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MUTUAL INFLUENCE REGRESSION MODEL

Xinyan Fan, Wei Lan*, Tao Zou* and Chih-Ling Tsai

Renmin University of China,
Southwestern University of Finance and Economics,
The Australian National University and University of California, Davis

Abstract: In this article, we propose the mutual influence regression (MIR) model to establish the relationship between the mutual influence matrix of actors and a set of similarity matrices induced by their associated attributes. This model is able to explain the heterogeneous structure of the mutual influence matrix by extending the commonly used spatial autoregressive model, while allowing it to change with time. To facilitate inferences using the MIR, we establish parameter estimation, weight matrices selection, and model testing. Specifically, we employ the quasi-maximum likelihood estimation method to estimate the unknown regression coefficients. Then, we demonstrate that the resulting estimator is asymptotically normal, without imposing the normality assumption and while allowing the number of similarity matrices to diverge. In addition, we introduce an extended BIC-type criterion for selecting relevant matrices from the divergent number of similarity matrices. To assess the adequacy of the proposed model, we propose an influence matrix test, and develop a novel approach to obtain the limiting distribution of the test. The results of our simulation studies support our theoretical findings, and a real example is presented to illustrate the usefulness of the proposed MIR model.

Key words and phrases: Extended Bayesian information criterion, mutual influence matrix, similarity matrices, spatial autoregressive model.

1. Introduction

The possibility of relationships between subjects (such as network connections or spatial interactions) means that the traditional data assumption of independent and identically distributed (i.i.d.) observations is no longer valid, and there can be a complex structure of mutual influence between subjects. Accordingly, understanding such mutual influence has become an important topic in fields such as business, biology, economics, medicine, sociology, political science, psychology, engineering, and science. For example, studying the mutual influence between actors can help to identify influential users within a network (see Trusov, Bodapati and Bucklin (2010)). In addition, investigating the mutual influence between geographic regions is essential for exploring spillover effects in spatial data (see Golgher and Voss (2016); Zhang and Yu (2018)). For example, this type of analysis is important to our understanding of how COVID-19 spreads

*Corresponding author.