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TIME-VARYING CORRELATION FOR NONCENTERED NONSTATIONARY TIME SERIES: SIMULTANEOUS INFERENCE AND VISUALIZATION

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Abstract: We consider simultaneous inference of the time-varying correlation, as a function of time, between two nonstationary time series, when their trend functions are unknown. Unlike the stationary setting, where the effect of precentering using the sample mean is trivially negligible, in the nonstationary setting, it is difficult to quantify the effect of precentering using nonparametric trend function estimators. This is mainly because the trend estimators are time-varying across different time points, which makes it difficult to quantify their cumulative interaction with the error process in a time series setting. We propose using a centering scheme that, instead of aligning with the time point at which the data are observed, aligns with the time point at which the local correlation estimation is performed. We show that the proposed centering scheme leads to simultaneous confidence bands with a solid theoretical guarantee for the time-varying correlation between two nonstationary time series when their trend functions are unknown. Lastly, we demonstrate the proposed method using numerical examples, including a real-data analysis.

Key words and phrases: Kernel smoothing, local linear estimation, noncentered data, simultaneous confidence band.

1. Introduction

The correlation coefficient is a popular metric for quantifying the dependence between two variables. In a time series setting, we can use the correlation between two observed time series to understand their relationship or co-movement over time, or the correlation between the time series and its lagged version to study the underlying dependence structure. The latter is often referred to as the autocorrelation; see Wu and Xiao (2012). In addition, we can use the correlation between a time series and the lagged version of another time series to understand the lagged effect of one on the other, referred to as the Granger causality in time series analysis. The problem of estimating the correlation and autocorrelation has been studied extensively for stationary time series; see, for example, Anderson (1971), Hannan (1976), Hall and Heyde (1980), Priestley (1981), Brockwell and Davis (1991), Phillips and Solo (1992), Hosking (1996), Wu and Min (2005), Wu (2009), Wu and Xiao (2012), and the references therein. In the aforementioned

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