Bhansali, Rajendra J. (University of Liverpool, UK)

Frequency Analysis of Chaotic Intermittency Maps with Slowly Decaying Correlations

Abstract: The established long-memory stochastic models are typically linear and envisage that the process obtained by taking the dth fractional difference, $0 < d < 0.5$, has short memory. A new and an entirely different approach to modeling phenomena exhibiting long-memory is by chaotic intermittency maps. These maps are typically non-linear and non-Gaussian and possess three important properties which qualify them as plausible class of models for physical phenomena exhibiting long memory: first, their correlations decay to 0 at a sub-exponential rate, meaning at a polynomial rate or slower; second, the invariant densities of these maps could approach infinity near the origin at a polynomial or slower rate; third, these maps display intermittent chaos and generate orbits which switch between laminar and chaotic regions. Asymptotic expressions describing the limiting behavior of the spectral density functions near the origin of the polynomial, logarithmic and symmetric cusp maps are developed. We show that for the symmetric cusp map, the spectral density, $f_y(\lambda)$, behaves like a logarithmic function near the origin, $f_y(\lambda) \quad log(|\lambda|)$, as $|\lambda| \to 0$. By contrast, for the logarithmic map, we show that $f_y(\lambda) \quad |\lambda|^{-1}|log(1/|\lambda|)$ as $|\lambda| \to 0$. An interpretation of these results is that although $d = 0$ for the symmetric cusp map, the map displays long-memory, and, similarly, $d = 0.5$ for the logarithmic map, and yet it generates a stationary process. Our results thus enable the behavior of the long-memory processes to be studied at the boundary between a long-memory and an intermediate-memory process on the one hand and between a long-memory and non-stationary process on the other hand. On the assumption that a realization of T consecutive observations from these maps is observed, with the initial value obtained from their invariant distributions, some of the asymptotic sampling properties of the low frequency periodogram ordinates, as $\mathrm{T} \to \infty$, for these maps are also examined; the results here generalize the earlier results of Hurvich and Beltrao(1993) , who considered a similar problem., but assume that $d \to (0, 0, 0.5)$. Simulation results investigating the empirical behavior of the low-frequency periodogram ordinates for these maps are also presented.