regression pro-
cesses based on a local linear approximation. Theoretically, we obtain the global
uniform Bahadur representation of the estimator with respect to the time/location
and the quantile level, and show that the estimator converges weakly to a two-
parameter continuous Gaussian process. We then derive the asymptotic bias and
mean integrated squared error of the smoothed individual functions and their uni-
form convergence rates under given quantile levels. Based on the theoretical results,
we introduce a global test for the coefficient functions and discuss how to construct
simultaneous confidence bands. We evaluate our method using simulations and by
applying it to diffusion tensor imaging data and ADHD-200 functional magnetic
resonance imaging data.

Key words and phrases: Functional data, global test statistic, simultaneous confi-
dence band, weak convergence.

1. Introduction

Functional data analysis deals with data in the form of functions, images,
and shapes, as well as more general objects (Wang, Chiou and Müller (2016)).
Functional regression models are widely used to model functional data, and in-
clude the functional linear regression (Ramsay and Dalzell (1991); Ramsay and
Silverman (2005); Yao, Müller and Wang (2005b)) and functional response re-
gression model (Ramsay and Silverman (2005)). The classical functional linear
regression describes the relationship between a scalar response and a functional
predictor. In contrast, a functional response regression characterizes the relation
between a functional response and scalar predictors.

The functional response regression model is defined as independent realiza-
tions of an underlying stochastic process

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