

CHANGE-POINT TESTS FOR THE TAIL PARAMETER OF LONG MEMORY STOCHASTIC VOLATILITY TIME SERIES

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Abstract: We consider a change-point test based on the Hill estimator to test for structural changes in the tail index of long memory stochastic volatility time series. In order to determine the asymptotic distribution of the corresponding test statistic, we prove a uniform reduction principle for the tail empirical process in a two-parameter Skorohod space. It is shown that such a process displays a dichotomous behavior according to an interplay between the Hurst parameter, that is, a parameter characterizing the dependence in the data, and the tail index. Our theoretical results are accompanied by simulation studies and an analysis of financial time series with regard to structural changes in the tail index.

Key words and phrases: Chaining, change-point tests, heavy tails, long-range dependence, stochastic volatility, tail empirical process.

1. Introduction

The tail behavior of the marginal distribution of time series is of major relevance for statistics in applied sciences such as econometrics and hydrology, where heavy-tailed data occur frequently. More precisely, time series from finance, such as the log-returns of exchange rates and stock market indices, display heavy tails; see Mandelbrot (1963). Furthermore, drastic events such as the financial crisis in 2008 substantiate the importance of studying time series models that underlie financial data. Against this background, identifying changes in the tail behavior of data-generating stochastic processes that result in an increase or decrease in the probability of extreme events is of utmost interest. In particular, analyzing of the tail behavior of financial data may pave the way for a corresponding adjustment of risk management for capital investments, thus preventing huge capital losses. Indeed, there is empirical evidence that the tail behavior of financial time series may change over time. Quintos, Fan and Phillips (2001) identify changes in the

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