

Testing Distributional Equality for Functional Random Variables

Bilol Banerjee

Indian Statistical Institute, Kolkata, India

Abstract

In this article, we consider testing whether two groups of functional observations are generated from the same distribution or not. Firstly, we present a general framework to build a measure that quantifies the distributional difference between two functional random variables. The measure is non-negative, and under fairly general assumptions it takes the value zero if and only if the distributions are equal. We consider such a measure based on the Maximum Mean Discrepancy (MMD) and study its theoretical properties. We also propose an estimator of the measure and study its asymptotic properties. Using the estimator as a test statistic, we propose a test for distributional equality for functional random variables. The estimator involves a kernel function with an associated bandwidth parameter. In practice, a data-based choice of bandwidth is used for implementing the test. Hence, we calibrate the test using the permutation method and establish that the proposed permutation test is consistent. Extensive simulation studies are conducted to compare the performance of our proposed test with some of the state-of-the-art methods.

Keywords:

Functional Data; Two Sample Test; Contiguity; Permutation Method; Maximum Mean Discrepancy.