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DE-BIASING PARTICLE FILTERING FOR A CONTINUOUS TIME HIDDEN MARKOV MODEL WITH A COX PROCESS OBSERVATION MODEL

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Abstract: We develop a (nearly) unbiased particle filtering algorithm for a specific class of continuous-time state-space models in which (a) the latent process X_t is a linear Gaussian diffusion, and (b) the observations arise from a Poisson process with intensity $\lambda(X_t)$. The likelihood and the posterior probability density function of the latent process include an intractable path integral. Our algorithm relies on using Poisson estimates to approximate this integral in an unbiased manner. We show how to tune these Poisson estimates to ensure that, with large probability, all but a few of the estimates generated by the algorithm are positive. Then setting the negative estimates to zero leads to a much smaller bias than that obtained using discretization. We quantify the probability of negative estimates for certain special cases, and show that our particle filter is effectively unbiased. We apply our method to a challenging 3D single molecule tracking example using a Born–Wolf observation model.

Key words and phrases: Continuous-time, Cox process, diffusions, hidden Markov model, particle filter, path integral, Poisson estimate, sequential Monte Carlo.

1. Introduction

1.1. Background

Diffusion processes have been used extensively to model continuous-time phenomena in a range of scientific areas, including finance (Merton (1975)), biochemistry (McAdams and Arkin (1997); Gillespie (1977)), and physics (Obukhov (1959)). These processes are usually applied to model both an observed process and an unobserved signal/state process in a hierarchical model.

This study develops novel methods for the optimal filtering of multivariate diffusion processes observed at irregular time instances, which follow a Cox process with intensity that is a (nonnegative) function of the state process. The complete data likelihood of such a model includes a path integral of the state trajectory (in the intensity function), which is intractable. This precludes using standard particle filters. Another common problem in continuous-time filtering

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