

Proximal Causal Inference for Interventional Indirect Effects under Intermediate Confounding

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Abstract

Unmeasured confounding remains a major challenge in causal mediation analysis, particularly when treatment affects intermediate variables that subsequently confound the mediator– outcome relationship. In such settings, traditional mediation methods generally fail unless strong no-unmeasured-confounding assumptions are imposed. We propose a nonparametric proximal identification framework using proxy-based adjustment to assess interventional indirect effects (IIEs) under simultaneous unmeasured and intermediate confounding. By extending the proximal g-formula to nested counterfactuals, we derive identification conditions through a sequence of bridge functions—solutions to Fredholm integral equations linking observed proxies to latent factors. We develop a triply robust, locally efficient estimator for the proximal IIE that generalizes semiparametric approaches for complex multivariate systems. Simulation studies demonstrate that our estimator remains unbiased in scenarios where standard methods ignoring unmeasured confounding exhibit significant bias.