On Multiplicative Ergodic Theorems for Diffusion Processes

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Abstract:

Let $X(t)$ be the diffusion process in R^d defined by

$$dX(t) = b(X(t))dt + \sigma(X(t))dB(t),$$

$B(t)$ is d-dim Brownian motion. Multiplicative (mean) ergodic theorems of $X(t)$ are results concerning the convergence of

$$\frac{1}{T} \log E_x[\exp(\int_0^T V(X(t))dt) f(X(T))]$$

as $T \to \infty$ for various V and f, or a stronger limiting result that states the convergence of

$$E_x[\exp(\int_0^T V(X(t))dt) f(X(T))] \exp(-\Lambda^* T)$$

as $T \to \infty$ for a suitable choice of Λ^*. This type of results relates to some limiting theorems of diffusion processes such as Central Limit Theorem, Large Deviation Principle. We shall show here the connection of this type of results with the generalized eigenvalue problem that the positive solutions of the following equation are considered,

$$L\psi + V\psi = \Lambda\psi,$$

where the operator L defined by

$$Lf = \frac{1}{2} a_{ij} D_{ij} f + b \cdot \nabla f$$

is the generator of the $X(t)$ and

$$a_{ij}(x) = \sum_k \sigma_{ik}(x)\sigma_{jk}(x).$$