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Selecting optimal multistep predictors for unstable autoregressive processes

Abstract: Most existing forecasting theory assume stationarity of the time series and knowing the true parameters. Not much is known about the case of nonstationary models with estimated parameters. To fill this gap, this paper investigates multistep forecast errors for autoregressive (AR) processes with unit roots. If a working AR model is adopted, then two completing types of multistep prediction methods, i.e., plug-in and direct methods, can be obtained from this model. We first obtain asymptotic expressions for the mean-squared prediction errors of these two methods up to terms of order $1/n$, where n is the number of observations. The expressions show that the best prediction result is not only determined by the prediction model, but also by the prediction method. To find the best combination, asymptotic properties of the accumulated squares of multistep prediction errors are investigated in the presence of unit roots. We show that the accumulated errors can be asymptotically decomposed into two components, one of which, arising from estimation uncertainty, is of order log n, and the other, arising from the disturbance term, is of order n but common for each candidate predictor. While the log n term is smaller compared to the other one, its associated constant crucially determines the candidate predictor's performances. This special feature enables us to construct a predictor selection criterion that can choose the best combination of the prediction model and the prediction method with probability tending to 1. Our theoretical findings are illustrated via simulation studies and analysis of several real data sets.