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A MORE EFFICIENT ISOMORPHISM CHECK FOR TWO-LEVEL NONREGULAR DESIGNS

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Abstract: In this paper, we propose some new necessary and sufficient conditions for identifying isomorphism in two-level fractional factorial designs, using a parallel flats structure. A new algorithm for checking isomorphism is provided accordingly. The proposed algorithm is simple and general, and can be used for either regular or nonregular designs. By taking advantage of the parallel flats structure when it exists, the method is much faster than current methods for assessing the isomorphism of nonregular two-level designs. Examples are given to illustrate the results. An efficient implementation of the proposed algorithm in Matlab can be found in the online Supplementary Material.

Key words and phrases: Algorithm, equivalent group, parallel flats, two-level fractional factorial.

1. Introduction

In this study, we restrict our attention to two-level fractional factorial designs, which are extremely popular screening designs. Two fractional factorial designs are called isomorphic if and only if one design can be obtained from the other by row permutations, column permutations, and level permutations within columns. Two isomorphic designs share the same statistical properties in some classical ANOVA models, and thus are considered essentially the same. Thus, determining isomorphism is important both in theory and in practice.

Given the simple algebraic structure of regular designs, the earliest studies on isomorphism checks focused on such designs. Draper and Mitchell (1967, 1968) proved that two isomorphic regular designs must have the same word length pattern. Draper and Mitchell (1970) further showed that two isomorphic regular designs must have the same letter pattern. The letter pattern counts the frequency of letters in words of different lengths. Agreement in letter pattern implies having the same word length pattern. Note that designs with the same letter pattern are not necessarily isomorphic; see Chen and Lin (1991), who disprove a conjecture of Draper and Mitchell (1970). Chen, Sun and Wu (1993) first proposed necessary and sufficient conditions by "applying some algebraic and combinatorial methods" to identify isomorphic regular designs. By matching the factors using their delete-one-factor projections, Xu (2009)

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