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TESTING FOR THRESHOLD REGULATION IN PRESENCE OF MEASUREMENT ERROR

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Abstract: Regulation is an important feature of dynamic phenomena, and is commonly tested within the threshold autoregressive setting, with the null hypothesis being a global nonstationary process. Nonetheless, this setting is debatable, because data are often corrupted by measurement errors. Thus, it is more appropriate to consider a threshold autoregressive moving-average model as the general hypothesis. We implement this new setting with the integrated moving-average model of order one as the null hypothesis. We derive a Lagrange multiplier test that has an asymptotically similar null distribution, and provide the first rigorous proof of tightness in the context of testing for threshold nonlinearity against difference stationarity, which is of independent interest. Simulation studies show that the proposed approach enjoys less bias and higher power in detecting threshold regulation than existing tests, especially when there are measurement errors. We apply the new approach to time series of real exchange rates of a panel of European countries.

Key words and phrases: Lagrange multiplier test, threshold autoregressive moving-average model, purchasing power parity.

1. Introduction

Regulation plays a fundamental role in fields such as economics, finance, biological growth, and population fluctuations, among others. Growth processes are generally regulation-free until they enter extreme phases. For instance, real exchange rates should be regulated through a threshold that triggers the mean reversion toward zero. However, existing tests fail to reject the null hypothesis of a random walk, resulting in the so called purchasing power parity (PPP) puzzle; see, for example, Taylor and Taylor (2004).

The random walk is a simple model for regulation-free dynamics. On the other hand, regulation from above (below) can be captured using a first-order threshold autoregressive (TAR) model that follows a random walk until the process crosses a certain threshold, above (below) which, mean-reversion takes place, while the process as a whole is *stationary*. In general, a nonlinear

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