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FIXED-DOMAIN ASYMPTOTICS UNDER VECCHIA'S APPROXIMATION OF SPATIAL PROCESS LIKELIHOODS

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Abstract: Statistical modeling for massive spatial data sets has generated a substantial body of literature on scalable spatial processes based on Vecchia's approximation. Vecchia's approximation for Gaussian process models enables fast evaluation of the likelihood by restricting dependencies at a location to its neighbors. We establish inferential properties of microergodic spatial covariance parameters within the paradigm of fixed-domain asymptotics when the parameters are estimated using Vecchia's approximation. We explore the conditions required to formally establish these properties, theoretically and empirically. Our results further corroborate the effectiveness of Vecchia's approximation from the standpoint of fixed-domain asymptotics.

Key words and phrases: Fixed-domain asymptotics, Gaussian processes, Matérn covariance function, microergodic parameters, spatial statistics.

1. Introduction

Geostatististical data are often modeled by treating observations as partial realizations of a spatial random field. We customarily model the random field $\{Y(s) : s \in \mathcal{D}\}$ over a bounded region $\mathcal{D} \in \mathbb{R}^d$ as a Gaussian process (GP), denoted as $Y(s) \sim GP(\mu_\beta(s), K_\theta(\cdot,\cdot))$, with mean $\mu_\beta(s)$ and covariance function $K_\theta(s, s') = \text{cov}(y(s_i), y(s_j))$. The probability law for a finite set $\chi = \{s_1, s_2, \ldots, s_n\}$ is given by $y \sim N(\mu_\beta, K_\theta)$, where $y = (y(s_i))$ and $\mu_\beta = (\mu_\beta(s_i))$ are $n \times 1$ vectors with elements $y(s_i)$ and $\mu_\beta(s_i)$, respectively, and $K_\theta = (K_\theta(s_i, s_j))$ is an $n \times n$ spatial covariance matrix in which the (i, j)th element is the value of the covariance function $K_\theta(s_i, s_j)$. We consider the widely employed stationary Matérn covariance function (Matérn (1986); Stein (1999b)) given by

$$K_\theta(s, s') := \frac{\sigma^2(\phi\|h\|)^\nu}{\Gamma(\nu)2^{\nu-1}}\mathcal{K}_\nu(\phi\|h\|), \quad \|h\| \geq 0\ , \tag{1.1}$$

where $h = s - s'$, $\sigma^2 > 0$ is called the *partial sill* or spatial variance, $\phi > 0$ is the scale or decay parameter, $\nu > 0$ is a smoothness parameter, $\Gamma(\cdot)$ is the gamma function, $\mathcal{K}_\nu(\cdot)$ is the modified Bessel function of order ν Abramowitz and Stegun (1965, Sec. 10), and $\theta = \{\sigma^2, \phi, \nu\}$. The spectral density corresponding to (1.1),

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