Optimal Change Point Detection In State Space Models

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Suppose there is a sequence of observations $\xi_0, \xi_1, \ldots$ from a state space model such that $\xi_0, \xi_1, \ldots, \xi_{\omega-1}$ are observations with probability distribution P^{θ_0}, and $\xi_\omega, \xi_{\omega+1}, \ldots$ are observations with probability distribution P^θ at some unknown time ω. The parameter of prechange θ_0 is given, while the parameter of after change θ is unknown. The problem is to raise an alarm as soon as possible after the distribution changes from P^{θ_0} to P^θ, but to avoid false alarms. Specifically, we seek a stopping rule N which allows us to observe the $\xi's$ sequentially, such that $P_\infty^{\theta_0}\{N < \infty\}$ is small, and subject to this constraint $\sup_k E_k^\theta(N - k|N \geq k)$ is, uniformly in θ, as small as possible. Here E_k^θ denotes expectation under the parameter is θ and the change point is k, and $P_\infty^{\theta_0}$ denotes probability under the parameter is θ_0 and the hypothesis of no change whatever.
In this paper we investigate theoretical properties of a mixture Shiryayev-Roberts-Pollak (SRP) change point detection rule in state space models. By making use of Markov chain representation for the likelihood function, exponential embedding of the induced Markovian transition operator, and sequential hypothesis testing theory for Markov random walks, we show that the mixture SRP procedure is second-order asymptotically optimal. To this end, we derive an asymptotic approximation for the expected stopping time of such a stopping scheme when $\omega = 1$. Finally, we apply these results to several types of state space models: general state Markov chains, linear state space models and $ARMA$ models.

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