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ESTIMATION AND INFERENCE FOR DYNAMIC SINGLE-INDEX VARYING-COEFFICIENT MODELS

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Abstract: Motivated by applications, we propose a class of dynamic single-index varying-coefficient models to explore the varying interaction effects on the response variable among a set of covariates. That is, the interaction effects are allowed to change with some factors of interest, such as time, spatial location, or other covariates. A spline-based approach is developed to estimate the index and varying-coefficient functions. The convergence rates and asymptotic normalities of the resulting estimators are established. It is also shown that the resulting estimators exhibit the oracle property. A penalized method is presented to select related covariates, and the consistency of the penalized estimator is proved. A test statistic is provided to check whether the interaction effect also varies with the factors of interest, and the asymptotic normality of the test statistic is established. Simulation studies and two real-data analyses illustrate the good performance of the proposed model and the corresponding statistical inference methods for finite samples.

Key words and phrases: Interaction effect, single-index varying-coefficient regression model, spline approximation, variable selection.

1. Introduction

Single-index varying-coefficient regression models (SICMs), proposed by Xia and Li (1999), have attracted much attention over the past few decades and have proven to be effective in practical applications; see Fan, Yao and Cai (2003), Ma and Song (2015), Liu, Cui and Li (2016), and the references therein. SICMs can capture the nonlinear relationship between a response and related covariates, and can achieve dimensional reduction by taking a linear combination of covariates as an index. A classical SICM is defined by

$$Y = \sum_{l=1}^{d} g_l(\mathbf{Z}^T\boldsymbol{\beta})X_l + \varepsilon, \tag{1.1}$$

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