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ON THE CONSISTENCY OF THE LEAST SQUARES ESTIMATOR IN MODELS SAMPLED AT RANDOM TIMES DRIVEN BY LONG MEMORY NOISE: THE RENEWAL CASE

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Abstract: In this study, we prove the strong consistency of the least squares estimator in a random sampled linear regression model with long-memory noise and an independent set of random times given by renewal process sampling. Additionally, we illustrate how to work with a random number of observations up to time $T = 1$. A simulation study is provided to illustrate the behavior of the different terms, as well as the performance of the estimator under various values of the Hurst parameter H.

Key words and phrases: Least squares estimator, long-memory noise, random times, regression model, renewal process.

1. Introduction

In many applications, data are observed at random times. This situation arises from a variety of causes, such as machinery faults or the inability to observe data in certain periods. In the financial field, the process often cannot be observed continuously (Duffie and Glynn (2004)). As a result, high-frequency financial data (a very large amount of data) tend to be sampled discretely in time, and the time separating successive observations is often random. For the random modeling of observations, the renewal case represents progressive randomness and distance from periodic sampling. Here Masry (1983) studied the problem of estimating an unknown probability density function, based on n independent observations sampled at random times. Vilar (1995) and Vilar and Vilar (2000), studied the nonparametric kernel estimator of the regression function under mixing dependence conditions, and the Ornstein–Uhlenbeck process driven by Brownian motion, respectively.

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