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Statistical Inference for time-varying ARCH processes

Abstract: In this talk the class of ARCH(∞) models is generalized to the nonstationary ARCH(∞) models with time-varying coefficients

$$\begin{aligned} X_{t,N} &= \sigma_{t,N} Z_t \\ \text{with} \quad \sigma_{t,N}^2 &= a_0(\tfrac{t}{N}) + \textstyle\sum_{j=1}^{\infty} a_j(\tfrac{t}{N}) X_{t-j,N}^2 \quad \text{for} \quad t = 1, \ldots, N, \end{aligned}$$

where Z_t are independent, identically distributed random variables with $\mathrm{E}Z_t = 0$, $\mathrm{E}Z_t^2 = 1$. In order to investigate the structure of such processes and to investigate the properties of estimators we use the idea of a stationary approximation and a stationary derivative process. The stationary approximation is defined by

$$\begin{aligned} \tilde{X}_t(u_0) &= \sigma_t(u_0) Z_t \\ \text{with} \quad \sigma_t(u_0)^2 &= a_0(u_0) + \textstyle\sum_{j=1}^{\infty} a_j(u_0) \tilde{X}_{t-j}(u_0)^2. \end{aligned}$$

We establish the relation

$$|X_{t,N}^2 - \tilde{X}_t(u_0)^2| \le K(|\frac{t}{N} - u_0| + \frac{1}{N}) U_t \quad a.s.$$

and use this relation for deriving our results. Moreover, stationary derivative processes are defined and a Taylor type expansion is proved for tvARCH processes. The existence of all these processes is proved via time-varying Volterra expansions.

For the estimation of the parameter curves of tvARCH(p)-processes weighted quasi-likelihood estimates are used. The asymptotic properties of such estimates are investigated. In particular asymptotic normality is proved and the extra bias due to nonstationarity of the process is determined.

Furthermore, recursive (online) estimates of the form

$$\underline{\hat{a}}_{t,N} = (I - \lambda \frac{\underline{X}_{t,N} \underline{X}_{t,N}^T}{|\underline{X}_{t,N}|^2}) \underline{\hat{a}}_{t-1,N} + \lambda \frac{X_{t,N}^2 \underline{X}_{t,N}}{|\underline{X}_{t,N}|^2}$$

for $\underline{a}(\frac{t}{N})$ where $\underline{a}(u)^T = (a_0(u), \ldots, a_p(u))$are introduced. Here $\underline{X}_{t,N} = (1, X_{t-1,N}^2, \ldots, X_{t-p,N}^2)^T$,$|\underline{X}_{t,N}| = 1 + \sum_{j=1}^{p} X_{t-j,N}^2$ andλ is the stepsize which controls the influence of the new observation. We give results on consistency and asymptotic normality of this recursive estimate.