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ROBUST OPTIMIZATION AND INFERENCE ON MANIFOLDS

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Abstract: We propose a robust and scalable procedure for general optimization and inference problems on manifolds, leveraging the classic idea of "median-of-means estimation". This is motivated by ubiquitous examples and applications in modern data science in which a statistical learning problem can be cast as an optimization problem over manifolds. Being able to incorporate the underlying geometry for inference, while addressing the need for robustness and scalability, presents great challenges. We address these challenges by first proving a key lemma that characterizes some crucial properties of geometric medians on manifolds. In turn, this allows us to prove the robustness and tighter concentration of our proposed final estimator in a subsequent theorem. This estimator aggregates a collection of subset estimators by taking their geometric median over the manifold. We illustrate bounds on this estimator using examples. The robustness and scalability of the procedure is shown in numerical examples on simulated and real data sets.

Key words and phrases: Geometric median on manifolds, median-of-means, optimization on manifolds, robust inference, robust principal geodesic analysis (RPGA), scalability.

1. Introduction

There is a rapidly growing collection of learning problems and applications in data science that can be formalized as optimization problems over non-Euclidean spaces, such as nonlinear Riemannian manifolds. Advancements in technology and computing have led to an increasing prevalence of complex data in non-Euclidean forms, such as positive-definite matrices (diffusion matrices) in diffusion tensor imaging (Alexander et al. (2007)), shape objects in medical vision (Kendall (1984)), network data objects (Kolaczyk et al. (2020)) and subspaces or orthonormal frames (Lin, Rao and Dunson (2017)). A proper statistical inference from such data involves optimizing over the underlying manifold to which the data are constrained. For example, there is a vibrant line of research on estimating Fréchet means (Fréchet (1948)), which are minimizers of Fréchet functions on manifolds (Bhattacharya and Bhattacharya (2012); Bhattacharya and Lin (2017)). In this case, both the data and the parameters of interest

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