

# **Nested Grassmannians for dimensionality reduction with applications**

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## **Abstract**

Recently, the nested structure of Riemannian manifolds has been studied in the context of dimensionality reduction as an alternative to the popular principal geodesic analysis (PGA) technique, for example, the principal nested spheres. A novel framework is proposed for constructing a nested sequence of homogeneous Riemannian manifolds. Common examples of homogeneous Riemannian manifolds include the spheres, the Stiefel manifolds, and the Grassmann manifolds. In particular, it is focused on applying the proposed framework to the Grassmann manifolds, giving rise to the nested Grassmannians (NG). An important application in which Grassmann manifolds are encountered is planar shape analysis. Specifically, each planar (2D) shape can be represented as a point in the complex projective space, a complex Grassmann manifold. Some salient features of the framework are: (i) it explicitly exploits the geometry of the homogeneous Riemannian manifolds, and (ii) the nested lower-dimensional submanifolds need not be geodesic. With the proposed NG structure, algorithms are developed for the supervised and unsupervised dimensionality reduction problems respectively. The proposed algorithms are compared with PGA via simulation studies and real data experiments and are shown to achieve a higher ratio of expressed variance compared to PGA.