Largest Entries of Sample Correlation Matrices from Equi-correlated Normal Populations

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Abstract

We the limiting distribution of the largest off-diagonal entry of the sample correlation matrices of high-dimensional Gaussian populations with equi-correlation structure. Assume the entries of the population distribution have a common correlation coefficient $\rho > 0$ and both the population dimension *p* and the sample size *n* tend to infinity with $\log p = o\left(n^{\frac{1}{3}}\right)$. As $0 < \rho < 1$, we prove that the largest off-diagonal entry of the sample correlation matrix converges to a Gaussian distribution, and the same is true for the sample covariance matrix as $0 < \rho < 1/2$. This differs substantially from a well-known result for the independent case where $\rho = 0$, in which the above limiting distribution is an extreme-value distribution. We then study the phase transition between these two limiting distributions and identify the regime of ρ where the transition occurs. It turns out that the thresholds of such a regime depend on *n* and converge to zero. If ρ is less than the threshold, larger than the threshold or is equal to the threshold, the corresponding limiting distribution is the extreme-value distribution, the Gaussian distribution and a convolution of the two distributions, respectively. The proofs rely on a subtle use of the Chen-Stein Poisson approximation method, conditioning, a coupling to create independence and a special property of sample correlation matrices. The results are then applied to evaluating the power of a high-dimensional testing problem of identity correlation matrix.