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PARTICLE-BASED, RAPID INCREMENTAL SMOOTHER MEETS PARTICLE GIBBS

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Abstract: The particle-based rapid incremental smoother (`PARIS`) is a sequential Monte Carlo technique that allows for efficient online approximations of expectations of additive functionals under Feynman–Kac path distributions. Under weak assumptions, the algorithm has linear computational complexity and limited memory requirements. It also comes with a number of nonasymptotic bounds and convergence results. However, being based on self-normalized importance sampling, the `PARIS` estimator is biased. This bias is inversely proportional to the number of particles, but has been found to grow linearly with the time horizon, under appropriate mixing conditions. In this work, we propose the Parisian particle Gibbs (`PPG`) sampler, which has essentially the same complexity as that of the `PARIS`, but significantly reduces the bias for a given computational complexity at the cost of a modest increase in the variance. This method is a wrapper, in the sense that it uses the `PARIS` algorithm in the inner loop of the particle Gibbs algorithm to form a bias-reduced version of the targeted quantities. We substantiate the `PPG` algorithm with theoretical results, including new bounds on the bias and variance, as well as deviation inequalities. We illustrate our theoretical results using numerical experiments that support our claims.

Key words and phrases: Bias reduction, particle filters, particle Gibbs, sequential Monte Carlo, smoothing of additive functionals, state space smoothing.

1. Introduction

Feynman–Kac formulae play a key role in many models used in statistics, physics, and many other fields; see Del Moral (2004), Del Moral (2013) and Chopin and Papaspiliopoulos (2020), and the references therein. Let $\{(\mathsf{X}_n, \mathcal{X}_n)\}_{n\in\mathbb{N}}$ be a sequence of measurable spaces and define, for every $n \in \mathbb{N}$, $\mathsf{X}_{0:n} := \prod_{m=0}^{n} \mathsf{X}_m$ and $\mathcal{X}_{0:n} := \bigotimes_{m=0}^{n} \mathcal{X}_m$. For a sequence $\{M_n\}_{n\in\mathbb{N}}$ of Markov kernels $M_n : \mathsf{X}_n \times \mathcal{X}_{n+1} \to [0,1]$, an initial distribution $\eta_0 \in \mathsf{M}_1(\mathcal{X}_0)$, and a sequence $\{g_n\}_{n\in\mathbb{N}}$ of bounded measurable potential functions $g_n : \mathsf{X}_n \to \mathbb{R}_+$, a sequence $\{\eta_{0:n}\}_{n\in\mathbb{N}}$ of *Feynman–Kac path measures* is defined by

$$\eta_{0:n} : \mathcal{X}_{0:n} \ni A \mapsto \frac{\gamma_{0:n}(A)}{\gamma_{0:n}(\mathsf{X}_{0:n})}, \quad n \in \mathbb{N}, \tag{1.1}$$

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